Using learners' responses to think about teaching mathematics

Resource materials based on the Annual National Assessments

Grade 6

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

JET EDUCATION SERVICES

unicef

Foreword from the Minister of Basic Education

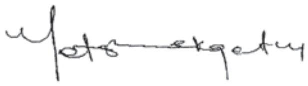
It is my pleasure to present this set of materials to be part of the range of resources made available for our teachers to improve the quality of their teaching. Since 2011, the Department of Basic Education (DBE) has been conducting the Annual National Assessment (ANA) on Grades 1-6 and 9 learners in Language and Mathematics.

The diagnostic reports produced after the administration of the ANA point to areas where individual teachers need specific support in terms of effective methods of facilitating learning. One of these areas is the utilisation of assessment data in a manner which will inform improved teaching.

This set of materials has therefore been developed to build the capacity of teachers to analyse the ANA and other tests in order to identify typical errors made by learners and thereafter select appropriate teaching strategies to correct these errors and improve the teaching of Mathematics. The materials give precise and useful guidelines, with regards to accurate identification of the challenging content and conceptual areas shown through the errors as well as using these errors to create opportunities for learners to improve their mathematical abilities.

I wish to express my sincere gratitude to our partners, the United Nations International Children's Emergency Fund (UNICEF) and JET Education Services for their invaluable contribution in making this resource available.

I am confident that teachers will find the materials useful and that this intervention will make a meaningful contribution to their teaching and professional development.



MRS ANGIE MOTSHEKGA, MP
MINISTER OF BASIC EDUCATION

General sections

Topics

Introduction

All South African learners in Grades 1 to 6 and 9 annually write the Annual National Assessments, which have become known colloquially as 'the ANAs'. The Department of Basic Education's diagnostic reports on the ANA results identified a need to strengthen the formative use of assessment data to support teaching. This resource book has been created to meet this need.

The material presented here focuses on mathematics topics covered in both the 2013 and 2014 ANAs and deals with the errors made by learners as shown in their responses to the ANA questions. In addition to highlighting learners' misconceptions, analysis of the learners' responses also reveals the correct understanding of the topics tested.

Following the error analysis of responses in the ANAs, teaching strategies to address these mistakes and misconceptions are presented. Both the error analysis and the teaching strategies in this resource book can be applied to other assessment questions that test the same topic, or used in the normal course of teaching the topic.

The materials presented here are, however, not meant to be exhaustive in terms of content covered in the mathematics curriculum, but are rather a living and growing resource that can be added to in response to future ANAs. Teachers are encouraged to add their own examples of learners' responses to assessment questions as well as to the teaching strategies which address the misconceptions noted in the learners' responses.

The following section of this book explains the concept of error analysis and how to conduct an analysis of learner's responses in an assessment or test. Next, guidelines on how to use these materials in conjunction with error analysis are presented. Finally, actual error analysis of learners' responses in the ANAs and ideas for teaching strategies to address the identified errors and misconceptions are presented per topic.

What is error analysis?

The formative use of assessment data to support teaching can be achieved by what researchers call "error analysis".

Error analysis, also referred to as error pattern analysis, is a multifaceted activity involving the study of errors in learners' work with a view to finding explanations for learners' reasoning errors.

It is important to note that not all errors are reasoning faults; some are simply careless errors which educational researchers have termed "slips". Slips can easily be corrected if the faulty process is pointed out to the learner. Slips are random errors that learners may make (e.g. reversing a number) and do not indicate systematic misconceptions or conceptual problems.

Error analysis is concerned with identifying and addressing the common errors (or 'bugs') which learners make due to their lack of conceptual or procedural understanding. These types of mathematical errors occur when the learner believes that what has been done is correct – showing that the learner's reasoning is faulty. Researchers have termed these errors systematic and persistent errors. Unless the misconceptions that cause the learners to make these errors are corrected, learners will repeat them over and over again and not be aware that they are using incorrect procedures to solve problems. These systematic errors can be said to be the result of the use of algorithms that lead to incorrect answers or the use of procedures that have not been fully understood.

Error analysis is important as it does not just mean analysing learners' steps in finding the solutions to a problem, but also involves finding the best ways to remediate the misconceptions the analysis shows. Rectifying learners' misconceptions enables teachers to make sure that learners can move on to the next step in the curriculum with a sound base.

Many writers on education have pointed out that the ability of teachers to understand and remediate common learner errors and misconceptions is an important part of what teachers should know, that is, of the pedagogical content knowledge that teachers should have. In order to conduct error analysis efficiently, teachers need to have a good knowledge of mathematical content, a good grasp of their learners' levels of mathematical understanding and a well-grounded understanding of the learner and how a learner learns.

Shulman (1986)[1] was a writer who developed a theory of teacher knowledge. He

[1] Shulman, L.S. 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4–14.

included teachers' knowledge of learners' levels of understanding as an important part of teacher knowledge and explained that this knowledge helps teachers to become aware of the process of learning mathematics as well as to understand the mathematical concepts that learners struggle to grasp. Other authors describe how error analysis and efforts to understand learners' levels of mathematical understanding build teacher's own knowledge of the underlying cognitive processes involved.

In conclusion, error analysis is a valuable activity that helps teachers to understand some of the thinking of their learners. Understanding the way learners think can assist teachers to adjust their teaching strategies and classroom and assessment practices and may ultimately lead to improvement in learner achievement.

How to analyse your learners' responses to assessment questions?

This section contains a fictional ten-question test which is used to illustrate how to design and use an assessment grid to analyse your class's results in a test or assessment. This is done in a step-by-step fashion.

The assessment grid, as the example given shows, reflects:

- The content addressed in each question;
- The question number;
- The marks allocated;
- The total of correct answers for each learner;
- The percentage each learner achieved;
- The learners' names; and
- The average score per question.

Step 1: Drawing up an assessment grid

- The first step in drawing up the grid is to identify the content addressed in each question in the assessment. The descriptions of this content make up the first heading row of the grid. It helps to be as specific as possible when identifying the content or skill being tested, as in the descriptions in blue in the example.
- You also need to record the marks (see red) allocated for each question and the total possible marks for the test (see green).

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	
Learners' name												
Average per question												

Step 2: Recording the results of each learner

In the next step you have to record the results per learner per question.

- First list all the learners in your class. We have 5 learners in our fictional class (see red).
- We have recorded the mark each learner got for each question (see blue).
- We also recorded the total marks each learner got (see green) and each learner's percentage (see pink).
- You might want to an additional column for the level code.

- Looking at the grid you will start to notice individual differences, e.g. learners that got the same total mark (see purple) did not get the same questions right or wrong.
- Also, you can identify which learners are doing well and which need additional support, e.g. Rosi and Lizzy are doing well, but Themba needs additional support.

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	Total correct	Percentage
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question												

Step 3: Working out the class average

The third step is to work out and record the class average for each question.

- To calculate your class average, add up all the learners' averages and then divide by the number of learners in the class, e.g, Class average = 67 + 33 + 50 + 50 + 83 (see pink) ÷ 5 learners in class x 100 = 57%

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question												60

Step 4: Working out the class average per question

In this step you want to determine the average your class got for each question. This will allow you to see the areas your learners are doing well in and the areas in which they need additional support.

- First work out the maximum possible marks the class could get for each question by multiplying the marks allocated to the question by the number of learners in the class, e.g., question 1 counts out of 3 marks, so the maximum possible total for the class would be 3 marks x 5 learners = 15 marks.
- Now add up all the learners' marks for question 1 (see red) to get the total marks achieved by the class for question 1, i.e. 2 + 1 + 3 + 1 + 3 = 10.
- To determine the class average for question 1 (see blue) divide the total marks achieved by the maximum possible marks and multiply the answer by 100, i.e. 10 ÷ 15 x 100 = 67% (see blue).

Content addressed in question	Simplification of expressions involving division	Equation involving exponents	Factorisation of expression	n^{th} term of a sequence	Simple interest	Compound interest	Congruent triangles	Drawing the reflected image	Surface area and volume of triangular prism	Determining gradient of line	**Total correct**	**Percentage**
Question number	1	2	3	4	5	6	7	8	9	10		
Mark	3	2	2	4	5	4	1	2	6	1	30	%
Learners' name												
Rosi Thladi	2	1	2	2	3	3	1	1	4	1	20	67
Themba Hlambelo	1	1	1	0	2	2	1	0	2	0	10	33
Luneta Petersen	3	2	2	1	5	2	0	0	0	0	15	50
Johan De Wit	1	2	1	1	2	2	1	1	3	1	15	50
Lizzy Gregory	3	2	2	2	4	3	1	2	4	1	24	80
Average per question	67	80	80	30	64	60	80	40	43	60		60

- Once you have done the same for all the questions, note which questions your class did well in (e.g. questions 2, 3 and 7) and which they did not do so well in (i.e. questions 8 and 9).
- Try to identify why this might be the case.

- Have you, for example, not spent enough time on these content areas?
- Do learners have a specific misunderstanding or misconception of this content area?
- Or are learners maybe making careless mistakes?

Step 5: Determine what learners should know to answer a specific question correctly

Let's consider what learners need to know to, for example, calculate the surface area of a triangular prism (question 9 in our fictional test). Learners should be able to:

- Deconstruct the prism into its net;
- Calculate the area of 2-D shapes, for example, triangles and rectangles;
- Write down the formulae for calculating the surface area of prisms;
- Substitute correctly within the formulae for calculating the surface area of prisms and cylinders.

You will note some of the content a learner needs to know in order to solve the problem correctly might have been covered in previous grades.

Step 6: Identify the typical errors learners make

Look at a selection of learners' responses and identify the typical errors they make. For example for question 9, you might note that learners:

- Neglected to calculate all the surface areas of the prism;
- Calculated the surface area of a specific face incorrectly; or
- Calculated all the surface areas of the prism correctly, but then added them incorrectly.

- It is useful here to have a discussion with specific learners or with the class to identify the reasoning learners used to solve the problem. By asking a learner to talk you through the process he/she followed to solve the problem, you could identify where the learner's logic went wrong or if the learner just made a careless calculation error.

Step 7: Determine appropriate teaching strategies to address learner errors and misconceptions

Once you have determined the reasoning or calculation errors learners made in a particular topic, you need to take steps to address these misconceptions or errors.

How to use these materials

The main section of this resource is organised according to topic in the following way:

- The topic and question used in the ANA to test it;
- The knowledge and skills required to answer the question and where the topic is located in the CAPS;
- Examples of learners' responses showing full, partial or no understanding of the question;
- Statistics showing the percentage of learners countrywide that answered the question correctly;
- Reasons the learners may have found it difficult to answer the question correctly;
- Teaching strategies to rectify the misconceptions that caused the errors; and (in some instances)
- Additional examples of how to test the topic.

To use the material

1. First determine which topic your class is experiencing difficulties with, then refer to the topic in the resource book.

- Take note of what learners need to know to answer the question correctly and where the topic is found in the CAPS.
- You should reflect critically on your teaching practice and the learners' knowledge with regard to the content and skills needed to answer the question. For instance, ask yourself:
  - Has sufficient time been spent on the required content and skills?
  - Have learners had sufficient time and practice to master the requisite skills?
- You might need to revise the content and skills needed for a particular topic before attempting to remediate the learners' performance.
- The content or skills might have been taught in a previous grade, but it is essential to make sure the learners have a solid foundation on which to build conceptual understanding of new topics.

2. Secondly, establish what kinds of slips and errors your learners make by either:

- Looking at their answers to the ANA question (remember, you could give the learners the question in class to solve as part of class work); or
- Looking at their answers to similar questions found in textbooks or class exercises.
- You can refer to the examples of typical learner responses to assist you.

3. Take note of the statistics that follow the examples of learner responses.

- The statistics will give you an indication of how difficult the questions should be for your class.

- If your class is doing better than the national sample, then it means you have started to lay a solid foundation and should continue your systematic teaching of the topic.

4. Lastly, look at and use the recommended teaching strategies to remedy the learners' errors and misconceptions as shown by the error analysis.
   - The teaching strategies are linked to the particular errors and misconceptions shown in this book – you may identify others made by your own learners.
   - The strategies are not meant as an exhaustive list of teaching strategies or exercises to do with learners on a specific topic, but as a starting point for remediation.
   - You will need to supplement the teaching strategies with your own ideas.

Note: Remember to teach **all the topics specified in the CAPS**, not just the ones featured in this resource.

Happy teaching!

Extra reading on error analysis

If you would like to know more about the theory of error analysis and how it can be used to aid you in your teaching, the following articles and books are suggested reading.

Allsopp, D.H., Kuger, M.H. & Lovitt, L.H. (2007). *Teaching mathematics meaningfully: Solutions for reaching struggling learners*. Baltimore: Paul H. Brooks.

Ashlock, R.B. (2006). *Error patterns in computation: Using error patterns to improve instruction. 9th edition*. New Jersey: Pearson.

Franke, M.L. & Kazemi, E. (2001). *Learning to teach mathematics: Focus on student thinking. Theory into Practice*, 40(2):102–109.

Herholdt, R & Sapire, I. (2014). An error analysis in early grades mathematics – A learning opportunity? *South African Journal of Childhood Education*, 4(1), 42-60.

Hill, H.C., Ball, D.L. & Schilling, S.C. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal of Research in Mathematics Education*, 39(4):372–400.

Ketterlin-Geller, L.R. & Yovanoff, P. (2009). Diagnostic assessments in mathematics to support instructional decision making. *Practical Assessment, Research &*

Evaluation, 14(16). Retrieved from http://pareonline.net/getvn.asp?v=14&n=16 (accessed on 19 April 2014).

McGuire, P. (2013). Using online error analysis items to support pre-service teachers' pedagogical content knowledge in mathematics. *Contemporary Issues in Technology and Teacher Education*, 13(3). Retrieved from http://www.citejournal.org/vol13/iss3/mathematics/article1.cfm (accessed on 19 April 2014).

Nesher, P. (1987). Towards an instructional theory: The role of students' misconceptions. *For the Learning of Mathematics*, 7(3):33–39.

Olivier, A. (1996). Handling pupils' misconceptions. *Pythagoras*, 21:10–19.

Radatz, H. 1979. Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3):163–172.

Riccomini, P.J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, 28(3):233–242.

Russell, M. & Masters, J. (2009). *Formative diagnostic assessment in algebra and geometry*. Paper presented at the annual meeting of the American Education Research Association. San Diego, California.

Sapire, I., Shalem, Y., & Reed, Y. (2013). *Assessment for Learning. Johannesburg: University of Witwatersrand and Saide.*

Shalem, Y., Sapire, I., & Sorto, M.A. (2014). Teachers' explanations of learners' errors in standardised mathematics assessments. *Pythagoras,* 35(1), Art. #254, 11 pages. http://dx.doi.org/10.4102/pythagoras.v35i1.254

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4–14.

Sousa, D.A. (2008). *How the brain learns mathematics*. California: Corwin Press.

Yang, C.W., Sherman, H. & Murdick, N. (2011). Error pattern analysis of elementary school-aged students with limited English proficiency. *Investigations in Mathematics Learning*, 4(1):50–67.

Topics

Number concept

ANA 2013 Grade 6 Items 2 and 4

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand, seven hundred and ninety-eight. [1]

4. What is the value of the underlined digit in 45 678 921? [1]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Work with place values up to hundreds of millions;
- Write number names up to hundreds of millions;
- Recognise the values of the digits in a nine-digit number.

Where is this topic located in the curriculum? Grade 6 Term 1 and Term 2

Content area: Numbers, Operations and Relationships.

Topic: Number concepts and number names.

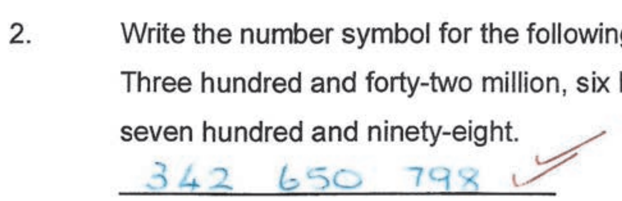
Concepts and skills:

- Order, compare and represent numbers to at least 9-digit numbers;
- Whole numbers: counting, ordering, comparing and representing digits, recognising place values of up to 9 digits.

What would show evidence of full understanding?

Item 2

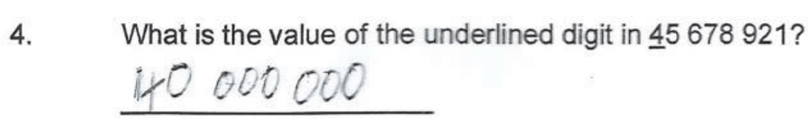
- If the learner gave the correct number symbol for the number given in words.



2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand, seven hundred and ninety-eight.
342 650 798

Item 4

- If the learner identified the value as 40 million, as in the answers shown.



4. What is the value of the underlined digit in 45 678 921?
40 000 000

4. What is the value of the underlined digit in <u>4</u>5 678 921?

Forty million. ✓

What would show evidence of partial understanding?

Item 2

- In the following examples the learners seem to understand the values of the numbers, but made other errors:

- In the first example the learner missed the millions digit. Instead of 342 million, the learner wrote 340 million.
- In the second the learner exchanged the digits 0 and 7.
- In the third example the learner wrote a 5 instead of a 6 in the hundred thousand place value.

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand,
seven hundred and ninety-eight.

340 650 798

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand,
seven hundred and ninety-eight.

342 657 098 ✗

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand,
seven hundred and ninety-eight.

342 550 798

Item 4

- If the learner gave a partly correct answer as shown in the next examples.

- In the first example the learner wrote two answers: TM which represents ten millions and the number 45 million. TM is correct but 45 million is not correct.

4. What is the value of the underlined digit in <u>4</u>5 678 921?

TM 45 Million ✗✗

- In the second example the learner just wrote the word millions instead of 40 million as required.

4. What is the value of the underlined digit in 45 678 921?

Millions

What would show evidence of no understanding?

Item 2

- If the learner demonstrated no knowledge of place values as in the examples that follow.

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand,
seven hundred and ninety-eight.

3004000060060070098

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand,
seven hundred and ninety-eight.

Eight hundred and nine

2. Write the number symbol for the following number:
Three hundred and forty-two million, six hundred and fifty thousand,
seven hundred and ninety-eight.

7000698

Item 4

- The following answers also demonstrate no knowledge of place values.

4. What is the value of the underlined digit in 45 678 921?

it is 65

4. What is the value of the underlined digit in 45 678 921?

Tens

4. What is the value of the underlined digit in 45 678 921?

400

What do the item statistics tell us?

Item 2

39% of learners answered the question correctly.

Item 4

55% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- The learners may not know how to convert words to symbols;
- The questions involved big numbers, e.g. millions;
- The questions involved numbers consisting of as many as 8 digits.

Teaching strategies

Revising the concept of place values by using place value worksheets

- In order to work efficiently with large numbers, learners need to have a good working knowledge of the place value system up to values of hundreds of millions.
- Place value is taught from the Foundation Phase. An understanding of place value is built up slowly over the years, from 2-digit numbers (in Grade 2) up to 9-digit numbers (in Grade 6).
- The number system that we work with uses place value in a base of ten. This means that successive places increase in multiples of ten. The first place is units or ones. The second place is tens, the third hundreds and so on.
- Place value refers to the value of a digit in a number according to its position in the number.
    - For example: In the number 13 450, the digit 4 is in the hundreds place.
- Number value is the actual value of the number, as determined by the digit and the place of the digit.
    - For example: In the number 13 450, the value of the '4' in the number is 400 because it is represented by a digit 4 in the hundreds place.
- You can revise the concept of place values by using place value worksheets where repetitive activities will give learners the opportunity to consolidate their knowledge of place value.
- When learners are familiar with placing the digits correctly in column form, the column method of addition will be much easier for them to understand and use.
    - The first worksheet that follows asks learners to identify the place values of digits in numbers from 2-digit numbers up to 8-digit numbers.
    - The second worksheet asks learners to line up numbers according to place value by using columns. There are sets of different combinations of numbers to give learners lots of practice in lining up numbers in columns according to place value (from 2-digit numbers up to 7-digit numbers).
    - The third worksheet allows learners the opportunity to write numbers in symbols.

Activities: Place values

Place value worksheet 1

For each of the numbers below write the value of the underlined digit.

	Number	Value of underlined digit
1).	87	____________________
2).	89	____________________
3).	98	____________________
4).	987	____________________
5).	798	____________________
6).	897	____________________
7).	3 482	____________________
8).	3 842	____________________
9).	3 284	____________________
10).	5 987	____________________
11).	8 957	____________________
12).	7 895	____________________
13).	43 489	____________________
14).	44 389	____________________
15).	44 839	____________________
16).	44 893	____________________
17).	34 489	____________________
18).	205 633	____________________
19).	246 917	____________________
20).	369 125	____________________
21).	975 312	____________________
22).	363 764	____________________
23).	766 321	____________________
24).	589 317	____________________
25).	1 234 567	____________________
26).	3 692 581	____________________
27).	8 794 561	____________________
28).	87 521 463	____________________
29).	98 653 214	____________________
30).	74 085 090	____________________

Solutions

	Number	Value of underlined digit
1).	87	7
2).	89	80
3).	98	8
4).	987	80
5).	798	8
6).	897	800
7).	3 482	400
8).	3 842	40
9).	3 284	4
10).	5 987	5 000
11).	8 957	50
12).	7 895	5
13).	43 489	400
14).	44 389	4 000
15).	44 839	40 000
16).	44 893	800
17).	34 489	30 000
18).	205 633	30
19).	246 917	40 000
20).	369 125	300 000
21).	975 312	10
22).	363 764	60 000
23).	766 321	6 000
24).	589 317	9 000
25).	1 234 567	1 000 000
26).	3 692 581	3 000 000
27).	8 794 561	8 000 000
28).	87 521 463	80 000 000
29).	98 653 214	8 000 000
30).	74 085 090	70 000 000

Place value worksheet 2: Numbers in columns

Line up the digits of the following numbers in columns according to place value.

The first one has been done for you.

a). 209; 33; 198 090

Answer:

HTH	TTH	TH	H	T	U
			2	0	9
				3	3
1	9	8	0	9	0

b). 10; 25 697; 3; 34 789; 564

c). 256; 2 547; 95; 87 546; 10 000 001

d). 202; 22 202; 2

e). 12 345; 1 234; 123; 12; 1

f). 389; 1 009; 2 378; 27

g). 698; 1; 55; 58 976; 9 875 689

h). 100; 10; 1 000; 1

i). 319; 2 568; 200 000

j). 3 112; 31 012; 3 210 123

Solutions

a). 209; 33; 198 090

HTH	TTH	TH	H	T	U
			2	0	9
				3	3
1	9	8	0	9	0

b). 10; 25 697; 3; 34 789; 564

TTH	TH	H	T	U
			1	0
2	5	6	9	7
				3
3	4	7	8	9
		5	6	4

c). 256; 2 547; 95; 87 546; 10 000 001

TM	M	HTH	TTH	TH	H	T	U
					2	5	6
				2	5	4	7
						9	5
			8	7	5	4	6
1	0	0	0	0	0	0	1

d). 202; 22 202; 2

TTH	TH	H	T	U
		2	0	2
2	2	2	0	2
				2

e). 12 345; 1 234; 123; 12; 1

TTH	TH	H	T	U
1	2	3	4	5
	1	2	3	4
		1	2	3
			1	2
				1

f). 389; 1 009; 2 378; 27

TH	H	T	U
	3	8	9
1	0	0	9
2	3	7	8
		2	7

g). 698; 1; 55; 58 976; 9 875 689

M	HTH	TTH	TH	H	T	U
				6	9	8
						1
					5	5
		5	8	9	7	6
9	8	7	5	6	8	9

h). 100; 10; 1 000; 1

TH	H	T	U
	1	0	0
		1	0
1	0	0	0
			1

i). 319; 2 568; 200 000

HTH	TTH	TH	H	T	U
			3	1	9
		2	5	6	8
2	0	0	0	0	0

j). 3 112; 31 012; 3 210 123

M	HTH	TTH	TH	H	T	U
			3	1	1	2
		3	1	0	1	2
3	2	1	0	1	2	3

Place value worksheet 3: Write the following numbers in symbols

The first one has been done for you.

a). Three thousand seven hundred and twenty-two

Answer:	Thousands:	3	[= 3 000]
	Hundreds:	7	[= 700]
	Tens:	2	[= 20]
	Units:	2	[= 2]

The number is the sum of all these place values:

3000 + 700 + 20 + 2 = **3 722**

b). Sixty-two

c). Nine thousand and nine

d). Nine thousand three hundred and sixty-five

e). Seventy-seven thousand five hundred

f). Four hundred and twenty thousand eight hundred and fifteen

g). Fifty-six million

h). Fifty-six million seven hundred and nine thousand

i). Fifty-six million seven hundred and nine thousand eight hundred and twenty

j). Four hundred and twenty thousand

Solutions

a). Three thousand seven hundred and twenty-two: [3 722]

b). Sixty-two: [62]

c). Nine thousand and nine: [9 009]

d). Nine thousand three hundred and sixty-five: [9 365]

e). Seventy-seven thousand five hundred: [77 500]

f). Four hundred and twenty thousand eight hundred and fifteen: [420 815]

g). Fifty-six million: [56 000 000]

h). Fifty-six million seven hundred and nine thousand: [56 709 000]

i). Fifty-six million seven hundred and nine thousand eight hundred and twenty: [56 709 820]

j). Four hundred and twenty thousand: [420 000]

Another example of how to test number concept

ANA 2014 Grade 6 Item 1.1

1 Circle the letter of the correct answer.

1.1 What is the value of the underlined digit in 249,15?

A 5
B 0,5
C 50
D 0,05 [1]

Notes:

Rounding

ANA 2013 Grade 6 Mathematics Item 1.6

1.6	Round 347 659 off to the nearest 100 000		
	A	300 000	
	B	348 000	
	C	350 000	
	D	400 000	[1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Round off numbers to the nearest hundred thousand;
- Work with place values up to 6-digits.

Where is this topic located in the curriculum? Grade 6 Term 1 and Term 2

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers.

Concepts and skills:

- Recognise the place value of digits in whole numbers to at least 9-digit numbers;
- Round off to the nearest 5, 10, 100 and 1 000.

What would show evidence of full understanding?

- If the learner answered A (300 000) this shows full understanding.

What would show evidence of partial understanding?

- If the learner answered B, C, or D this shows partial understanding as all the options involved some rounding off.
- If the learner answered B (348 000) or C (350 000) this shows the learner could round off a number, but not correctly to 100 000 according to the question.
- If the learner selected D (400 000) this shows the learner realised that the number had to be rounded off the nearest 100 000, but did it incorrectly.

What would show evidence of no understanding?

- It could be that learners who chose the incorrect options did not understand the question at all. Teachers would need to interview their learners to find out the real reasons for the incorrect choices in this instance.

What do the item statistics tell us?

35% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not understand the concept of rounding off to a given number.
- Learners may not understand place value.

Teaching strategies

Rounding in real life situations

Rounding off makes addition in real life situations easier. It is much easier to use rounded off numbers than decimals or numbers not rounded off.

Example

1). You want to buy three items at the grocery store. Their prices are R2,25, R0,88 and R2,69. By rounding off to the nearest rand, decide if R5 will be enough for you to buy these three items.

Solution

R2,25 ≈ R2,00
R0,88 ≈ R1,00
R2,69 ≈ R3,00
R6,00

Five rand will not be enough to pay for the items.

2). 45 482 tickets were sold for the soccer match. Three reporters said the following over the radio:

Reporter A: About 46 000 spectators attended the match.
Reporter B: About 45 000 spectators attended the match.
Reporter C: About 50 000 spectators attended the match.

Which of the reporters was the closest to the actual attendance?

Solution

Reporter A tried to round off to the nearest thousand, but did it incorrectly.
Reporter B rounded off to the nearest thousand and did so correctly.
Reporter C rounded off to the nearest ten thousand and did so correctly.

Therefore Reporter B was closest to the actual figure of attendance.

Understanding what it means to "round off"

- To round off a number we have to be able to see that number's position on the number line. Any number always lies between two other numbers. If we are asked to round a number off to the nearest 100, 1 000 and so on we must decide which two other numbers that number is closest to.
- We then "round off" the number by writing it as that "closest" number, rather than the number itself: for example, 157 can be rounded off to 160 when we round it off to the nearest 10.

Examples

1). If we are asked to round a number off to the nearest 5, we must locate its position between two multiples of 5.
  - A number which has been rounded off to the nearest 5 will always end in a 5 or a 0.
  - For example:
    122 783 rounded off to the nearest 5 is 122 785
    457 801 rounded off to the nearest 5 is 457 800

2). If we are asked to round a number off to the nearest 10 we must locate its position between two tens.
  - A number which has been rounded off to the nearest 10 will always end in a 0.
  - For example:
    122 783 rounded off to the nearest 10 is 122 780
    457 801 rounded off to the nearest 10 is 457 800

3). If we are asked to round a number off to the nearest 100 we must locate its position between two hundreds.
  - A number which has been rounded off to the nearest 100 will always end in a 00.
  - For example:
    122 783 rounded off to the nearest 100 is 122 800
    457 801 rounded off to the nearest 100 is 457 800

4). If we are asked to round a number off to the nearest 1 000 we must locate its position between two thousands.
   - A number which has been rounded off to the nearest 1 000 will always end in a 000.
   - For example:
     122 783 rounded off to the nearest 1 000 is 123 000
     457 801 rounded off to the nearest 1 000 is 458 000

5). If we are asked to round a number off to the nearest 10 000 we must locate its position between two ten thousands.
   - A number which has been rounded off to the nearest 10 000 will always end in a 0 000.
   - For example:
     122 783 rounded off to the nearest 10 000 is 120 000
     457 801 rounded off to the nearest 10 000 is 460 000

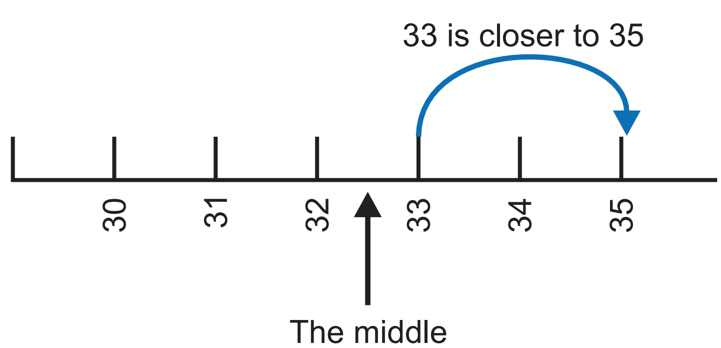
- This process may be continued to higher place values. Discuss general rounding off with your class using lots of examples to allow them to show you that they have understood your explanations.

Using the number line to round numbers off

It is useful to use the number line to locate a number so that we can see which two numbers it is the closest to.

Examples

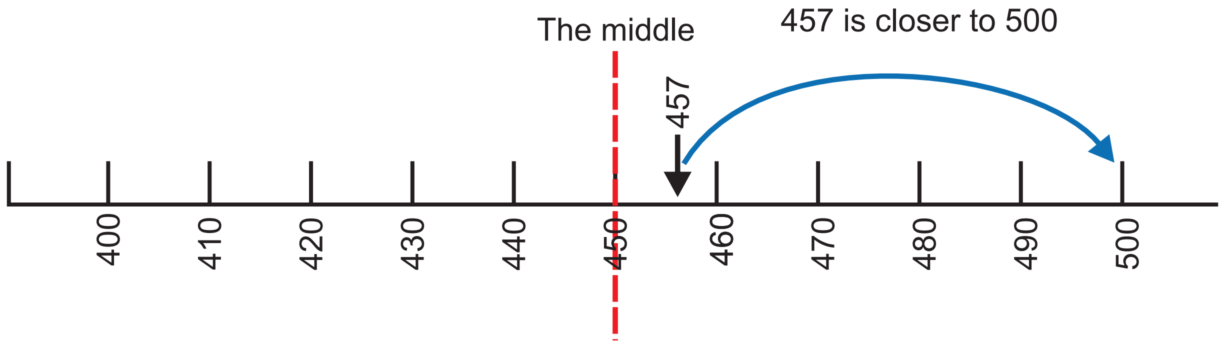
1). Round 33 off to the nearest 5
   - Ask: Between which two multiples of 5 does 33 lie? (33 lies between 30 and 35).
   - Draw a number line demarcated in units showing numbers between 30 and 35.
   - Locate 33 on the number line.



   - Can you see that 33 is closer to 35? We write: 33 = 35 (to the nearest 5).
   - If we do not add **"to the nearest ___"** we cannot write: 33 = 35.
   - If we have rounded off and we do not say "to the nearest" then we must use the approximate sign $\approx$ and write 33 $\approx$ 35.

2). Round 457 off to the nearest 100

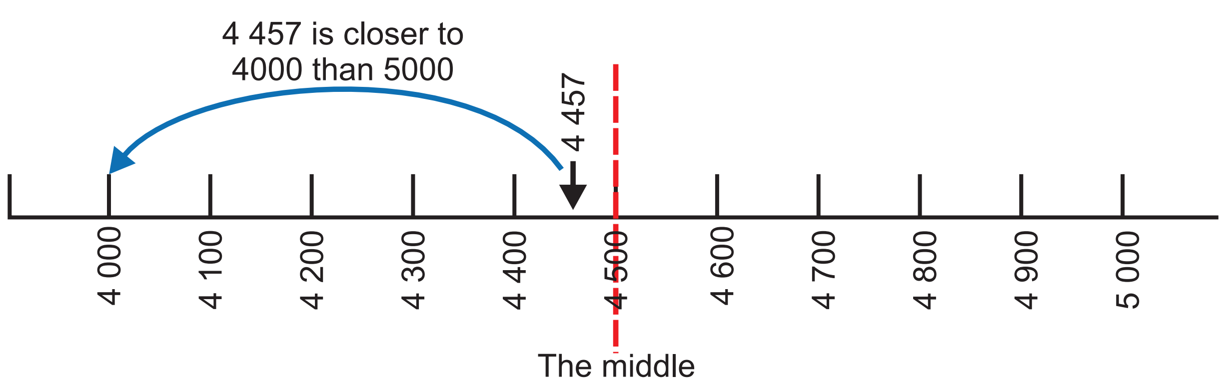
- First decide which two hundreds 457 lies between. Discuss this with your class to involve them in the decision.
- Ask: Between which two hundreds does 457 lie? (457 lies between 400 and 500).
- Draw a number line demarcated in 10s from 400 to 500.
- Locate 457 on the number line.



- 457 rounded off to the nearest 100 is 500.
- We write 457 ≈ 500.

3). Round 4 457 off to the nearest 1 000

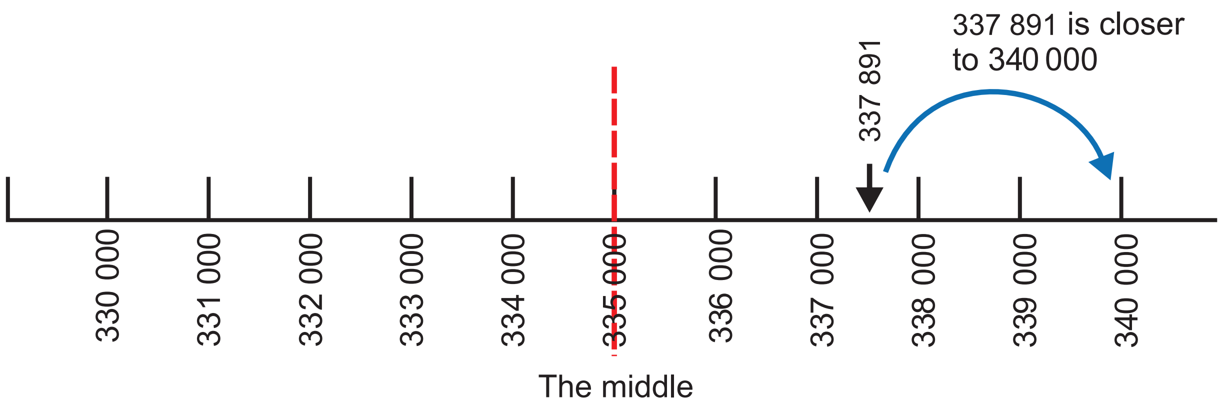
- First decide which two thousands 4 457 lies between.
- Ask: Between which two thousands does 4 457 lie? (4 457 lies between 4 000 and 5 000).
- Draw a number line demarcated in 100s from 4 000 to 5 000.
- Locate 4 457 on the number line.



- 457 rounded off to the nearest 1 000 is 4 000.
- We write 4 457 ≈ 4 000.

4). Round 337 891 off to the nearest 10 000

- First decide between which two ten thousands 337 891 lies.
- Ask: Between which two ten thousands is the number 337 891 ? (337 891 lies between 330 000 and 340 000).
- Draw a number line demarcated in 10 000s from 330 000 to 340 000.
- Locate 337 891 on the number line.



- 337 891 rounded off to the nearest 10 000 is 340 000.
- We write 337 891 ≈ 340 000.

- Discuss the above examples with your class.

Activity: Rounding off

Write the following question on the board. Give your learners time to answer on their own and then go over the solutions together.

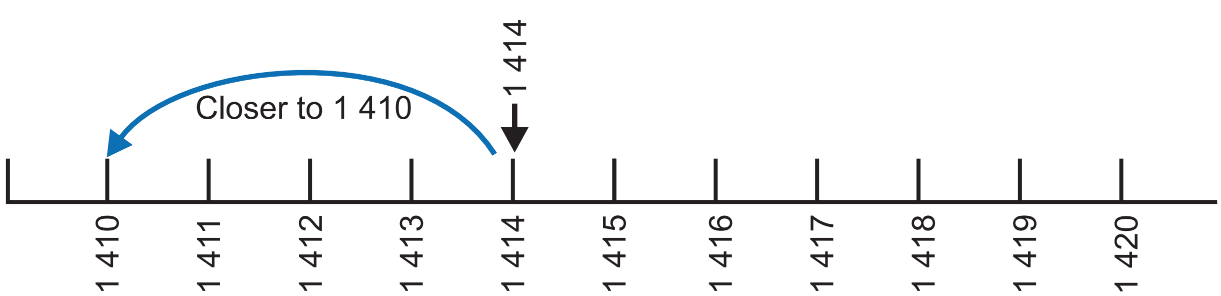
Use the number line to round off the following:

1). 1 414 to the nearest 10

2). 1 449 to the nearest 100

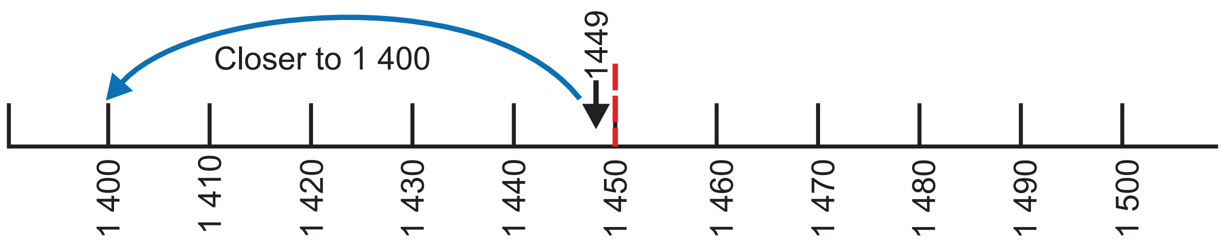
Solutions

1.



1 414 = 1 410 (rounded off to the nearest 10)

2).



1 449 = 1 400 (rounded off to the nearest 100)

Using the place value chart

The place value chart is a useful instrument that you can use to help learners to understand rounding off.

Examples

1). Round 347 659 off to the nearest 10 000

- Draw a blank place value chart up to hundred thousands on the board.

HT	TT	T	H	T	U

- Write the number on the chart so that the digits are in the correct places.

HT	TT	T	H	T	U
3	4	7	6	5	9

- Look at the ten thousand digit: it is 4.
- Now look at the digit to the right, the thousand digit: it is 7.
- If the thousand digit is 5 or more then the ten thousand digit **becomes one bigger**.
- All the other digits to the right of the ten thousand digit become zeroes.
- 347 659 rounded off to the nearest 10 000 will be 350 000.
- We write: 347 659 = 350 000 (rounded off to the nearest 10 000).

2). Round 347 659 off to the nearest 100 000

- Follow the same procedure as above.

HT	TT	T	H	T	U
3	4	7	6	5	9

- Look at the hundred thousand digit: it is 3
- Then look at the ten thousand digit: it is 4 and is smaller than 5. Now the hundred thousand digit stays the same.
- Drop all the other digits to the right of the hundred thousand digit (they become zeroes).
- 347 659 rounded off to the nearest 100 000 will be 300 000.
- We write: 347 659 = 300 000 (rounded off to the nearest 100 000).

- Now ask your learners to do the following to consolidate this learning.

Activity: Rounding off using the place value chart

Use a place value chart to round off the following:

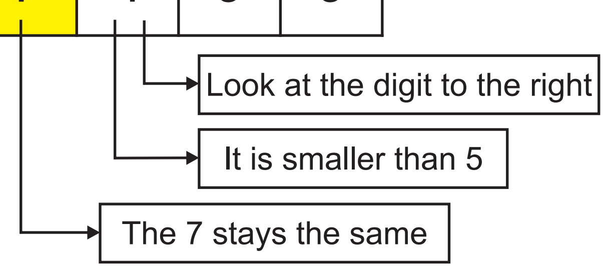
1). 77 499 to the nearest 1 000

2). 1 578 499 to the nearest 100 000

Solutions

1.

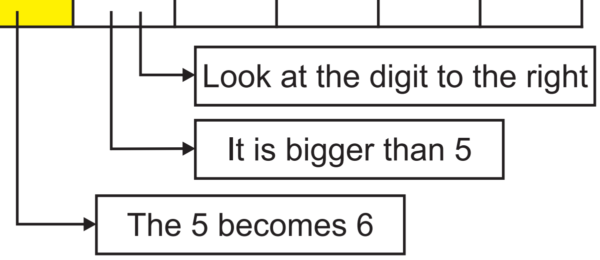
TT	T	H	T	U
7	7	4	9	9



77 499 = 77 000 (to the nearest 1 000)

2).

M	HT	TT	T	H	T	U
1	5	7	8	4	9	9



1 578 499 = 1 600 000 (to the nearest 100 000)

Some useful hints to remember

- If the number you are rounding is followed by 5, 6, 7, 8, or 9, round the number up.
- If the number you are rounding is followed by 0, 1, 2, 3, or 4, round the number down.
- All the numbers to the right of the place you are rounding to become zeroes.

Notes:

Addition and subtraction

ANA 2013 Grade 6 Mathematics Items 5.1, 5.2 and 8

5.1 Calculate the answer: 43 489 + 345 987 + 307	[2]
5.2. Calculate the answer: 495 089 – 85 847	[2]
8. Susan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?	[2]

What should a learner know to answer these questions correctly?

Learners should be able to:

Items 5.1 and 5.2

- Add and subtract numbers of up to 6 digits with and without borrowing across place values;
- Work with numbers with place values up to hundred thousands;
- Competently perform vertical (and horizontal) subtraction;
- Carry over digits: linking knowledge of place value to operation strategies.

Item 8

- Add and subtract in the context of a word problem relating to money;
- Use their general knowledge of the buying process (paying and getting change) to solve a word problem in the context of money.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers: addition and subtraction.

Concepts and skills

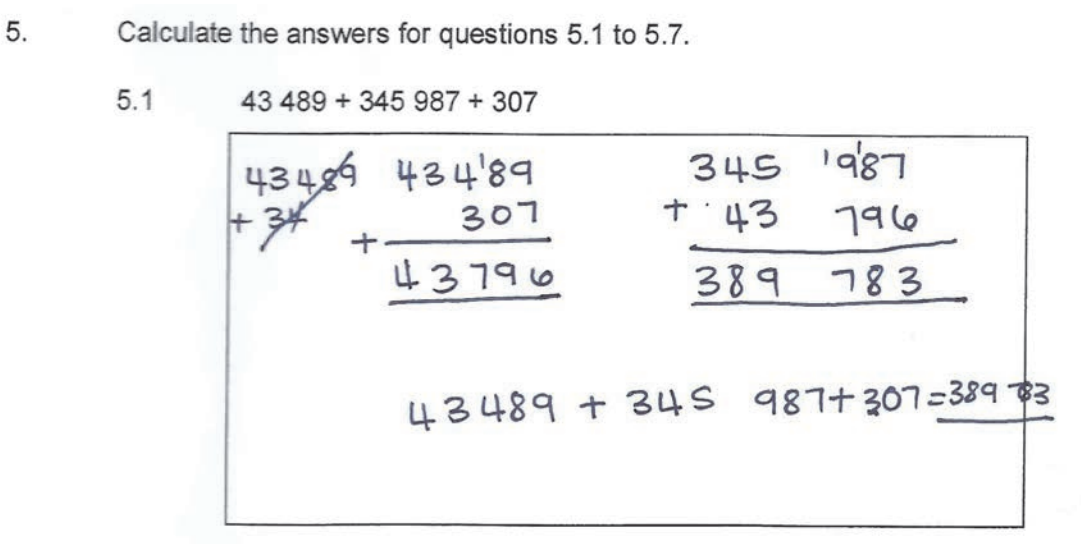
- Addition and subtraction of whole numbers of at least 6 digits;
- Solve problems involving whole numbers and decimal fractions, including in the context of money.

What would show evidence of full understanding?

Item 5.1

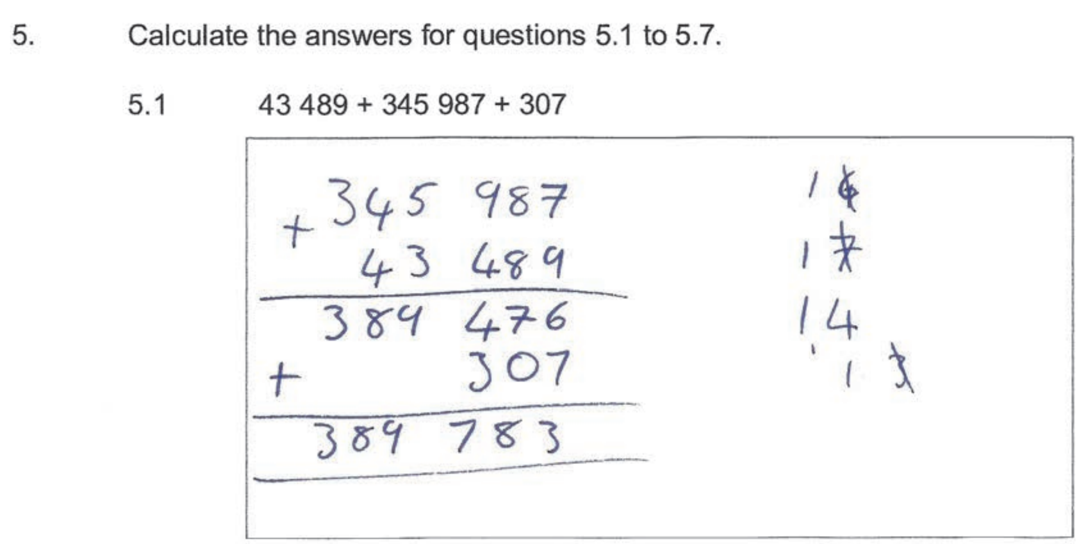
- If the learner answered 389 783 and showed the correct working as demonstrated in the following examples.

- In the first examples the learners added the first two values and then added the third value correctly, with digits carried over clearly shown.



5. Calculate the answers for questions 5.1 to 5.7.

5.1 43 489 + 345 987 + 307



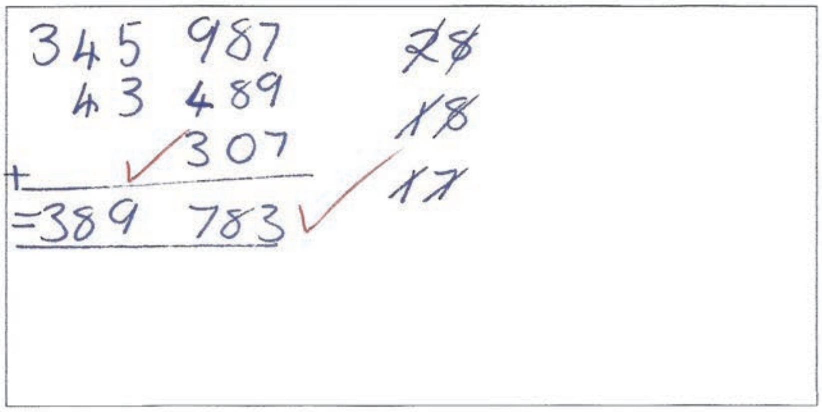
5. Calculate the answers for questions 5.1 to 5.7.

5.1 43 489 + 345 987 + 307

- In the next examples the learners first arranged the numbers to be added, with the biggest at the top to the lowest at the bottom (the first example) or the lowest at the top to the biggest at the bottom (the second example) and then added correctly.
- In some cases the learners kept the numbers in the order given in the question and added correctly.

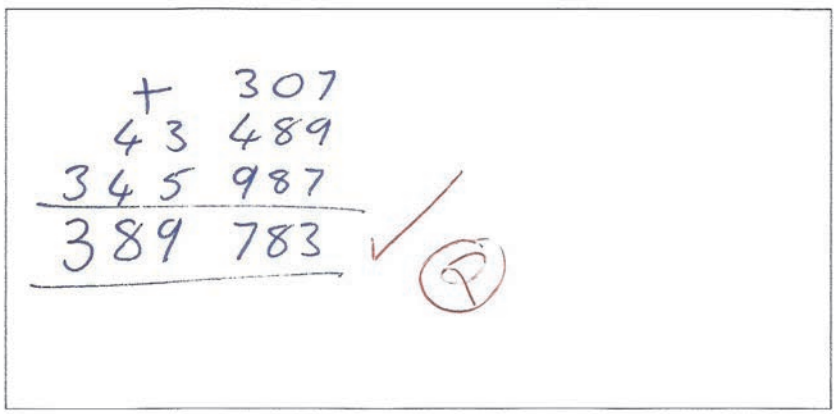
5. Calculate the answers for questions 5.1 to 5.7.

5.1 43 489 + 345 987 + 307



5. Calculate the answers for questions 5.1 to 5.7.

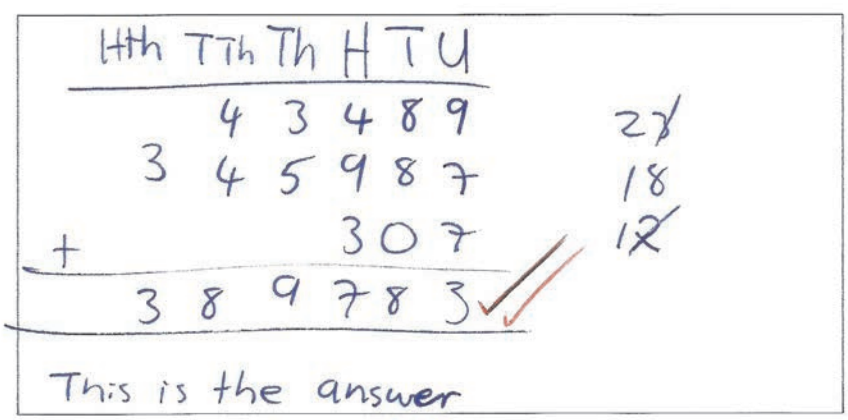
5.1 43 489 + 345 987 + 307



- In the next examples the learners either indicated placed values above the numbers (shown in the first illustration) or inserted zeros to occupy place values (shown in the second illustration).

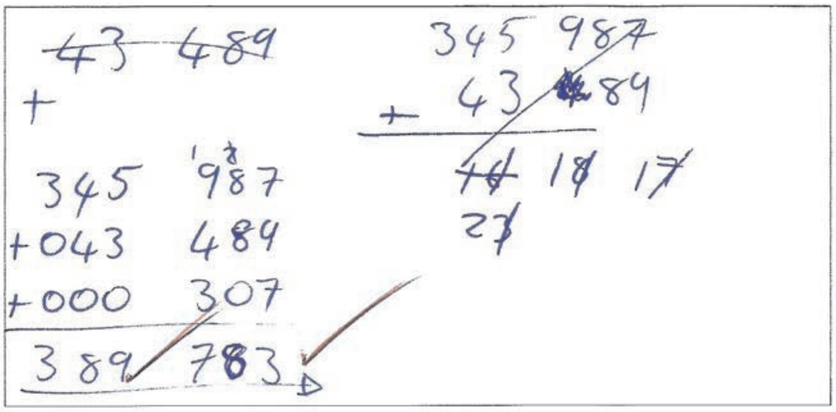
5. Calculate the answers for questions 5.1 to 5.7.

5.1 43 489 + 345 987 + 307



5. Calculate the answers for questions 5.1 to 5.7.

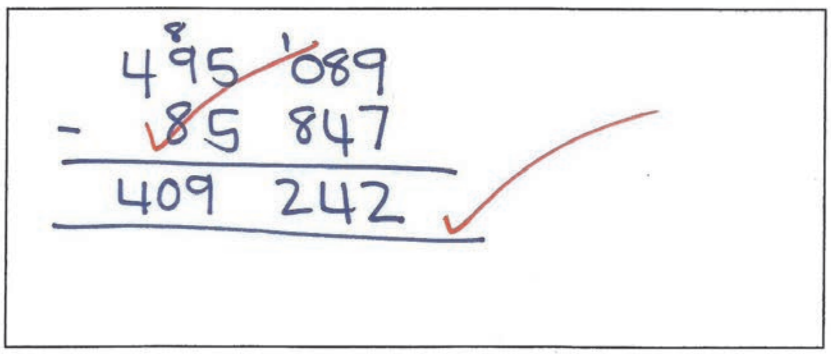
5.1 43 489 + 345 987 + 307



Item 5.2

- If the learner answered 409 242 and the working shown is correct as in the following answer.

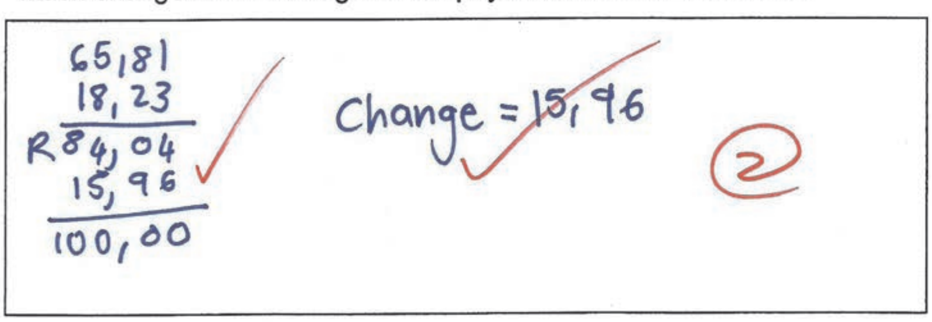
5.2 495 089 – 85 847



Item 8

- If the learner added R65,81 + R18,23 correctly and subtracted the answer from R100 correctly to calculate the amount of change. In the example that follows the learner worked out the amount of change by calculating how much to add to R84,04 to get R100.

8. Suzan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?



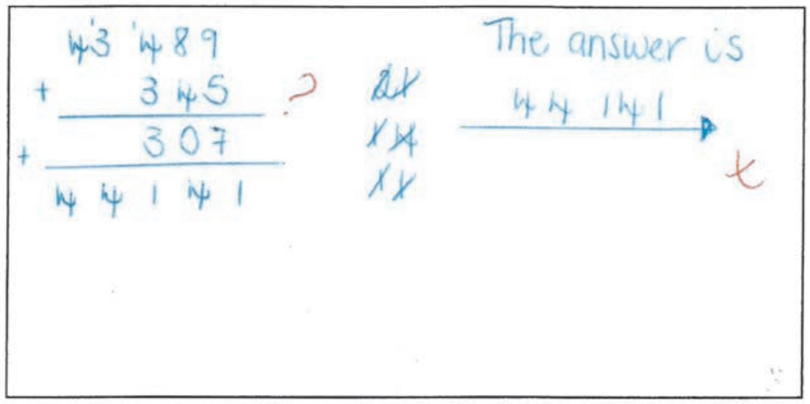
What would show evidence of partial understanding?

Item 5.1

- If the learner added correctly, but did not copy down the sum correctly: in this example the learner copied 345 987 as 345.

5. Calculate the answers for questions 5.1 to 5.7.

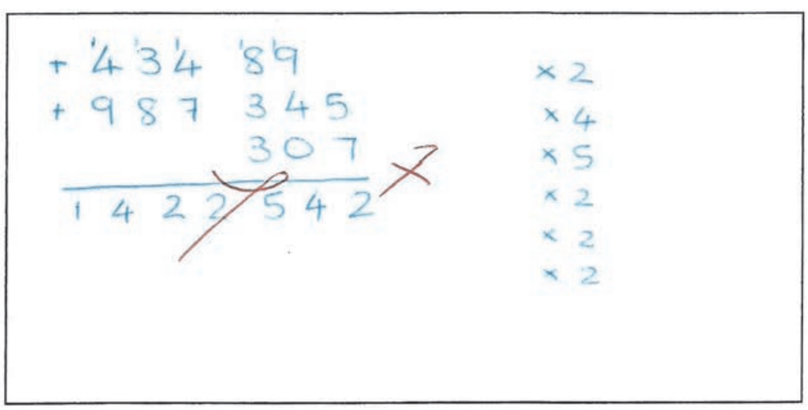
5.1 43 489 + 345 987 + 307



- If the learner did not use place value correctly when doing the addition: in the following example the learner placed 43 489 incorrectly as 434 890, but then did the addition and carry over correctly. Also, the learner wrote the number 345 987 as 987 345.

5. Calculate the answers for questions 5.1 to 5.7.

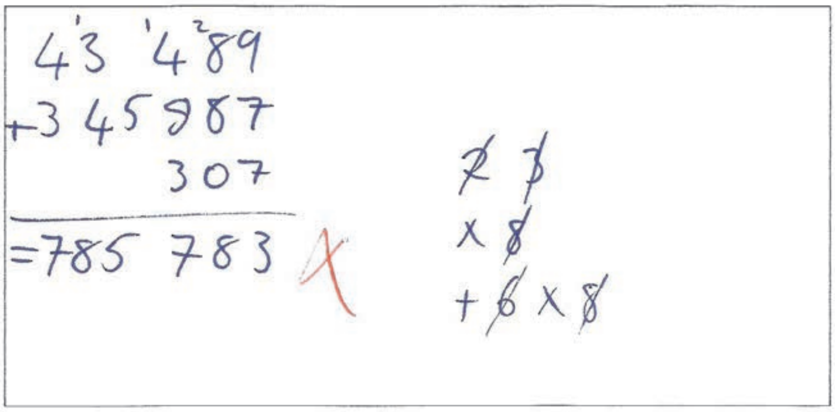
5.1 43 489 + 345 987 + 307



- In the next example the learner shows confusion about the arrangement of place values. The addition and carry over were both done correctly, but the learner wrote 43 489 as 430 489 (430 489 + 345 987 + 307 = 785 783).

5. Calculate the answers for questions 5.1 to 5.7.

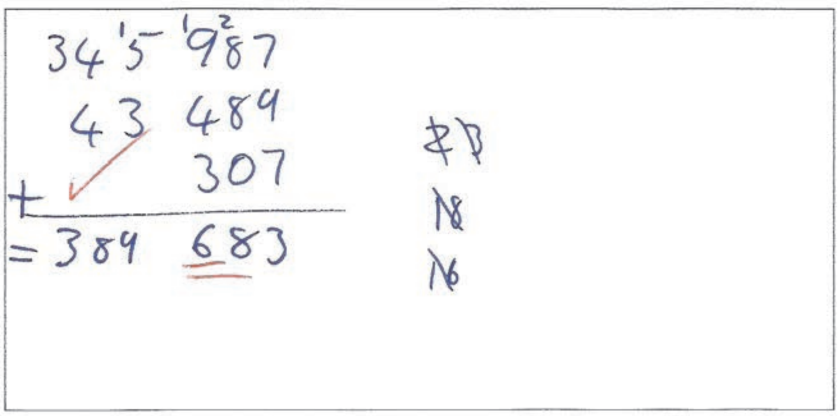
5.1 43 489 + 345 987 + 307



- The last examples show addition that is partly correct. 389 is correct, but the 6 should be a 7. In the second example 50 should be 78. These learners seem to understand addition, but made errors in their working.

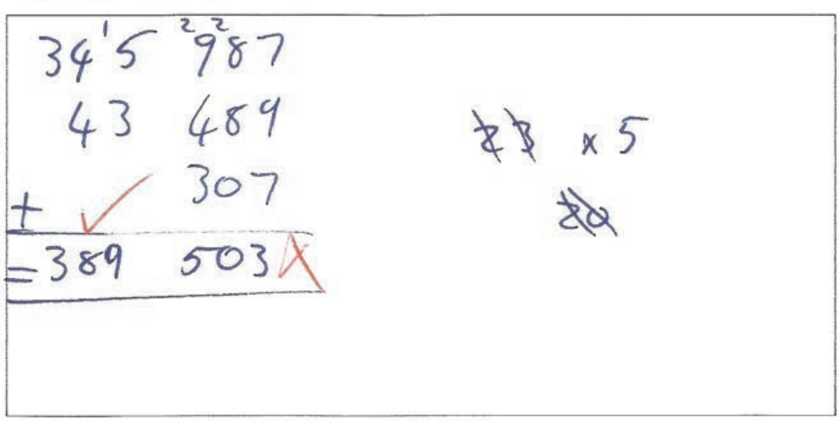
5. Calculate the answers for questions 5.1 to 5.7.

5.1 43 489 + 345 987 + 307



5. Calculate the answers for questions 5.1 to 5.7.

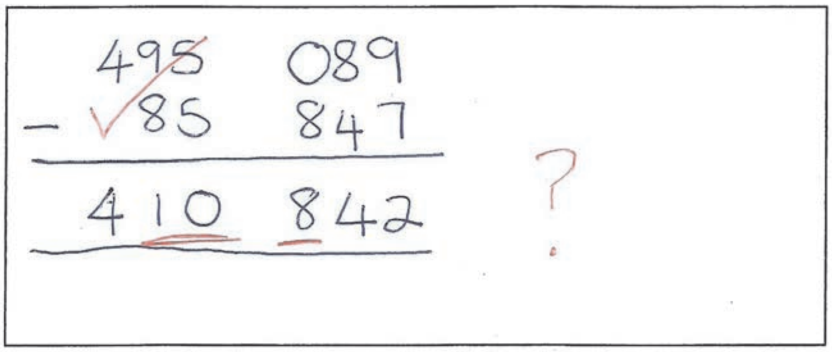
5.1 43 489 + 345 987 + 307



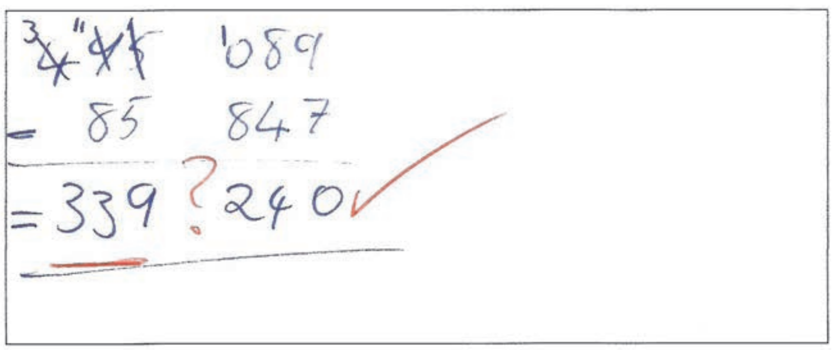
Item 5.2

- In the examples that follow the learners appear to know how to arrange the place values and how to subtract individual digits, but may not know how to do borrowing.

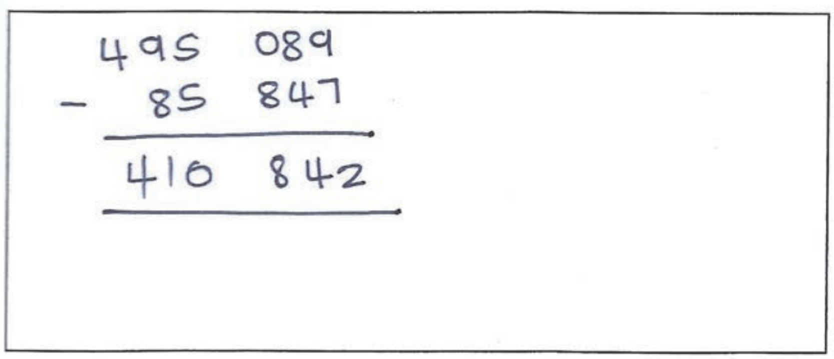
5.2 495 089 – 85 847



5.2 495 089 – 85 847

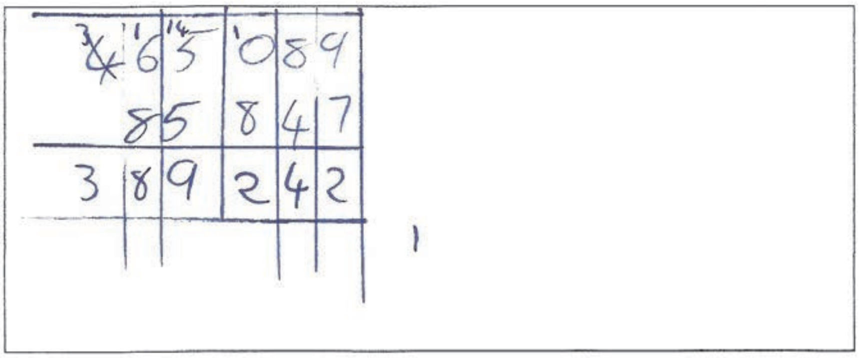


5.2 495 089 – 85 847



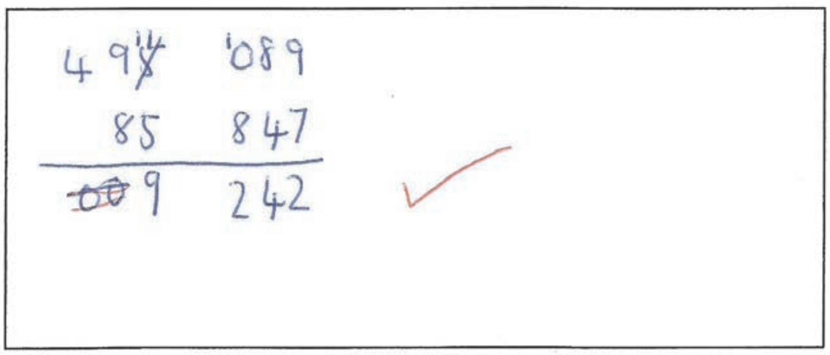
- The following answer illustrates the learner's correct subtraction of an incorrectly copied question. The number 495 089 was wrongly copied as 465 089, but the digits are well arranged with borrowing shown (although there is one error in the ten thousands place).

5.2 495 089 – 85 847



- The last example shows incomplete subtraction. The learner stopped the subtraction with the ten thousands digit.

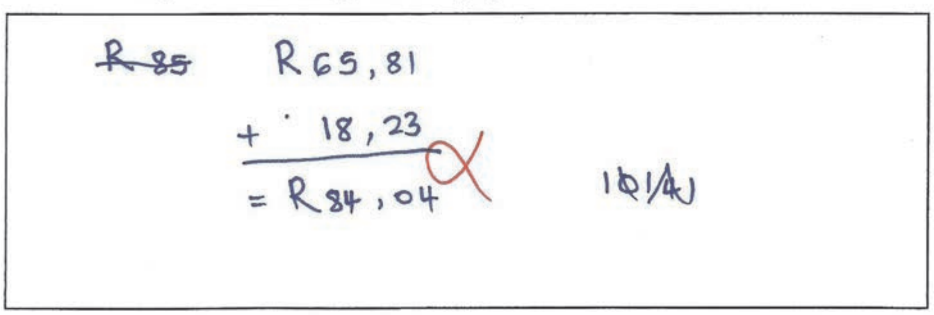
5.2 495 089 – 85 847



Item 8

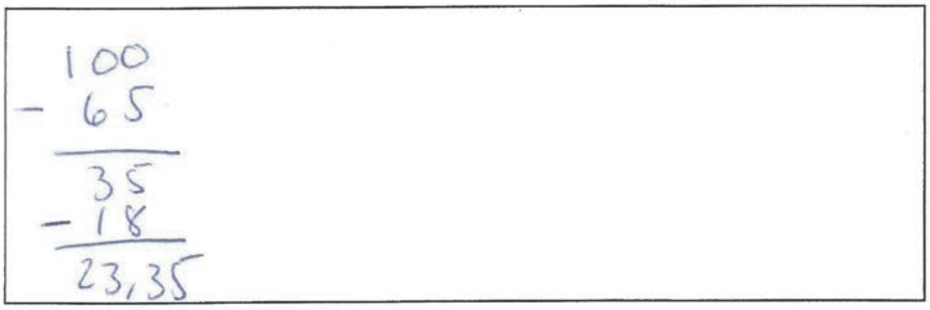
- If the learner added R65,81 + R18,23 correctly to get R84,04 but did not go on to subtract from R100 to calculate the change;

8. Suzan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?

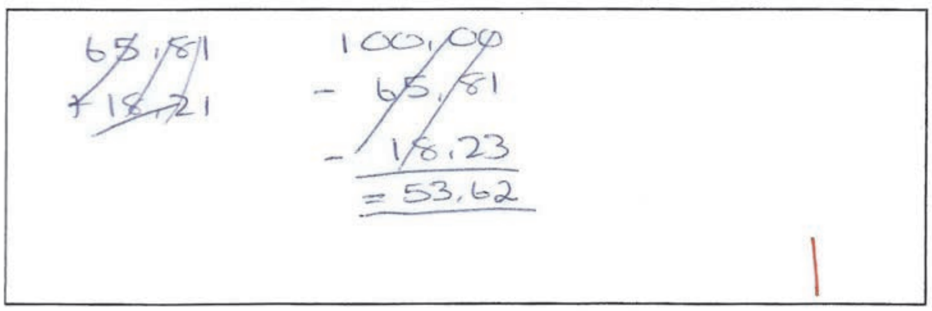


- If the learner subtracted the two amounts from R100, but ignored the cents given in the question;
- If the learner's working shows an attempt to subtract the two amounts (R65,81 and R18,23) from R100.

8. Suzan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?



8. Suzan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?



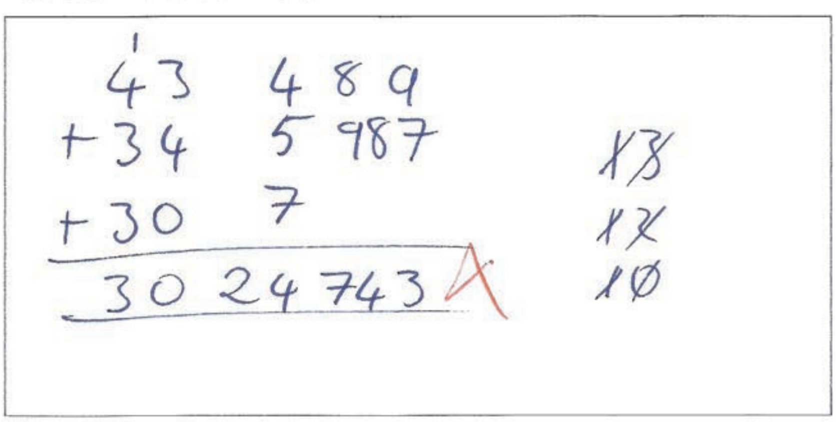
What would show evidence of no understanding?

Item 5.1

- If the learner showed working that indicates no understanding of the addition process;
- If the leaner placed the digits wrongly and added incorrectly.

5. Calculate the answers for questions 5.1 to 5.7.

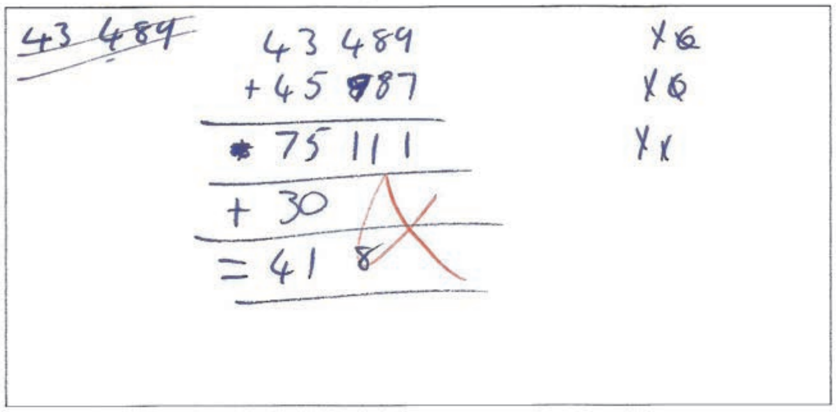
5.1 43 489 + 345 987 + 307



- In the example that follows the learner copied the number 345 987 incorrectly by omitting the 3, then wrote down the tens digit carrying over the units and did not add the units in the next step. There appears to be addition and subtraction being done in this working.

5. Calculate the answers for questions 5.1 to 5.7.

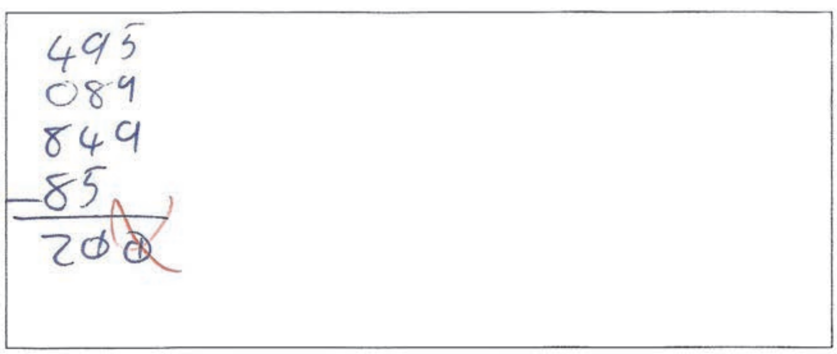
5.1 43 489 + 345 987 + 307



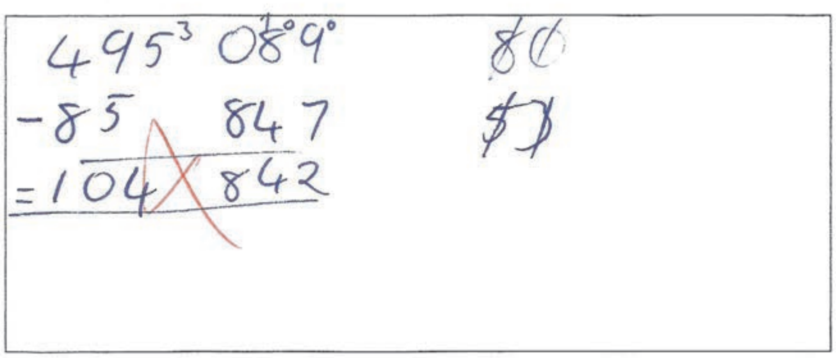
Item 5.2

- If the learner placed the digits wrongly (not using their place values correctly) and subtracted incorrectly as shown in these examples.

5.2 495 089 – 85 847



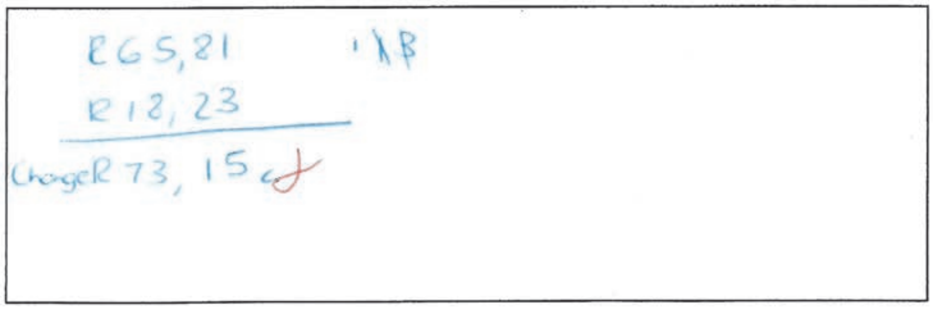
5.2 495 089 – 85 847



Item 8

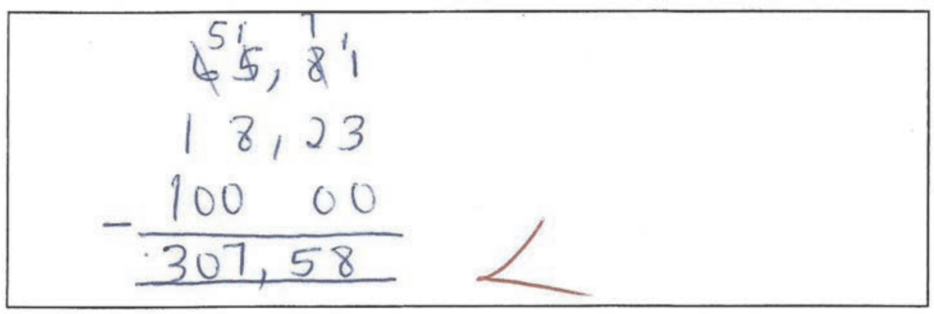
- If, as shown, the learner subtracted to find the difference between the two values R65,81 and R18,23, instead of subtracting their sum from R100.

8. Suzan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?



- In the next example three numbers were written in column form and some subtraction was done (although not the correct subtraction for this problem). In the tens and hundreds column it is not clear what the learner was attempting to do.

8. Suzan buys a school bag for R65,81 and a pencil case for R18,23. How much change should she get if she pays with a R100 banknote?



What do the item statistics tell us?

Item 5.1

53% of learners answered this question correctly.

Item 5.2

42 % of learners answered this question correctly.

Item 8

20% of learners answered this question correctly.

Factors contributing to the difficulty of the items

Item 5.1

- There were three numbers be added, not all with the same number of digits.
- The question was given as a horizontal number string.
- The three numbers in the string lead to several instances where learners need to be able to bridge 10 (by carrying/trading).

Item 5.2

- The subtraction needed was a higher order subtraction of a 6-digit number minus a 5-digit number.
- The subtraction involved borrowing of place values.
- Learners may not understand how to arrange the place values correctly.
- The item required learners to copy the question accurately.

Item 8

- A multistep procedure was required.
- Learners had to relate the problem to a real life buying and selling situation.
- Learners may not know how to add and subtract decimals.

Teaching strategies

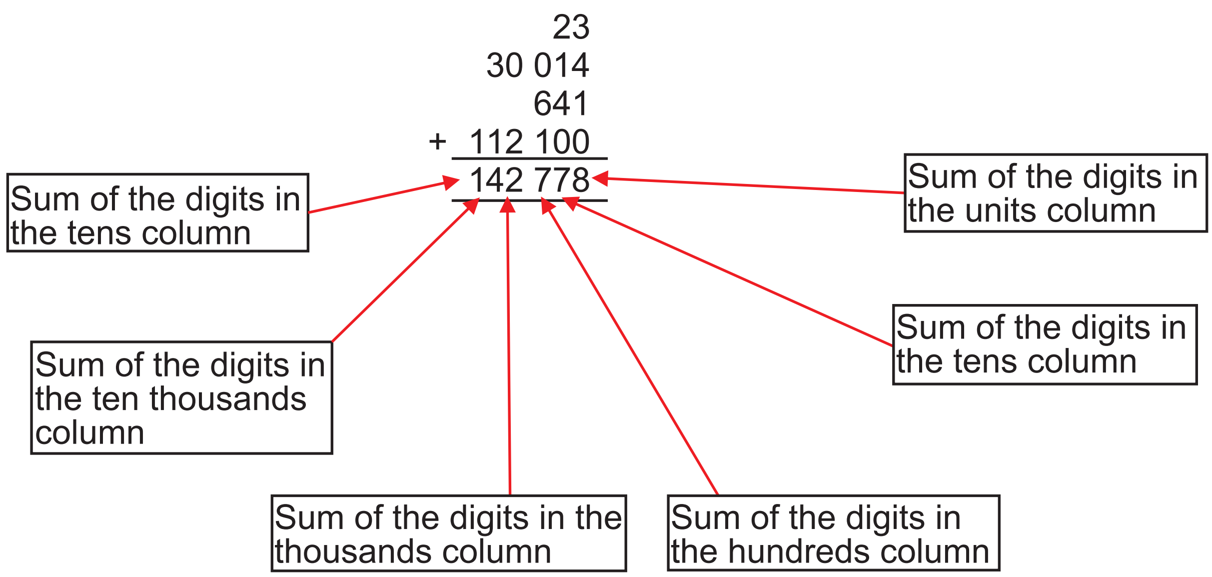
Column addition of both large and small numbers, but with no carrying over

- Explain to the learners that to add large numbers we can list the numbers in columns and then add the numbers using the place value of the columns.
- When numbers are aligned according to place value it becomes easy to add the digits in the place value columns.
- The sorting of the numbers into columns is a way of helping us to add units to units, tens to tens, hundreds to hundreds and so on.
- Learners need to understand the way in which the process works and not just how to do it, so your explanations in which you talk about the place values and how they work together are very important.

- Here is an example that you can use to work through with your class.

Example

Add the following numbers: 23 + 30 014 + 641+ 112 100



Solution

We first have to line up the numbers according to the place values. Notice how this happens in the example provided, because the numbers in the string do not all have the same number of place values.

- In the units column we add 3 + 4 + 1 to get 8. This answer is then written below the units column. This is the sum of all of the units. There is no carry-over to the tens place because the total number of units is less than 10.
- When we add up the digits in the tens column we get 2 + 1 + 4 = 7. We then write 7 under the tens column. This is the sum of all of the tens. There is no carry-over to the hundreds place because the total number of tens is less than 10.
- The numbers 0, 6, and 1 occupy the hundreds column. When we add them up we get 7. This 7 is written under the hundreds column. This is the sum of all of the hundreds. There is no carry-over to the thousands place because the total number of hundreds is less than 10.
- If we add up numbers in the thousands column we get 0 + 2 = 2. The answer 2 is written below the thousands column. This is the sum of all of the thousands. There is no carry-over to the ten thousands place because the total number of thousands is less than 10.
- 3 and 1 are found in the ten thousands column. We add these up to get 4. 4 is written below the ten thousands column. This is the sum of all of the ten-thousands. There is no carry-over to the hundred thousands place because the total number of ten thousands is less than 10.
- There is only one digit in the hundred thousands column, so we simply bring it down as 1 in the hundred thousands column.
- We have now added all of the numbers according to place value by using place value columns to break the number up.

Column addition of numbers with carry overs

In the activity above the numbers were chosen so that there would not be the complication of carrying across places. But the reality is that when we add numbers, we will have often to carry over when we do calculations.

- You can use the simple column method as shown above, explaining carefully to your class about what is happening when you add and re-group with carrying over.
- If learners have problems understanding carrying over in vertical addition, then the expanded vertical column method can be shown to your class. This process is more cumbersome and learners as a rule are not required to work this way. However, this method could be used until the learners realise what happens when they carry over across places.
- Your goal should be to teach learners how to use efficient algorithms (methods for adding/subtracting etc.) with understanding.

Expanded column method for addition

Example

Let us consider the sum: 43 489 + 245 987 + 307.

This can be written as:

43 489	→			40 000	+	3 000	+	400	+	80	+	9
245 987	→	200 000	+	40 000	+	5 000	+	900	+	80	+	7
+ 307	→						+	300	+	00	+	7

Adding the numbers according to place value gives us:

200 000 + 80 000 + 8 000 + 1 600 + 160 + 23

Writing this sum in columns we get:

```
  200 000
   80 000
    8 000
    1 600
      160
+      23
  289 783
```

- This method is useful initially as it demonstrates the addition process more fully (and it eliminates most carry-over mistakes).

Simple column algorithm

When learners are ready show them the simple column algorithm which is neat and efficient. But make sure that they understand the full process of the algorithm and can explain to you what they are doing when they go through the steps.

Example

Find the sum of 5 897, 78, 726 and 8 569.

Using the simple column method:

$$\begin{array}{cccccc} & & 5 & 8 & 9 & 7 \\ & & & & 7 & 8 \\ & & & 7 & 2 & 6 \\ + & & 8^{2} & 5^{2} & 6^{3} & 9 \\ \hline & 1 & 5 & 2 & 7 & 0 \\ \hline \end{array}$$

- Write the numbers in columns with the thousands, hundreds, tens and units lined up.

- 7 + 8 + 6 + 9 = 30. Thus, the sum of the digits in the units column is 30. So we write 0 in the units position of the answer and carry 3 to the tens column for further addition.
- The sum of the digits in the tens column after adding 3 is 27. So we place 7 in the tens position of the answer and carry 2 to the hundreds column to be added to the other hundreds.
- The sum of the digits in the hundreds column after adding 2 is 22. So we place 2 in the hundreds position of the answer and carry 2 to the thousands column to be added to the other thousands.
- The sum of the digits in the thousands column after adding 2 is 15. This is the last place value column in which we are adding. We can write 15 thousands to complete the answer to the question.
- NOTE that the digits that have been carried have been written along the bottom line of place values. The carried digits are also often written along the top line of the columns. Learners can decide on their preference as long as they develop a systematic and correct way of recording partial steps or the carrying over in the working as they add across all the columns.

Activity: Arranging numbers in columns for addition

In each of the following, arrange the numbers in columns for addition.

a). 15 + 300 + 3 489

b). 7 + 4 006 + 53 200

c). 15 700 + 874 + 10 + 345 126

d). 178 964 + 799 +5 855

e). 69 + 30 068

f). 756 542 + 4 299 + 808 + 6

g). 900 + 69 312 + 538 + 75 932

h). 1 + 21 + 211 + 3 211

i). 515 547 + 5 678 + 17 529 + 9

j). 456 879 + 2 145 + 26

Solutions

- The solutions given here show both simple column working/recording and expanded column addition.
- Learners might find the expansions useful but they should be encouraged to use correct, efficient methods to record their addition working.

a). 15 + 300 + 3 489		
15 300 + 3 489 3 804	[U] [T] [H] [TH]	5 + 0 + 9 = (1)4 (1) + 1 + 0 + 8 = (1)0 (1) + 3 + 4 = 8 3 = 3
b). 7 + 4 006 + 53 200		
7 4 006 + 53 200 57 213	[U] [T] [H] [TH] [TTH]	7 + 6 + 0 = (1)3 (1) + 0 + 0 = 1 0 + 2 = 2 4 + 3 = 7 5 = 5
c). 15 700 + 874 + 10 + 345 126		
15 700 874 10 + 345 126 361 710	[U] [T] [H] [TH] [TTH] [HTH]	0 + 4 + 0 + 6 + 0 = (1)0 (1) + 0 + 7 + 1 + 2 = (1)1 (1) + 7 + 8 + 1 = (1)7 (1) + 5 + 5 = (1)1 (1) + 1 + 4 = 6 3 = 3
d). 178 964 + 799 + 5 855		
178 964 799 + 5 855 185 618	[U] [T] [H] [TH] [TTH] [HTH]	4 + 9 + 5 = (1)8 (1) + 6 + 9 + 5 = (2)1 (2) + 9 + 7 + 8 = (2)6 (2) + 8 + 5 = (1)5 (1) + 7 = 8 1 = 1

e). 69 + 30 068		
69	[U]	9 + 8 = (1)7
+ 30 068	[T]	(1) + 6 + 6 = (1)3
30 137	[H]	(1) + 0 = 1
	[TH]	0 = 0
	[TTH]	3 = 3
f). 756 542 + 4 299 + 800 + 6		
756 542	[U]	2 + 9 + 0 + 6 = (1)7
4 299	[T]	(1) + 4 + 9 + 0 = (1)4
800	[H]	(1) + 5 + 2 + 8 = (1)6
+ 6	[TH]	(1) + 6 + 4 = (1)1
761 647	[TTH]	(1) + 5 = 6
	[HTH]	7 =s 7
g). 900 + 69 312 + 538 + 75 932		
900	[U]	0 + 2 + 8 + 2 = (1)2
69 312	[T]	(1) + 0 + 1 + 3 + 3 = 8
538	[H]	9 + 3 + 5 + 9 = (2)6
+ 75 932	[TH]	(2) + 9 + 5 = (1)6
146 682	[TTH]	(1) + 6 + 7 = 14
h). 1 + 21 + 211 + 3 211		
1	[U]	1 + 1 + 1 + 1 = 4
21	[T]	2 + 1 + 1 = 4
211	[H]	2 + 2 = 4
+ 3 211	[TH]	3 = 3
3 444		
i). 515 547 + 5 678 + 17 529 + 9		
515 547	[U]	7 + 8 + 9 + 9 = (3)3
5 678	[T]	(3) + 4 + 7 + 2 = (1)6
17 529	[H]	(1) + 5 + 6 + 5 = (1)7
+ 9	[TH]	(1) + 5 + 5 + 7 = (1)8
538 763	[TTH]	(1) + 1 + 1 = 3
	[HTH]	5 = 5
j). 456 879 + 2 145 + 26		
456 879	[U]	9 + 5 + 6 = (2)0
2 145	[T]	(2) + 7 + 4 + 2 = (1)5
+ 26	[H]	(1) + 8 + 1 = (1)0
459 050	[TH]	(1) + 6 + 2 = 9
	[TTH]	5 = 5
	[HTH	4 = 4

Use of expanded notation in columns for subtraction

- The column method of addition can also be used effectively for subtraction.
- The difference is that when we subtract we don't have the need to "carry" (which is re-grouping to cope with more than "ten" of anything in a particular place).
- Instead, we might have to "borrow" (which is when we encounter an impasse – a situation where we do not have a big enough value in a certain place to do the subtraction that needs to be done).
- The best way to explain this to your learners is to use examples.

Example

495 089 – 85 847

This can be written as:

495 089 →	400 000	+	90 000	+	5 000	+	000	+	80	+	9
- 85 847 →			80 000	+	5 000	+	800	+	40	+	7
	400 000	+	80 000	+	**14 000**	+	**1 000**	+	80	+	9
-			80 000	+	5 000	+	800	+	40	+	7
	400 000	+	0	+	9 000	+	200	+	40	+	2
= 409 242											

- Explain the working to your learners so that they understand how to work with the place values of the digits as you do the subtraction.
- You start subtracting from the smallest place, the units.
- In the units place we have to subtract 7 from 9. The answer is 2, which we can write in the units position of the answer.
- In the tens place, we have to subtract 40 from 80. The answer is 40, which we can write in the tens position of the answer.
- So far, we have encountered no problems as we subtract.
- In the hundreds place we have to subtract 800 from 0 hundreds. We have a problem! This is called an impasse. It is a temporary problem that we can overcome.
- To overcome the problem we have to go to the next highest place (thousands). In the thousands place, we have 5 thousands. We "borrow" one of those thousands and break it down into 10 hundreds (remind the learners that 1 thousand = 10 hundreds). We leave behind 4 thousands in the thousands column. We now have 10 hundreds or 1 000 in the hundreds column. We can now subtract 800 leaving us with 200. We can record this as the answer in the hundreds place.
- Now we need to subtract the thousands. We have 4 thousand. We want to subtract 5 thousand. We have reached another impasse. To overcome this we go to the next highest place, the ten thousands, and we borrow again. This leave us with 80 000 in the ten thousands column and gives us 14 000 in the thousands column and we can subtract: 14 000 – 5 000 = 9 000.

- In the ten thousands place we now subtract 80 000 from 80 000 which leaves 0 in that column, and we record it in this place.
- In the hundred thousands place we now subtract 0 from 400 000 which leaves 400 000 in that column, and we record it in this place.
- This gives us a final answer of 409 242 if we read the answer across all of the columns.

Activity: Using the expanded column method

Use the expanded column method to calculate the answers for the following sums. The first one has been done for you. Use the simple column method if you feel comfortable to do so.

	289 →		200	+	80	+	9
-	65 →			-	60	+	5
			200	+	20	+	4
		=	224				

a). 369 + 79 =

b). 742 - 83 =

c). 1 221 + 754 =

d). 3 981 - 992 =

e). 7 457 + 163 =

f). 84 998 - 2 987 =

g). 65 740 + 5 630 =

h). 92 000 - 7 568 =

i). 300 000 + 99 897 =

j). 564 098 - 53 728 =

k). 109 752 + 99 468 =

Solutions

- Solutions are shown with both the simple and the expanded column methods.
- Encourage your learners to work efficiently, but they must be able to explain their working to you, showing that they understand how the place values work to get the final answer.

a).

	369 →		300 +	60 +	9
+	79 →	+		70 +	9
			300 +	130 +	18
	= 448				

b).	742 →				700 +	40 +	2
	- 83 →			-		80 +	3
					600 +	50 +	9
	= 659						

c).	1 221 →			1 000 +	200 +	20 +	1
	+ 754 →		+		700 +	50 +	4
				1 000 +	900 +	70 +	5
	= 1 975						

d).	3 981 →			3 000 +	900 +	80 +	1
	- 992 →		-		900 +	90 +	2
				2 000 +	900 +	80 +	9
	= 2 989						

e).	7 457 →			7 000 +	400 +	50 +	7
	+ 163 →		+		100 +	60 +	3
				7 000 +	500 +	110 +	10
	= 7 620						

f).	84 998 →		80 000 +	4 000 +	900 +	90 +	8
	- 2 987 →	-		2 000 +	900 +	80 +	7
			80 000 +	2 000 +	000 +	10 +	1
	= 82 011						

g).	65 740 →		60 000 +	5 000 +	700 +	40 +	0
	+ 5 630 →	+		5 000 +	600 +	30 +	0
	= 71 370		60 000 +	10 000 +	1 300 +	70 +	0

h).	92 000 →		90 000 +	2 000 +	000 +	00 +	0
	- 7 568 →	-		7 000 +	500 +	60 +	8
			80 000 +	4 000 +	400 +	30 +	2
	= 84 432						

i).	300 000 →	300 000 +	00 000 +	0 000 +	000 +	00 +	0
	+ 99 897 →	+	90 000 +	9 000 +	800 +	90 +	7
		300 000 +	90 000 +	9 000 +	800 +	90 +	7
	= 399 897						

j).	564 098 →	500 000 +	60 000 +	4 000 +	000 +	90 +	8
	- 53 728 → -		50 000 +	3 000 +	700 +	20 +	8
		500 00 +	10 000 +	0 000 +	300 +	70 +	0
	= 510 370						

k).	109 752 →	100 000 +	00 000 +	9 000 +	700 +	50 +	2
	+ 99 468 → +		90 000 +	9 000 +	400 +	60 +	8
		100 000 +	90 000 +	18 000 +	1 100 +	110 +	10
	= 209 220						

Use of place value grids to help with place values

- The use of direct column subtraction and addition should follow after using the expanded notation method. It is very important at this stage to inform learners that numbers in column computations are written and subtracted from right to left – starting with the units on the far right and moving across to the highest place according to the numbers in the question.
- Revise the concept of place values with the learners by using place value grids. Most of the learners who did not get maximum marks did not align the place values correctly.
- Grids can be used to help learners stick to the correct place values.
- Headings for the columns of the grids may or may not be used: it could be that simple columns may be enough to guide learners.
- The grid below demonstrates the layout of the previous example in grid form.
- Recording of the "borrowing" and exchange is shown above the top number in the calculation.

Example

495 089
- 85 847

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
4	8 9	14 5	10 0	8	9
-	8	5	8	4	7
4	0	9	2	4	2

Activity: Using place value grids

Calculate the answers for the following questions using place value grids. The first two have been done for you

1). 987 + 98 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			9	8	7
		+		9	8
		1	0	8	5

2). 321 – 85 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			3	2	1
		-		8	5
			2	3	6

3). 456 – 78 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)

4). 219 + 63 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)

5). 784 – 59 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)

6). 5 987 + 489 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)

7). 9 001 – 875 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)

8). 19 354 + 7 200 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)

9). 74 562 – 8 921 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)

10). 63 800 + 7 977 =

11). 400 500 – 46 547 =

12). 129 622 + 75 801 =

13). 857 495 – 77 632 =

14). 511 321 + 63 718 =

Solutions

Notice that the solutions are given in simpler format as they progress. Make sure that learners understand all of the working and can explain it to you. This will confirm that they are able to use the algorithm correctly and efficiently.

1). 987 + 98 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			9	8	7
	+			9	8
		1	**0**	**8**	**5**

2). 321 – 85 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			3	2	1
		-		8	5
			2	**3**	**6**

3). 456 – 78 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			4	5	6
		-		7	8
			3	**7**	**8**

4). 219 + 63 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			2	1	9
		+		6	3
			2	**8**	**2**

5). 784 – 59 =

Hundred Thousands (HTH)	Ten Thousands (TTH)	Thousands (TH)	Hundreds (H)	Tens (T)	Units (U)
			7	8	4
		-		5	9
			7	**2**	**5**

6). 5 987 + 489 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)
		5	9	8	7
	+		4	8	9
		6	**4**	**7**	**6**

7). 9 001 – 875 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)
		9	0	0	1
	-		8	7	5
		8	**1**	**2**	**6**

8). 19 354 + 7 200 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)
	1	9	3	5	4
+		7	2	0	0
	2	**6**	**5**	**5**	**4**

9). 74 562 – 8 921 =

(HTH)	(TTH)	(TH)	(H)	(T)	(U)
	7	4	5	6	2
-		8	9	2	1
	6	**5**	**6**	**4**	**1**

10). 63 800 + 7 977 =

	6	3	8	0	0
+		7	9	7	7
	7	**1**	**7**	**7**	**7**

11). 400 500 – 46 547 =

4	0	0	5	0	0
-	4	6	5	4	7
3	**5**	**3**	**9**	**5**	**3**

12). 129 622 + 75 801 =

1	2	9	6	2	2
+	7	5	8	0	1
2	**0**	**5**	**4**	**2**	**3**

14). 857 495 – 77 632 =

8	5	7	4	9	5
-	7	7	6	3	2
7	**7**	**9**	**8**	**6**	**3**

15). 511 321 + 63 718 =

5	1	1	3	2	1
+	6	3	7	1	8
5	**7**	**5**	**0**	**3**	**9**

Real life scenarios

- Learners have to be given many real life examples to help them become familiar with interpreting and solving these problems.
- The following exercise will help elucidate the concepts which the learners should know.
- Learners should be encouraged to give the answers to word problems using words. This is not always credited for marks but it promotes the development of reasoning skills.
- Learners should be required to give the units in the answer.
- More information on the solution of word problems is given in item 5.4.

Word problems in financial mathematics

Example

Thuto and her two friends want to have pizza for lunch. Thuto buys 2 small pizzas at R24,90 per pizza. How much is Thuto going to remain with if she had R200 to start with?

Solution

Cost of the 2 pizzas = R24,90 + R24,90 = R49,80

This means the shop will subtract R49,80 from the money that Thuto gives them. The money remaining will be given back to Thuto as change.

Money Remaining = R200 – R49,80 = R150,20

Activity: Word problems in financial mathematics

1). Emma bought a R5 ice-lolly using a R10 note. How much change did she get?

2). During a Spring Day dance at Rethabile Primary School cold drinks are sold for R15. If a student pays with a R20 note for 1 cold drink, how much change will she get?

3). Apples cost R4 each at a local Fruit and Veg City. You buy 4 of them. How much must you pay for the apples? How much change do you get from a R50 note?

4). After you bought 4 apples at R4 each your friend also bought 4 bananas at R3 each. How much money did you both spend?

5). Your mom goes to a local bookshop to buy stationery for you. She buys 2 pencils at R3,20 each, 2 black pens at R4,50 each and a ruler for R5. How much money did she spend at the bookshop?

6). After a heavy training session at the sports ground, Tsakani and his two friends feel really thirsty. They decide to buy 3 soft drinks costing R7 each, only to discover that the money they have falls short by R6. How much money did they have?

7). The Lion Park Resort requires children to pay R30 and adults to pay R50 to enter. In Tsakani's family there are 2 children and 2 adults. How much is the family going to pay to get into the Lion Park Resort?

8). A farmer slaughtered a lot of goats. The total mass of the goats was 14 359 kg. He also slaughtered sheep with a total mass of 7 958 kg. How much more was the mass of the goats?

Solutions

1). R10,00 – R5,00 = R5,00
Emma got R5 change

2). R20,00 – R15,00 = R5,00. Therefore the student will get R5 change.

3). Amount paid = R4,00 x 4 =R16,00
Change: R50,00 – R16,00 = R34,00

4). Money spent = 4 x R4,00 + 4 x R3,00
= R16,00 + R12,00
= R28,00

5). Money spent = 2 x R3,20 + 2 x R4,50 + R5,00
= R6,40 + R9,00 + R5,00
= R20,40

6). Total cost = 3 x R7,00 = R21,00
Money short = R6,00
Money they had = R21,00 – R6,00
= R15,00

7). Amount to pay = R30,00 x 2 + R50,00 x 2
= R60,00 + R100,00
= R160,00

8). Mass of goat meat = 14 359 kg
Mass of sheep meat = 7 958 kg
Difference = 14 359 kg – 7 958 kg
= 6 401 kg

Notes:

Other examples of how to test addition and subtraction of numbers up to 6-digit numbers

ANA 2014 Grade 6 Mathematics Items 4.1 and 4.2

4 Calculate the answers for Questions 4.1 - 4.8.

4.1 42 152 + 28 945 + 76 361

[2]

4.2 87 546 – 43 968

[2]

Notes:

Addition and subtraction: using inverses

ANA 2013 Grade 6 Mathematics Item 9

9.	Complete: If 787 – 614 = 173, then 173 + ______ = 787	[1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Use addition to check subtraction;
- Use addition and subtraction as inverse operations in order to find solutions to equations.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Patterns, Functions and Algebra.

Topic: Number sentences: Introduction to algebraic expressions.

Concepts and skills:

- Addition and subtraction: using addition and subtraction as inverse operations.

What would show evidence of full understanding?

- If the learner realised that the right-hand equation could be completed by using the inverse relationship between addition and subtraction, i.e. that the same three numbers are involved in the two equations.

9. Complete: If 787 – 614 = 173, then 173 + 614 = 787

What would show evidence of partial understanding?

- The answer that follows shows that the learner knew what should have been written but did not copy 614 correctly. 614 was copied as 164.

9. Complete: If 787 – 614 = 173, then 173 + 164 = 787

What would show evidence of no understanding?*

- If the learner failed to realise that 173 + 614 = 787, as shown in the following answers.

9. Complete: If 787 – 614 = 173, then 173 + 84 = 787

9. Complete: If 787 – 614 = 173, then 173 + 160 = 787

9. Complete: If 787 – 614 = 173, then 173 + 20 = 787

What do the item statistics tell us?

59% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- This topic (properties of numbers and laws of operations) is specified in the CAPS document, but it may not be fully addressed in classrooms;
- The question needed higher levels of thinking for learners to realise that adding 614 to 173 would give 787. Learners need to show an understanding of the relational nature of equations and not just find an answer to a given sum where both addends are on the same side of the equal sign.

Teaching strategies

Using addition to check subtraction and vice-versa

- Learners need to be able to apply their knowledge of addition and subtraction to equations to balance the equations.
- The rule to bear in mind is that the left-hand side must equal the right-hand side for the equation to be balanced.
- When solving equations addition is removed by its inverse operation, which is subtraction, and subtraction is removed by addition.
- Errors in answering this item show that learners were not able to balance the equation.

The following examples illustrate the reasoning behind balancing the type of equation in the ANA item.

Examples

1).

$$3 + 2 = 5$$

$$\Rightarrow 3 + 2 - 2 = 5 - 2$$

$$\Rightarrow 3 = 3$$

- This is often summarised by the saying that "what we do to the left, we do to the right" in solving the equations.

2).

$$50 - 10 = 40$$

$$\Rightarrow 50 - 10 + 10 = 40 + 10$$

$$\Rightarrow 50 = 50$$

- Normally part of step 2 in both examples is omitted such that the two examples could be represented as:

3 + 2 = 5 ⇒ 3 = 5 - 2 ⇒ 3 = 3 Hence we can see that if 3 + 2 = 5, then 5 – 2 = 3.	50 - 10 = 40 ⇒ 50 = 40 + 10 ⇒ 50 = 50 Hence we can see that if 50 – 10 = 40, then 40 + 10 = 50.

- In both of these examples we are writing the equations out in different ways, but using the same numbers.

Activity: Using addition to check subtraction and subtraction to check addition

Consider the following pairs of equations: Note that in one equation the balancing is done using subtraction and in the corresponding equation the balancing is done using addition. Hence we say we are using inverse operations.

If 3 + 1 = 4, then 3 = 4 – 1

- On the left hand side 1 is added to 3 to get 4. On the right hand side 1 is subtracted from 4 to get 3. In both these equations 3 and 4 are fixed. What "moves" is the 1 which changes sign (from + to –) when it "crosses the equal sign".

If 8 – 3 = 5, then 8 = 5 + 3

- On the left hand side 3 is subtracted from 8 to get 5. On the right hand side 3 is added to 5 to get 8. In both these equations 8 and 5 are fixed. What "moves" is the 3 which changes sign (from – to +) when it "crosses the equal sign".

Complete the following:

1). If 4 + 5 = 9, then 4 = 9 – ________
2). If 6 + 7 = 13, then 6 = 13 – ________
3). If 37 + 26 = 63, then 37 = 63 – ________
4). If 71 + 51 = 122, then 71 = 122 – ________
5). If 100 + 52 = 152, then 100 = 152 – ________
6). If 220 + 330 = 550, then 220 = 550 – ________
7). If 215 + 185 = 400, then 215 = 400 – ________
8). If 713 + 37 = 750, then 750 – ________ = 713
9). If 360 + 55 = 415, then 415 – ________ = 360
10). If 5 – 3 = 2, then 5 = 2 + ________
11). If 4 – 1 = 3, then 4 = 3 + ________
12). If 9 – 6 = 3, then 9 = 3 + ________

13). If 37 – 11 = 26, then 37 = 26 + ________
14). If 71 – 20 = 51, then 71 = 51 + ________
15). If 155 – 55 = 100, then 155 = 100 + ________
16). If 354 – 120 = 234, then 354 = 234 + ________
17). If 759 – 275 = 484, then 759 = 484 + ________

Solutions

1). If 4 + 5 = 9, then 4 = 9 – 5
2). If 6 + 7 = 13, then 6 = 13 – 7
3). If 37 + 26 = 63, then 37 = 63 – 26
4). If 71 + 51 = 122, then 71 = 122 – 51
5). If 100 + 52 = 152, then 100 = 152 – 52
6). If 220 + 330 = 550, then 220 = 550 – 330
7). If 215 + 185 = 400, then 215 = 400 – 185
8). If 713 + 37 = 750, then 750 – 37 = 713
9). If 360 + 55 = 415, then 415 – 55 = 360
10). If 5 – 3 = 2, then 5 = 2 + 3
11). If 4 – 1 = 3, then 4 = 3 + 1
12). If 9 – 6 = 3, then 9 = 3 + 6
13). If 37 – 11 = 26, then 37 = 26 + 11
14). If 71 – 20 = 51, then 71 = 51 + 20
15). If 155 – 55 = 100, then 155 = 100 + 55
16). If 354 – 120 = 234, then 354 = 234 + 120
17). If 759 – 275 = 484, then 759 = 484 + 275

Another example of how to test addition and subtraction using inverses

ANA 2014 Grade 6 Mathematics Item 9

9.	Complete: If 336 ÷ 14 = 24, then 24 x 14 = __________	[1]

Multiplication

ANA 2013 Grade 6 Mathematics Item 5.3

5.3	Calculate the answer: 3 097 × 249	[4]

What should a learner know to answer this question correctly?

Learners should be able to:

- Know and use the multiplication tables of 1 to 10;
- Perform column multiplication of numbers with and without carry-overs;
- Perform column multiplication and long multiplication (arranging of numbers and placing of digits before multiplication);
- Multiply 4-digit numbers by 3-digit numbers.

Where is this topic located in the curriculum? Grade 6 Term 4

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers.

Concepts and skills:

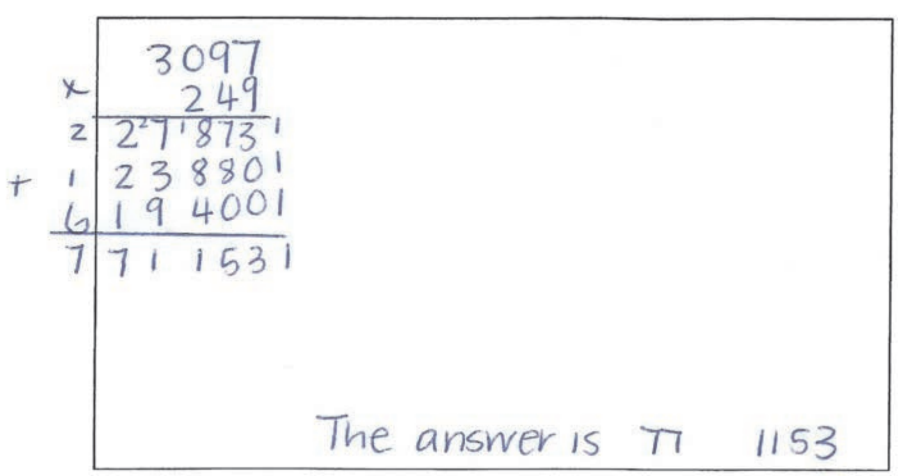
- Multiplication (4-digit numbers by 3-digit numbers).

What would show evidence of full understanding?

- If the learner obtained the correct answer, 771 153, by any mathematically correct method.

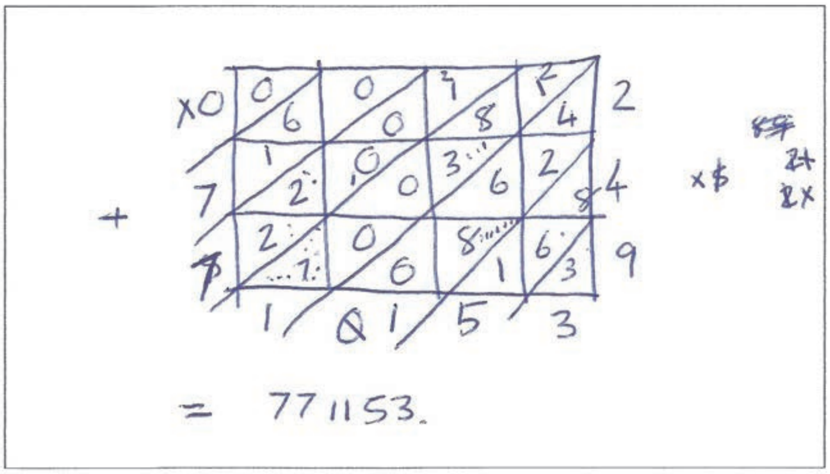
- In this first example the learner used a vertical algorithm.

5.3 3 097 x 249



- In the second example the learner used Napier's rods – strips which list the two-digit multiples of numbers. This is a less common method, but it is correct and acceptable.

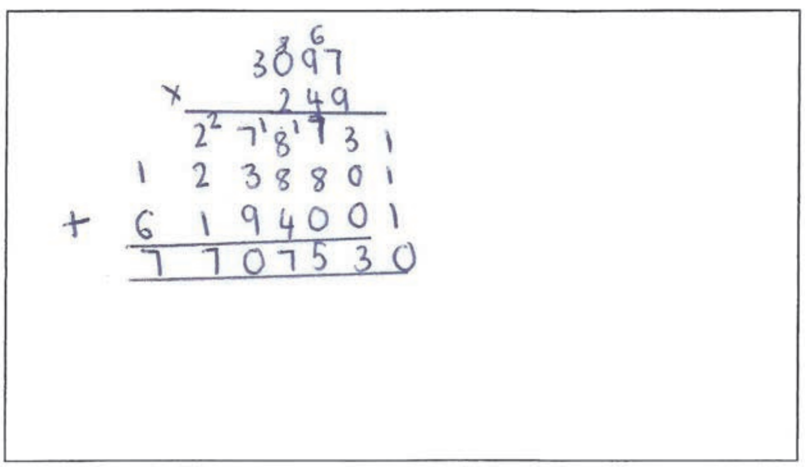
5.3 3 097 x 249



What would show evidence of partial understanding?

- If the learner arranged the digits correctly and multiplied correctly, but added wrongly at the end.

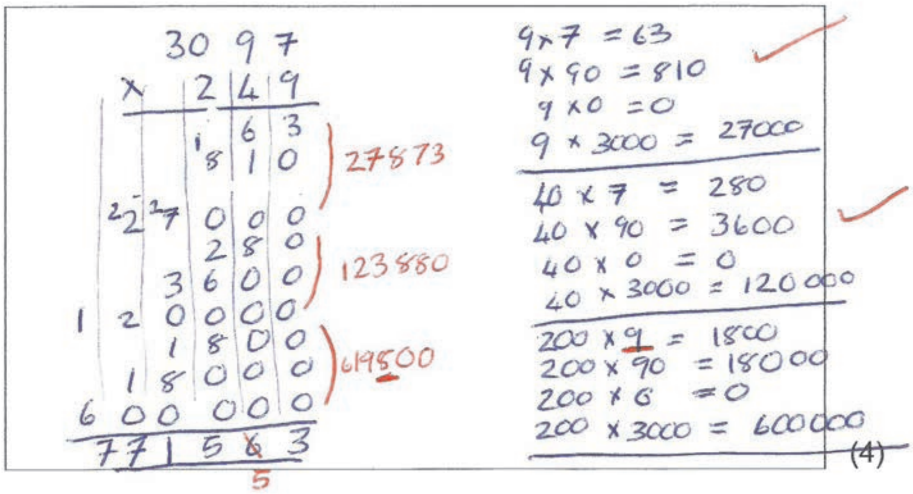
5.3 3 097 x 249



- In both of the following examples the learners had an idea of how to break down the bigger number into smaller values that are easier to work with. Multiplication with multiples of 10 proved to be easier for these learners.

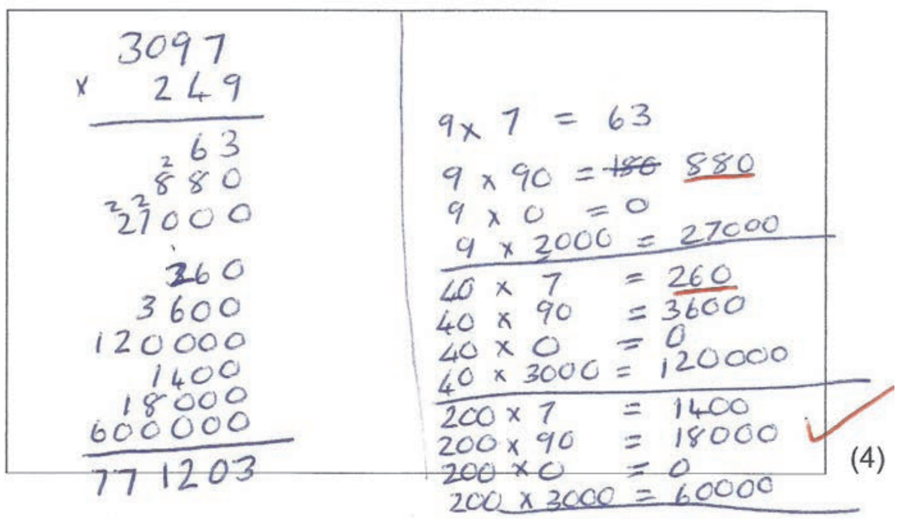
- The problem in the first solution shown is the learner's knowledge of place values.

5.3 3 097 x 249



- In this second solution shown, 2 mistakes were made: 9 x 90 = 810 NOT 880 and 40 x 7 = 280 NOT 260.

5.3 3 097 x 249

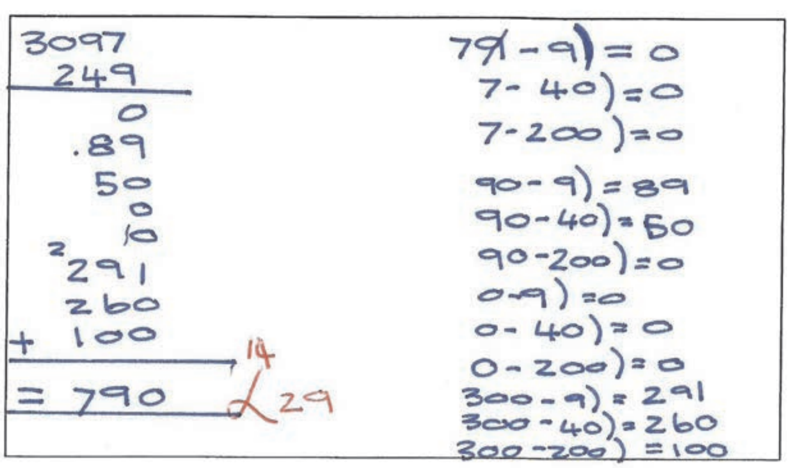


(4)

What would show evidence of no understanding?

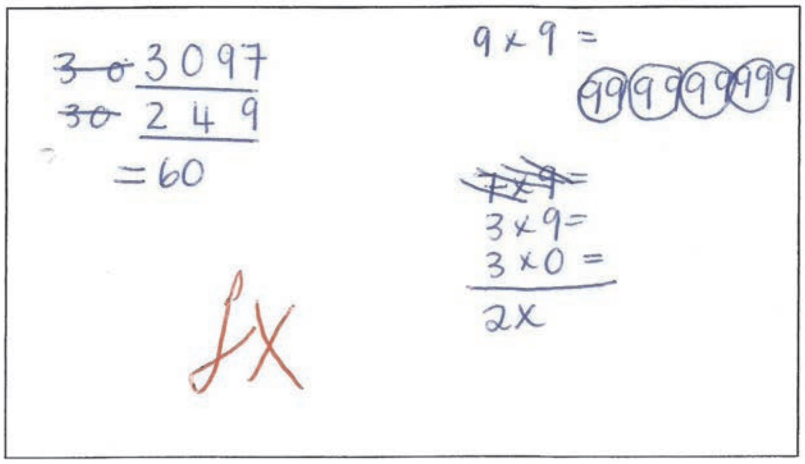
- In the example below the learner presented the work as multiplication, but no multiplication was done. The subtraction and final addition were done incorrectly.

5.3 3 097 x 249

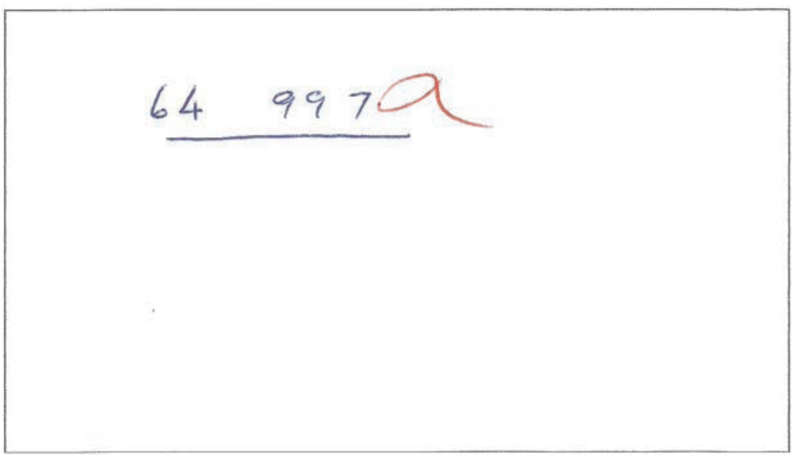


- In the following examples no reasoning or working is shown to justify the answers obtained.

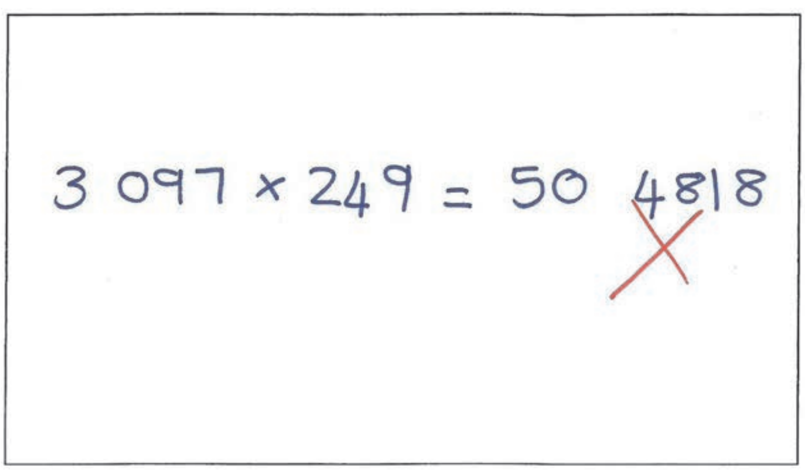
5.3 3 097 x 249



5.3 3 097 x 249

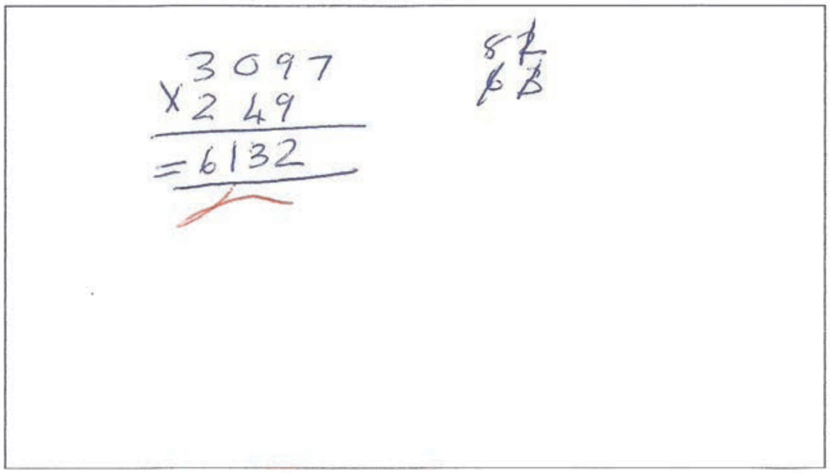


5.3 3 097 x 249



- In the last example the learner did not observe place value and multiplied incorrectly.

5.3 3 097 x 249



What do the item statistics tell us?

18% of learners answered the question correctly.

Factors contributing to the difficulty of the item:

- This is higher order multiplication involving a 4-digit number and a 3-digit number;
- Multiplication involved carry-overs;
- This area of the work schedule is supposed to be done in Term 4, hence it is possible that in some schools the concepts and skills required had not been adequately covered.

Teaching strategies

Learning basic multiplication

- Learning of multiplication tables of 1 to 9 can be done initially by repetitive addition of 2s, 3s, 4s, up to 9s.
- For example, this can be illustrated by finding the multiples of 9, which can be done by repetitive addition of 9 starting from 0.

i.e. 0 + 9 = 9
9 + 9 = 18
18 + 9 = 27
27 + 9 = 36
36 + 9 = 45 … this process continues.

- All this can be summarised in the multiplication table below.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

- Multiplication using multiples of 10 is easier to understand for many learners.
- It may be easier to start using this basic process before moving on to more difficult calculations.
- If learners become familiar with certain processes, relating them to new and unfamiliar situations will be easier for them.

Consolidation of multiplication methods for bigger numbers

- This can only be done after the basic multiplication tables are known thoroughly by learners.
- Place values need to be emphasised as they provide a basis for the direct method of column multiplication.
- Column multiplication can be done using any selection of numbers:
  - 2-digit numbers by 1-digit numbers,
  - 3-digit numbers by 1-digit numbers,
  - 3-digit numbers by 2-digit numbers,
  - Up to 4-digit numbers by 3-digit numbers (each time with and without carry-overs).

- There are many ways to record working when doing calculations.
- Multiplication can be recorded efficiently using place values and columns.
  - Place value is used when multiplying with bigger numbers, because multiplication has to be broken into steps.
  - Because multiplication is commutative we can multiply starting from the digit in the units place, or from a digit in another place.

- We show examples of multiplying starting with the digit in the units place as well as examples starting from the hundreds place.
- Learners can choose either method.
- Fluency in the method is essential and lots of practice is needed.

- We have provided an activity with many examples that you could use to give learners the necessary practice they need to consolidate their ability to multiply numbers.

Examples

- Two worked examples with explanations are given below.
- You should work through these and many other similar examples, explaining all of the steps as you go along so that your learners understand why and how the process works and not just how do to it.

1). Column multiplication: 3-digit number by 2-digit number starting with the units

	397
×	69
	3 573
+	23 820
	27 393

Multiplying by 69 is the same as multiplying by 60 + 9. One can multiply first by the unit 9 and then by 6 tens.

First we multiply the multiplicand 397 by the unit 9. The product of 397 and 9 is 3 573. We write this number starting from the left, observing the place values.

Next we multiply by 397 by 60. We first write 0 to represent multiplication by 10, then we multiply by 6. 397 multiplied by 6 is 2 382. From the left we write 0 first, then 2 382 to make it 23 820.

Finally we add 3 573 + 23 820 to get 27 393. Therefore 397 x 69 = 27 393.

2). Column multiplication: 4-digit number by 3-digit number starting with the units

	9 412
×	798
	75 296
	847 080
+	6 588 400
	7 510 776

In this example the multiplier is 798 (=700 + 90 + 8)

We start by multiplying by 8 units. 9 412 x 8 = 75 296.

Next we multiply the multiplicand 9 412 by 9 tens. We will write 0 just below 6 to represent multiplication by 10, then we multiply 9 412 by 9 (to get 84 708). The total product from the left will be 847 080.

We multiply further by 7 hundreds. We have to write down two zeroes first to represent multiplication by 100. We then multiply by 7. 9 412 x 7 = 65 884. The product, then of 9 412 by 700 will be 6 588 400.

The product of 9 412 by 798 is found by adding 75 296 + 847 080 + 6 588 400 which is 7 510 776.

3). Column multiplication: 3-digit number by 2-digit number starting with the hundreds

```
      5 6 7
  ×     2 9
  ---------
  1 1 3 4 0
+   5 1 0 3
  ---------
  1 6 4 4 3
```

Multiplying by 29 is the same as multiplying by 20 + 9. First we have to recognise the place values of 20 (tens) and 9 (units). Then we multiply by 1 digit at a time.

If we start from the left of the multiplier (29), then we start by multiplying by 20: we use 0 to represent the 'tens' in the multiplication and we multiply 567 by 2 because there are 2 tens.

The product of 567 and 20 is 11 340. This is written in the first row of multiplication.

Next we multiply 567 by the 9 (the units in the multiplier 29). This answer is 5 103 and is written in the second row of multiplication.

Finally, we must add the two answers obtained above to get the total of 567 x 20 and 567 x 9. The final answer is the sum of the 2 answer rows. It is 11 340 + 5 103 which equals 16 443.

4). Column multiplication: 4-digit number by 3-digit number starting with the hundreds

```
       8 349
  ×      789
  ----------
   5 844 300
     667 920
+     75 141
  ----------
   6 587 361
```

In this example the multiplier is 789. Again we have to recognise the place values first: 700 (hundreds) and 80 (tens) and 9 (units). (789 = 700 + 80 + 9)

We start by multiplying by 700. We use the 00s to signify the hundreds of this multiplier and we multiply by 7. The answer in the first row is the product of 8 349 and 700.

Next we multiply 8 349 by 80. We use 0 to signify the tens and we multiply by 8, which gives us 667 920. This is written in the second row of the multiplication.

The last digit that we use in the multiplication is 9, which is used directly, without zeros, (since it is in the units place, no hundreds or tens apply with this last digit).
9 x 8 349 = 75 141

The final answer will be the sum of the 3 products as calculated above.

Activity: Multiplication practice

1). 21 × 2	2). 34 × 2	3). 43 × 3	4). 71 × 5	5). 82 × 4
6). 25 × 4	7). 52 × 6	8). 63 × 5	9). 79 × 2	10). 99 × 7
11). 324 × 4	12). 112 × 9	13). 246 × 7	14). 926 × 3	15). 987 × 2
16). 753 × 10	17). 573 × 11	18). 434 × 22	19). 211 × 45	20). 113 × 33
21). 832 × 98	22). 567 × 29	23). 447 × 81	24). 977 × 55	25). 784 × 47
26). 4 413 × 28	27). 3 312 × 32	28). 2 112 × 43	29). 1 111 × 76	30). 3 213 × 31
31). 3 674 × 59	32). 9 220 × 46	33). 8 697 × 24	34). 1 977 × 26	35). 1 983 × 69
36). 3 321 × 231	37). 2 311 × 311	38). 1 221 × 443	39). 2 333 × 332	40). 1 234 × 123
41). 2 457 × 275	42). 8 349 × 789	43). 1 642 × 983	44). 3 098 × 394	45). 5 278 × 333

Solutions

1). 21 × 2 42	2). 34 × 2 68	3). 43 × 3 129	4). 71 × 5 355	5). 82 × 4 328
6). 25 × 4 100	7). 52 × 6 312	8). 63 × 5 315	9). 79 × 2 158	10). 99 × 7 693
11). 324 × 4 1 296	12). 112 × 9 1 008	13). 246 × 7 1 722	14). 926 × 3 2 778	15). 987 × 2 1 974
16). 753 × 10 7 530	17). 573 × 11 5 730 + 573 6 303	18). 434 × 22 8 680 + 868 9 548	19). 211 × 45 8 440 +1 055 9 495	20). 113 × 33 3 390 + 339 3 729
21). 832 × 98 74 880 + 6 656 81 536	22). 567 × 29 11 340 + 5 103 16 443	23). 447 × 81 35 760 + 447 36 207	24). 977 × 55 48 850 + 4 885 53 735	25). 784 × 47 31 360 + 5 488 36 848
26). 4 413 × 28 88 260 + 35 304 123 564	27). 3 312 × 32 99 360 + 6 624 105 984	28). 2 112 × 43 84 480 + 6 336 90 816	29). 1 111 × 76 77 770 + 6 666 84 436	30). 3 213 × 31 96 390 + 3 213 99 603
31). 3 674 × 59 183 700 + 33 066 216 766	32). 9 220 × 46 368 800 + 55 320 424 120	33). 8 697 × 24 173 940 + 34 788 208 728	34). 1 977 × 26 39 540 + 11 862 51 402	35). 1 983 × 69 118 980 + 17 847 136 827
36). 3 321 × 231 664 200 99 630 + 3 321 767 151	37). 2 311 × 311 693 300 23 110 + 2 311 718 721	38). 1 221 × 443 488 400 48 840 + 3 663 540 903	39). 2 333 × 332 699 900 69 990 + 4 666 774 556	40). 1 234 × 123 123 400 24 680 + 3 702 151 782
41). 2 457 × 275 491 400 171 990 + 12 285 675 675	42). 8 349 × 789 5 844 300 667 920 + 75 141 6 587 361	43). 1 642 × 983 1 477 800 131 360 + 4 926 1 614 086	44). 3 098 × 394 929 400 278 820 + 12 392 1 220 612	45). 5 278 × 333 1 583 400 158 340 + 15 834 1 757 574

Another example of how to test multiplication

ANA 2014 Grade 6 Mathematics Item 5.3

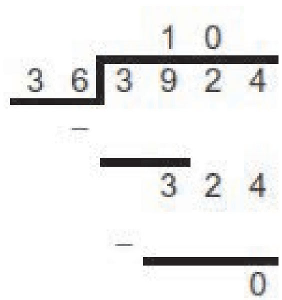
4.3	3 107 x 35	[3]

Notes:

Division

ANA 2013 Grade 6 Mathematics Items 5.4 and 11

5.4 Fill in the missing digits in the division sum.



[3]

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners? [3]

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 5.4

- Multiply 1-digit numbers by 2-digit numbers;
- Calculate multiples or repetitive additions of 36;
- Perform column addition and subtraction;
- Apply their knowledge of place values so that addition and subtraction becomes easier;
- Divide a 4-digit number by a 2-digit number;
- Perform the long division algorithm.

Item 11

- Divide 785 by 65 (division of 3-digit numbers by 2-digit numbers);
- Apply deductive reasoning;
- Convert word sentences to mathematical sums.

Where is this topic located in the curriculum? Grade 6 Term 4

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers.

Concepts and skills:

- Whole numbers: Division: long division, multiplication, subtraction;
- Whole numbers: Division: grouping and equal sharing with remainders.

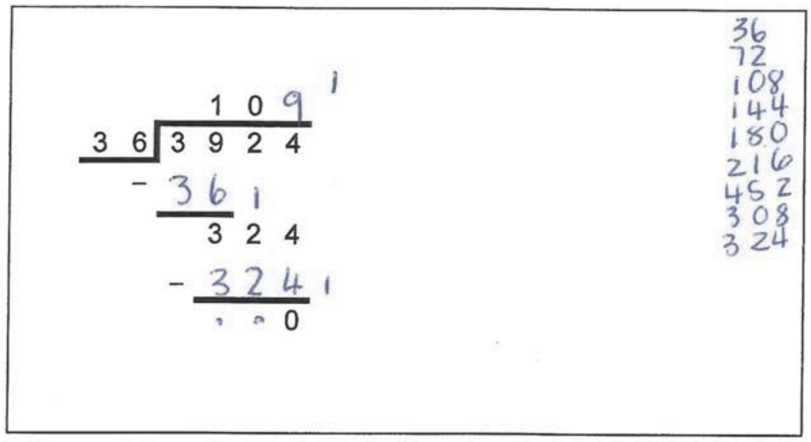
What would show evidence of full understanding?

Item 5.4

- The two examples that follow show full understanding.

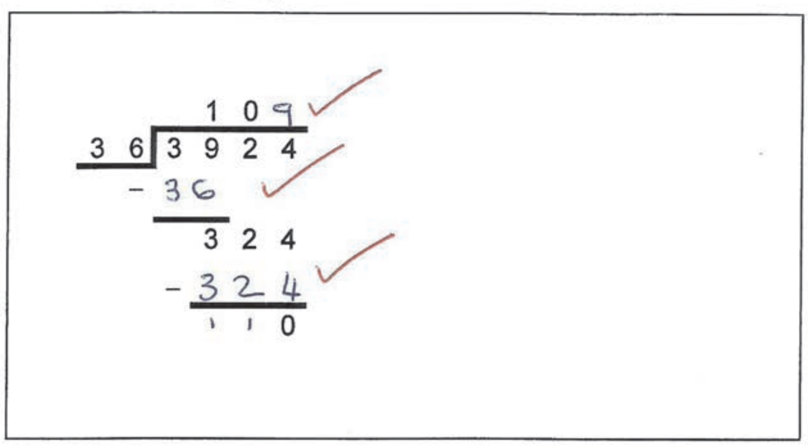
- In the first example the learner wrote multiples of 36 on the right-hand side of the figure and then used those to aid in multiplication. Place values were correctly arranged and the long division method correctly used. Computations were done correctly.
- In the second example the learner simply performed the correct long division method without first writing down the multiples of 36.

5.4 Fill in the missing digits in the division sum.



(3)

5.4 Fill in the missing digits in the division sum.

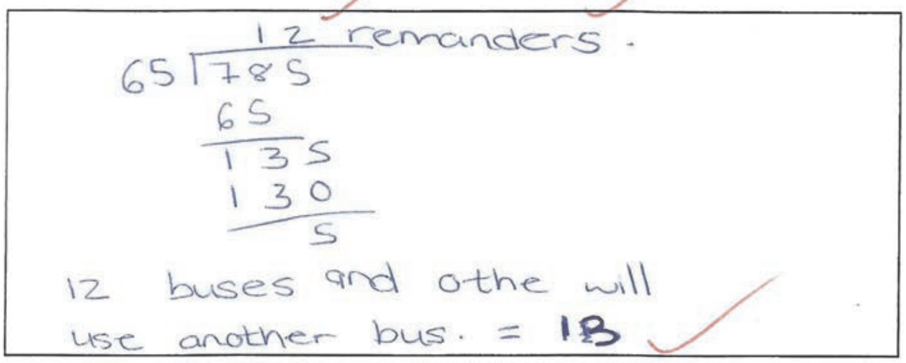


(3)

Item 11

- Full understanding is shown if the learner divided 785 by 65 to get 12 with remainder 5 and reasoned that since a bus can accommodate a maximum of 65 people 13 buses are needed.

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



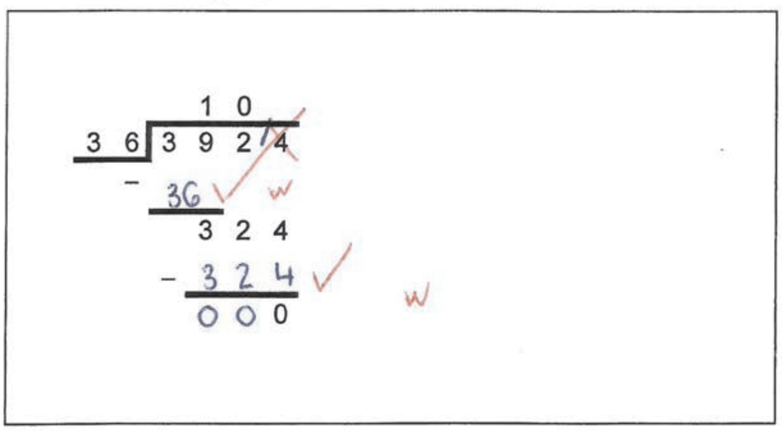
(3)

What would show evidence of partial understanding?

Item 5.4

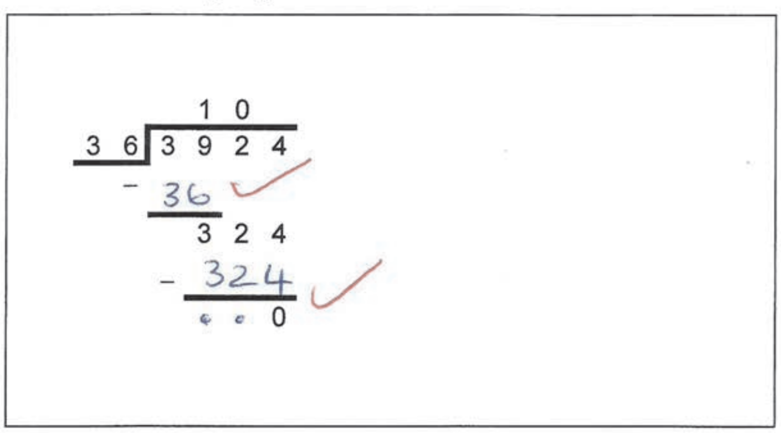
- In the next examples the learners might understand place values and know how to do the computations correctly, but failed to link the final answer to the workings done.

5.4 Fill in the missing digits in the division sum.



(3)

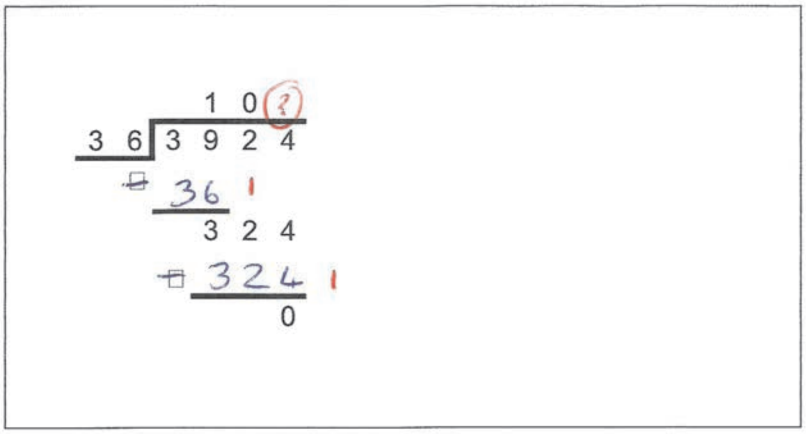
5.4 Fill in the missing digits in the division sum.



(3)

- In the following example the learner managed to multiply 36 x 1 to get 36 and also realised that 32 divided by 36 cannot be done and hence the need to bring down 4 to make the number 324. However, the learner failed to divide 324 by 36.

5.4 Fill in the missing digits in the division sum.

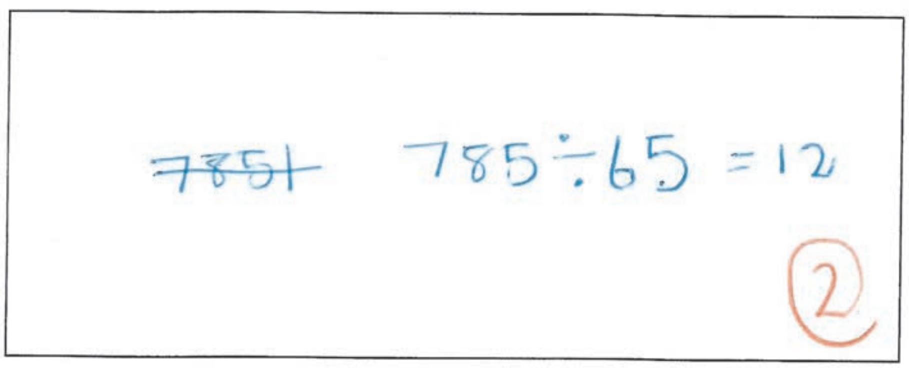


(3)

Item 11

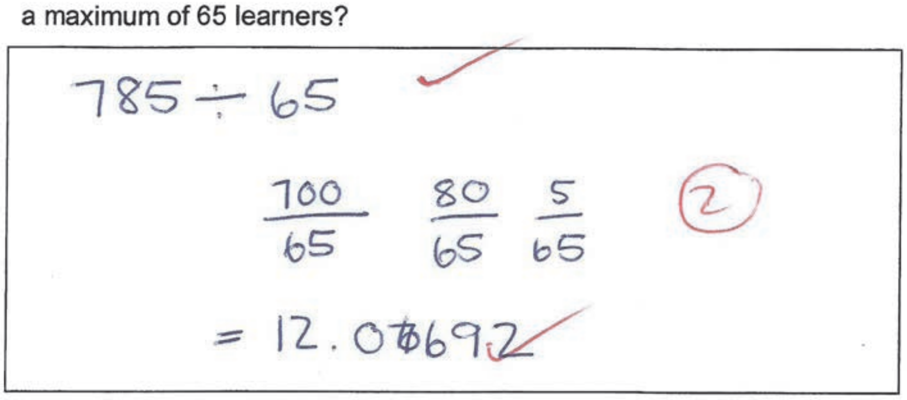
- In the next examples the learners divided 785 by 65 to get 12 remainder 5, but did not reach the final answer of 13 buses.

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



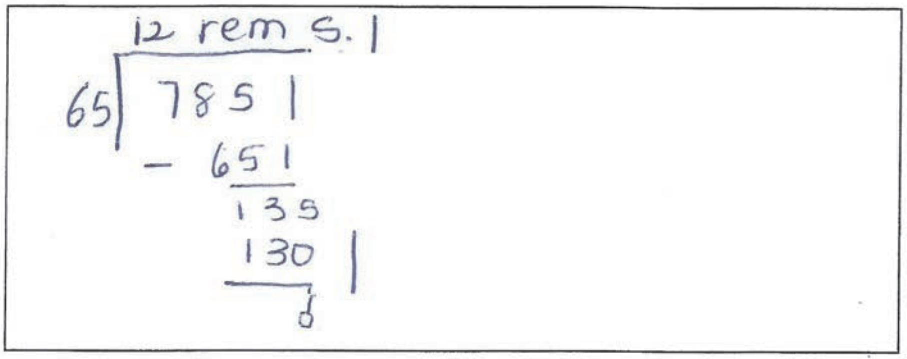
(3)

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



(3)

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



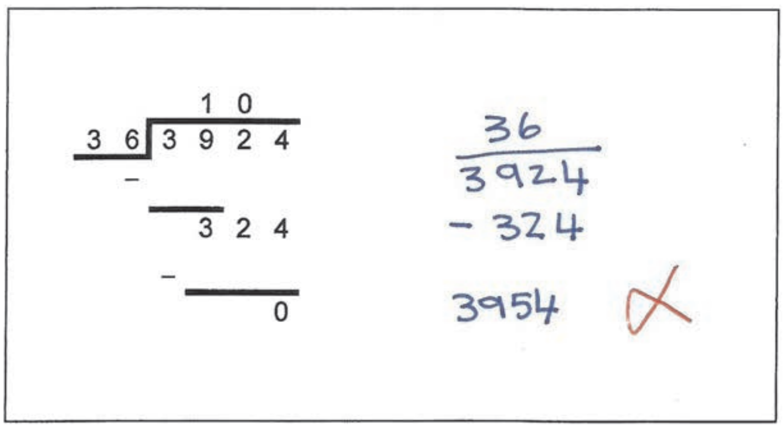
(3)

What would show evidence of no understanding?

Item 5.4

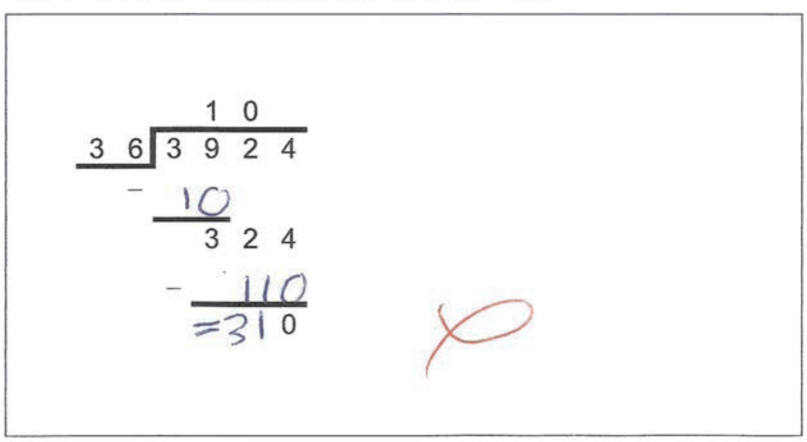
- If no understandable working is shown as in the following examples which demonstrate that these learners have very limited knowledge of the long division process.

5.4 Fill in the missing digits in the division sum.



(3)

5.4 Fill in the missing digits in the division sum.

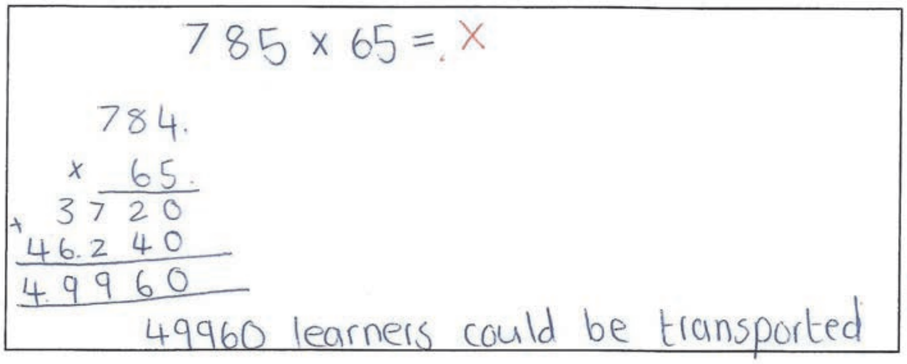


(3)

Item 11

- In this example the learner did not understand what to do with the two values 785 and 65 and multiplied instead of divided.

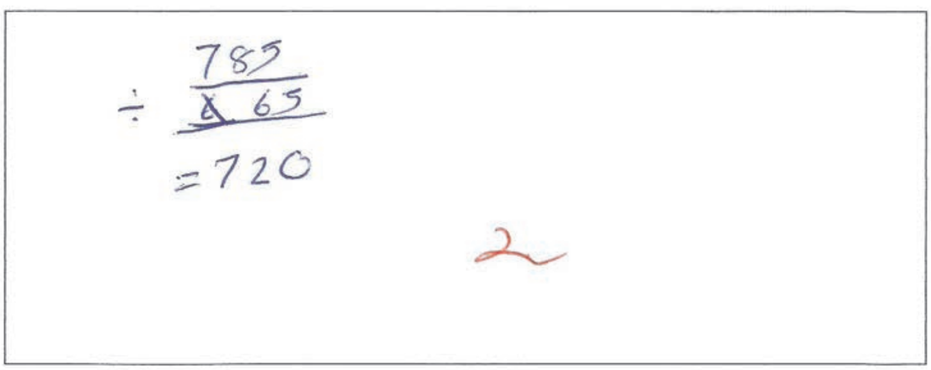
11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



(3)

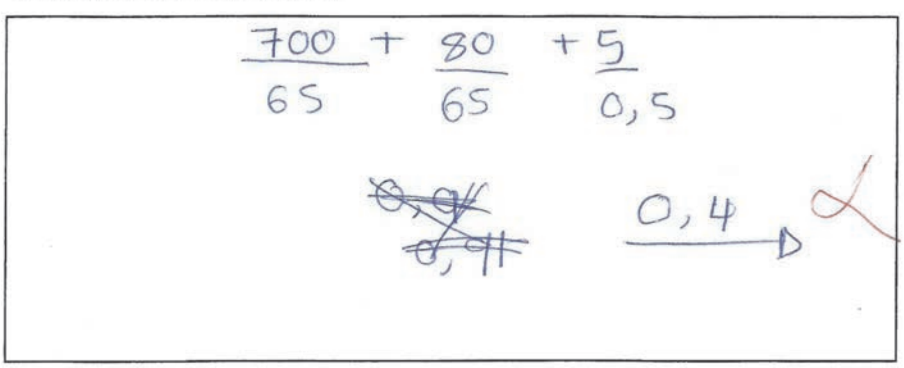
- The next examples demonstrate that some learners knew division was required, but did not know how to do it.

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



(3)

11. During a school trip 785 learners were transported in buses. How many buses were used to transport all the learners if each bus could transport a maximum of 65 learners?



(3)

What do the item statistics tell us?

Item 5.4

26% of learners answered the question correctly.

Item 11

13% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- This section of the work schedule is towards the end of Term 4, so coverage of this section in class might not have been sufficient.
- Some learners identified 10 in item 5.4 as the quotient and did not check to establish whether 10 was the correct answer. These learners may have assumed that the working shown was the working the required to find the final solution to the question.
- Learners may lack sound knowledge of the long division process.
- This was a higher order question which required deductive reasoning to find the correct answer.
- In Item 11, after dividing by 785 by 65 to get 12, learners did not know what to do with the remainder of 5. Some learners found it difficult to work out that an extra bus, the 13th bus, was needed to accommodate the remainder.

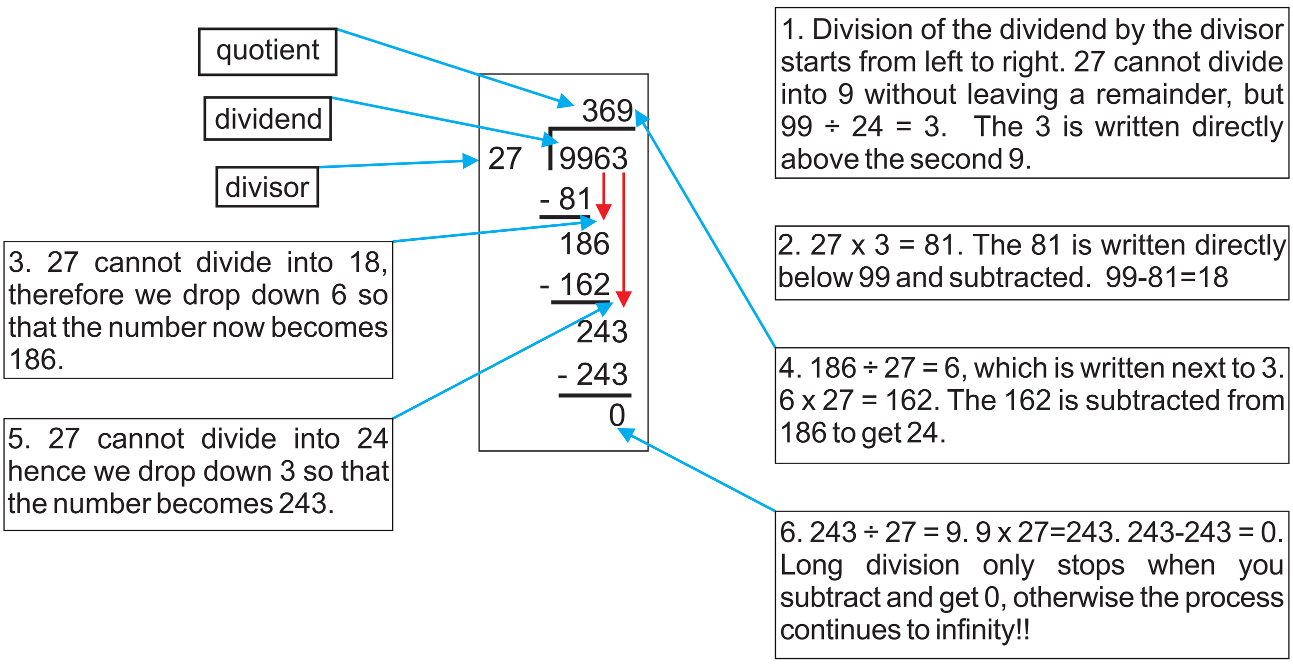
Teaching strategies

The long division process

- The long division process must be clearly explained to the learners and they must then be allowed to practise the process on their own and in small groups.
- Worksheets, using the following method, are always a good way to allow learners to develop the required skill in a topic.

Example

Consider 9 963 ÷ 27 using the process of long division.



- It is wise to allow learners to write multiples of the divisor first for easy reference during the long division process, thus:

The following nine multiples of 27 helped in answering the example above.

1	2	3	4	5	6	7	8	9
27	54	81	108	135	162	189	216	243

Activity: Long division

Answer the following questions using the long division process.

1). $12\overline{)420}$
2). $9\overline{)63}$
3). $15\overline{)210}$
4). $8\overline{)1128}$
5). $9\overline{)2124}$
6). $23\overline{)4025}$
7). $68\overline{)6327}$
8). $94\overline{)2924}$
9). $123\overline{)1567}$
10). $571\overline{)4282}$

Solutions

1). 35
2). 7
3). 14
4). 141
5). 236
6). 175
7). 93 r 3
8). 31 r 10
9). 12 r 91
10). 7 r 285

Problem solving: operations in context

- Solving problems is one of the best ways to exercise your brain. Mathematics is a subject that lends itself to problem-solving activities. We can exercise our own brains (and those of our learners) if we apply our minds to mathematical problems.
- "Word Problems" (often spoken about in maths) are branded by some people as impossible and not worth spending time on. This is an attitude that limits the potential of mathematics to positively impact the cognitive development of learners.
- Problem solving is not an activity which should be reserved for a few. We can all develop our skills in problem solving through following guidelines for the problem solving process and through perseverance. We must not expect to solve all problems in a few minutes: this would be unrealistic. Not all problems are so simple! Solving some problems requires deep thought and careful consideration.
- This is where problem solving and mathematical thinking train us for real life and equip us to deal with some of the real-life problems we are confronted with in everyday life.

- Problems need to be approached systematically and the working done to obtain the solution can be expanded and explained progressively, as suggested by the steps in the example that follows.

Example

In the following exercise which you can do with your learners, consider the steps which can guide you towards solving the problem successfully.

Bogotle Primary School has 3 Grade 7 classes with 31 learners in each class. There are only 2 Grade 4 classes, each with 52 learners. Are there more Grade 4 or Grade 7 learners? How many more?

Step 1:

Read the problem carefully and ensure that you understand what the problem is about.

- Restating the problem in your own words is a good exercise, which will make it clear to you whether or not you have understood the meaning of the problem.
- It is often a good idea to try and sketch a diagram that assists you to illustrate the problem. The problem is about the number of learners in two different grades at a school.
- The number of learners per class and the number of classes per grade are given.

Step 2:

Once you have understood what the problem is asking, you have to think of your strategy for solving the problem.

- Think about what operations you may have to use in the solution.
- Have you got all the information that you need in order to solve the problem; have you solved other similar problems which can guide your solution to the current problem?
- Can the problem be broken up into smaller parts if it seems too big to solve all at once?
- We need to use multiplication to work out the number of learners per grade first.
- We can compare the number of learners per grade to see which grade has the most learners.
- Then we will use subtraction to find out how many more learners there are in the grade with the most learners.

Step 3:

This step should not present any difficulties if step 2 has prepared you adequately to solve the problem.

- In this step you implement your problem-solving strategy to find the actual solution to the problem.
- It is important that you realise the difference between devising a strategy to solve a problem and finding the actual solution to the problem. Both are important activities.
- In finding the solution it will become clear whether you need to change your strategy or find a new one or if your original strategy was appropriate.
  - Grade 7 learners = 3 x 31 = 93
  - Grade 4 learners = 2 x 52 = 104
  - There are more Grade 4 learners.
  - Difference: 104 – 93 = 11. There are 11 more Grade 4 learners than Grade 7 learners.

Step 4:

Once you have solved the problem, a final "logic check" of your solution is never a waste of time. Careless errors can slip into your working (though your strategy may be correct) and lead you to an answer which is not correct.

- Re-read your work just to be sure that it makes sense and presents a valid, satisfactory solution to the problem.
- This step of verification may seem like a waste of time, but will often prove its usefulness when you make small changes and improvements to your answers.
- Cross check that numbers were written down correctly and the final answer is the answer required. All correct!

Activity: Contextual word problems

1). Usain Bolt is a male athlete who holds a 100 m sprint record of nine and fifty-eight hundredths seconds. Write this world record as a number.

2). Jane bought 5 loaves of bread. Only 2 loaves of bread can fit into one paper bag. How many paper bags does Jane need?

3). Thuto can only cycle 20 km per day. On which day does Thuto complete cycling 70 km?

4). About 7 classes at a certain school are going on a sports trip. That means the sports department must hire buses for all 226 learners and staff. Each bus can carry a maximum of 48 people. How many buses must the school hire?

5). The Feed a Child food initiative collected 360 cans of food. Your class needs to pack them into packs of 45 for each benefiting family.

a). How many families can benefit from the collection?

b). How many boxes will you need if each box takes a dozen cans?

6). You have twin sisters Thuto and Thato. On their birthday you buy Thuto 3 gifts costing R24 each and Thato 5 gifts costing R18 each. How much money did you spend on the two of them?

7). The average of the following 5 mathematics marks (which are all percentages) will be recorded as your Mathematics Term Mark: 68, 75, 86, 94 and 50. What mark will appear on your term report?

8). Four friends are eating four pizzas. After the meal, Thabo has $\frac{3}{4}$ left, Thomas had $\frac{3}{5}$ left, Thuto had $\frac{2}{3}$ left and Cindy had $\frac{4}{7}$ left. Who has more pizza left?

9). Tsakani has 50 marbles in his school bag. 30% of the marbles are blue. How many marbles are blue?

Solutions

1). 9,58 seconds.

2). $\frac{5\ loaves}{2\ loaves/paperbag}$ = 2,5 ($2\frac{1}{2}$) paper bags.

Since you cannot have half a paper bag 3 paper bags are needed.

3). $\frac{70\ km}{20\ km/day}$ = $3\frac{1}{2}$ days.

Thutho will complete 70 km on day 4.

4). $\frac{226\ people}{48\ people/bus}$ = 4 remainder 34.

4 buses will be full and another one will have 34 people in it.

Therefore 5 buses will be needed.

5). Can collection and packing:

a). $\frac{360\ cans}{45\ per\ family}$ = 8 .

8 families will be able to benefit, receiving 45 cans each.

b). $\frac{45\ cans/family}{12\ cans/dozen}$ = 3 remainder 9.

3 full boxes and one box with 9 cans in it need to be packed for each family. That means 4 boxes per family. 8 × 4 = 32.

That means that 32 boxes will be needed to pack all of the cans for all of the families.

6). Money spent on Thuto = 3 × R24 = R72,00

Money spent on Thato = 5 × R18 = R90,00

Total money spent = R72 + R90 = R162,00

7). Average = $\frac{68+75+86+94+50}{5}$ = $\frac{373}{5}$ = 74,6

The mark that appears will be 75.

8).

Thabo	Thomas	Thuto	Cindy
$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{4}{7}$

Writing all fractions over the same denominator:

Denominator = 4 × 5 × 3 × 7 = 420

$\frac{315}{420}$	$\frac{252}{420}$	$\frac{280}{420}$	$\frac{240}{420}$

Therefore Thabo has more pizza left.

9). $\frac{30}{100}$ × 50 = 15

15 marbles are blue.

Another example of how to test division

ANA 2014 Grade 6 Mathematics Items 4.1

4. Calculate the answers for Questions 4.1 – 4.8

   4.1 42 152 + 28 945 + 76 361 [2]

Notes:

Prime numbers

ANA 2013 Grade 6 Mathematics Item 3

3.	Write down the next prime number.	
	19 , 23 , 29 , __________	[1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Identify prime numbers;
- Understand that prime numbers have only two factors, namely 1 and the number itself.

Where is this topic located in the curriculum? Grade 6 Term 1 and Term 3

Content area: Numbers, Operations and Relationships.

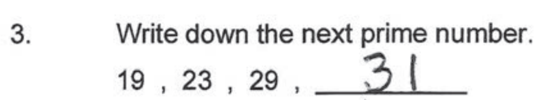
Topic: Whole numbers.

Concepts and skills:

- Represent prime numbers to at least 100.

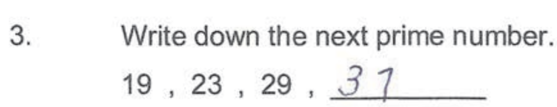
What would show evidence of full understanding?

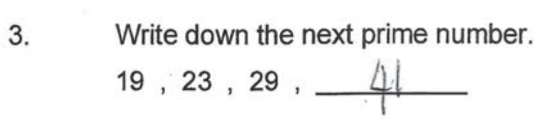
- If the learner gave the correct answer, 31.


3. Write down the next prime number.
19 , 23 , 29 , 31

What would show evidence of partial understanding?

- If the learner gave another prime number that comes after 29, for example 37, 41, etc., as the answer, we can consider this as a demonstration of partial understanding. This shows that the learner realised that another prime number was required, but gave a prime number that does not follow directly after 29.


3. Write down the next prime number.
19 , 23 , 29 , 37


3. Write down the next prime number.
19 , 23 , 29 , 41

What would show evidence of no understanding?

- If the learner gave a number that is not a prime number as the answer: this could indicate that the learner does not understand prime numbers or that the learner does not know his/her tables.

- In the example that follows the number 32 is even and thus not prime;

3. Write down the next prime number.
   19 , 23 , 29 , 32

- In the next example the number 35 is a multiple of 5 and thus not prime;

3. Write down the next prime number.
   19 , 23 , 29 , 35

- The number 33 is a multiple of 3 and thus is also not prime.

3. Write down the next prime number.
   19 , 23 , 29 , 33

- It is possible that the learner who gave 33 as the solution could have looked at the tens and unit digits and identified the pattern: 1**9** and 2**9** and then 2**3** and 3**3**. This shows an understanding of patterns but not of the pattern of prime numbers which was required to answer this question.

What do the item statistics tell us?

18% of learners answered the question correctly.

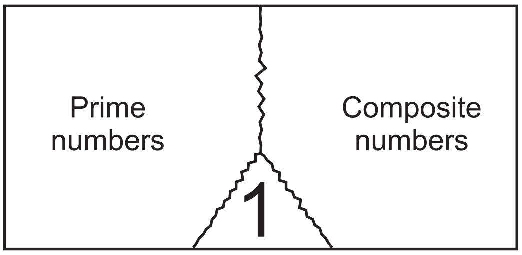
Factors contributing to the difficulty of the item

- Learners may lack knowledge of prime numbers.
- Learners may be unable to recall factors and multiples.
- Learners may have a weak knowledge of multiplication tables.

Teaching strategies

Understanding prime numbers

- A prime number is a number that has only two different factors, namely 1 and itself.
- If a number is not a prime number (or 1), then it is called a composite number.
- One is not a prime number, because it does not have two different factors.
- When we think about whole numbers in relation to their factors, we can divide them into three groups – as shown in the diagram:



- Here is the list of the 25 prime numbers smaller than 100. Grade 6 learners have to be able to identify all of them.

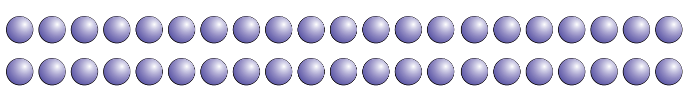
2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47; 53; 59; 61; 67; 71; 73; 79; 83; 89; 97

Rectangular arrays

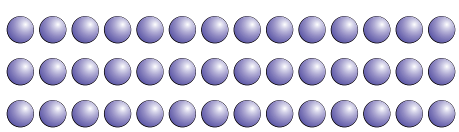
- Composite numbers can be represented in rectangular arrays with more than one row or column.
- Prime numbers cannot be represented as rectangular arrays with more than one row or column because they do not have any factors other than 1 and the number itself.

Examples

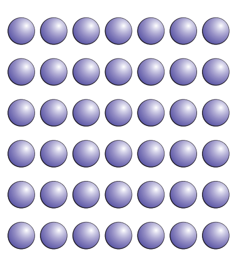
Look at the composite number 42. You can make the following arrays.



2 by 21 array

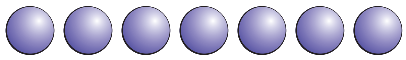


3 by 14 array

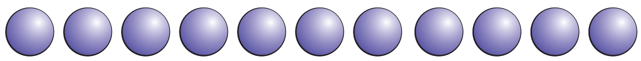


6 by 7 array

- Note that prime numbers cannot be represented in this type of rectangular array.



7 is a prime number. ONLY ONE ROW



11 is a prime number: ONLY ONE ROW

Activity: Identifying prime or composite numbers

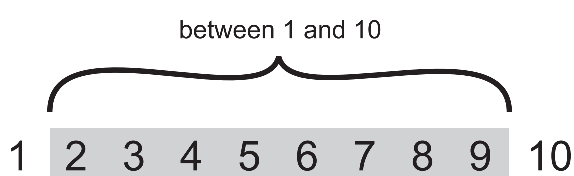
Call out the following numbers and ask learners whether the numbers are prime or composite. Ask the learners to explain their answers.

a). 12 (It is an even composite number, with factors, 2 ; 3 ; 4 ; 6)
b). 19 (It is a prime number)
c). 27 (It is a composite number and a multiple of 3)
d). 47 (It is a prime number)

Prime numbers between two other numbers

Explain to your leaners the meaning of the word "between".

Example



- The numbers between 1 and 10 are: 2; 3; 4; 5; 6; 7; 8; 9
- The prime numbers between 1 and 10 are: 2; 3; 5; 7

Activity

List the prime numbers between:

a). 10 and 20
b). 20 and 30
c). 30 and 40
d). 40 and 50
e). 50 and 60
f). 60 and 70
g). 70 and 80
h). 80 and 90
i). 90 and 100

Solutions

a). Prime numbers between 10 and 20 are 11; 13; 17; 19.

b). Prime numbers between 20 and 30 are 23; 29.

c). Prime numbers between 30 and 40 are 31; 37.

d). Prime numbers between 40 and 50 are 41; 43; 47.

e). Prime numbers between 50 and 60 are 53; 59.

f). Prime numbers between 60 and 70 are 61; 67.

g). Prime numbers between 70 and 80 are 71; 73; 79.

h). Prime numbers between 80 and 90 are 83; 89.

i). The prime number between 90 and 100 is 97.

Rules of divisibility for 2, 3, 4, 5 and 6

Rule of divisibility for 2

A number is divisible by 2 if it is even. Even numbers have 2, 4, 6, 8, or 0 as the unit digit.

Example

- 7 238 is divisible by 2 because it ends with a multiple of 2 in the unit place.

Rule of divisibility for 3

A number is divisible by 3 if the digits in the number add up to a multiple of 3.

Example

- 12 is divisible by 3 because 1 + 2 = 3
- 12 345 is divisible by 3 because 1 + 2 + 3 + 4 + 5 = 15 and again 1 + 5 = 6, which is a multiple of 3

Rule of divisibility for 4

A number is divisible by 4 if the last two digits are divisible by 4.

Example

- If you have a big number such as 564 738 212, you can simply look at the last two digits (12) and check if 4 goes into that number. 4 divides into 12, so 564 738 212 is divisible by 4
- You can also check that the number is divisible by 2 twice, for example,
- 1 026 ÷ 2 = 513, but 2 does not divide into 513, so 1 026 is not divisible by 4.

Rule of divisibility for 5

A number is divisible by 5 if it ends in a 0 or a 5. These numbers can be easily identified on the 100-chart.

Example

- 413 26**5** and 8 989 89**0** are divisible by 5 because they end with a 5 and a 0 respectively.

Rule of divisibility for 6

- A number is divisible by 6 if it is divisible by both 2 and 3. So you must check for divisibility by both 2 and 3.

Example

- The number 4 122 ends with an even number, so it is divisible by 2.
- When you add the digits: 4 + 1 + 2 + 2, you get 9 which is a multiple of 3, so it is divisible by 3.
- Therefore, 4 122 is divisible by 6.

Activity: Recognising different types of numbers

The table below contains some prime and some composite numbers. Tick the boxes to show the numbers' divisibility by 2, 3, 4, 5 or 6, or if the number is prime.

	Test for divisibility					
The number	**by 2**	**by 3**	**by 4**	**by 5**	**by 6**	**Prime**
a). 224						
b). 40						
c). 1 731						
d). 101						
e). 2 340						
f). 97						
g). 57						
h). 181						

Solutions

	Test for divisibility					
The number	**by 2**	**by 3**	**by 4**	**by 5**	**by 6**	**Prime**
a). 224	✓		✓			
b). 40	✓		✓	✓		
c). 1 731		✓				
d). 101						✓
e). 2 340	✓	✓	✓	✓	✓	
f). 97						✓
g). 57		✓				
h). 181						✓

The Sieve of Eratosthenes

The sieve of Eratosthenes is a method of filtering out the composite numbers less than 100 and leaving the prime numbers "in the sieve".

- Draw up a table with numbers 1 – 100 (a 100-chart).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Assist your learners to work through the following activity steps which act as the sieving process:
  - Cross out 1 (it is not prime)
  - Circle 2, and then cross out all the multiples of 2
  - Circle 3 and then cross out all the multiples of 3
  - Circle 5, and then cross out all the multiples of 5
  - Circle 7, and cross out all the multiples of 7 still remaining: 49; 77; 91
  - Circle all the numbers that were not crossed out
  - These will be the prime numbers between 1 and 100

Activity: Finding prime numbers

Ask questions referring to the 100 square, for example:

1). How many prime numbers are even?
(Only one prime number is even – 2 is the only even prime number).

2). Which row has the most prime numbers?
(Rows 1 and row 2: both have four prime numbers).

- Explain the words: "the most" and "the least" to the learners.

3). Which row has the least prime numbers?
(The bottom row: it has only one prime number).

4). Which column has the most prime numbers?
(Column 3: it has seven prime numbers).

5). Which columns have only one prime number? Can you explain why?
(Columns 2 and 5 have only one prime number, two in column 2 and five in column 5. The rest of the numbers in those columns are all multiples of either 2 or 5, that is, the numbers in column 2 are all even numbers and the numbers in column 5 are all multiples of 5).

6). Which columns have no prime numbers? Can you explain why?
(Columns 4, 6, 8 and 10 have no prime numbers. They contain even numbers, and/or multiples of 5).

Another example of how to test prime numbers

ANA 2014 Grade 6 Mathematics Item 1.2

1.2 What is the next prime number?

3 , 5 , 7 , ________

A 9
B 11
C 8
D 15 [1]

Notes:

Factors and multiples

ANA 2013 Grade 6 Mathematics Items 1.7 and 10

1.7 Which one of the following numbers is a factor of 81?

A 7
B 8
C 18
D 27 [1]

10. Will I count the number 104 if I count in multiples of 16 up to 160?

Answer YES or NO. ______________ [1]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Find the multiples and factors of a number;
- Factorise a number;
- Count in multiples of a number.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers – multiplication and division.

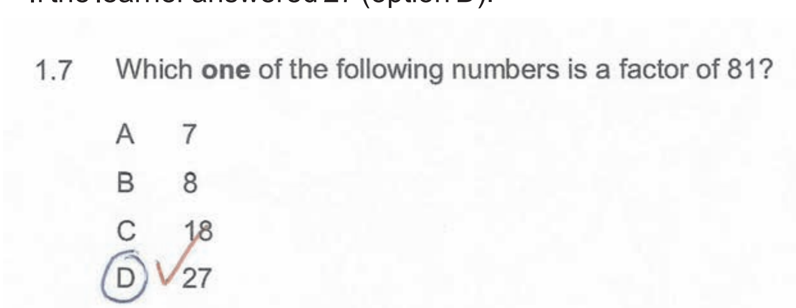
Concepts and skills:

- Factorisation.

What would show evidence of full understanding?

Item 1.7

- If the learner answered 27 (option D).

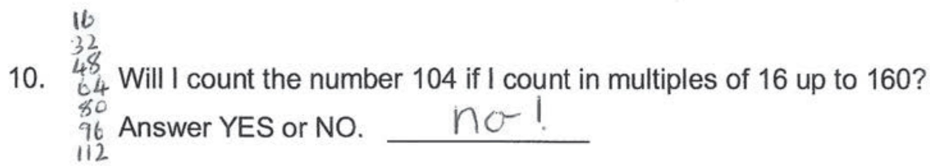


1.7 Which **one** of the following numbers is a factor of 81?

A 7
B 8
C 18
D 27

Item 10

- Since this was a yes/no question we can assume that if the learner answered “no” this shows full understanding. In the example shown, the learner wrote out the multiples of 16 up to 112 to illustrate the “counting in 16s” which does not include 104.



16
32
48
64
80
96
112

10. Will I count the number 104 if I count in multiples of 16 up to 160?
Answer YES or NO. no!

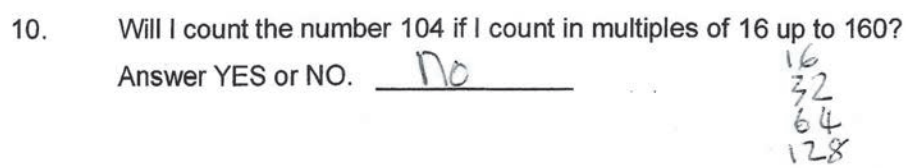
What would show evidence of partial understanding?

Item 1.7

- If the learner answered C (18) as both 81 and 18 are multiples of 9.

Item 10

- If the learner gave the correct answer “No”, but used the incorrect method. In the example below the learner doubled to get 16, 32, 64 and 128, instead of looking at the multiples of 16 namely 16, 32, 48, etc. Although the answer is correct, the learner did not test for divisibility by 16.



10. Will I count the number 104 if I count in multiples of 16 up to 160?
Answer YES or NO. No

16
32
64
128

What would show evidence of no understanding?

Item 1.7

- If the learner answered A or B as neither 7 nor 8 has any factors in common with 81.

Item 10

- If a learner gave "Yes" as an answer it shows that the learner either does not understand factors and multiples or does not know the multiplication tables.

What do the item statistics tell us?

Item 1.7

38% of learners answered the question correctly.

Item 10

45% of learners answered the question correctly.

Factors contributing to the difficulty of the items

Item 1.7

- Learners may be unfamiliar with the process of factorisation.

Item 10

- The wording of this question was somewhat different to what learners might hear in class ("count in 16's"). Learners are more familiar with questions such as “Is 104 a multiple of 16?” or “Is 16 a factor of 104?”

Teaching strategies

Understanding links between multiples and factors

- A factor is a number that can divide exactly into another number, leaving no remainder.
- For example, the factors of 25 are 1, 5 and 25.
- A number only has a limited number of factors.

- A multiple is a whole number product of another given number.
- For example, the multiples of 25 are 25; 50; 75; 100; 125; …
- A number can have an infinite number of multiples.

- Ask the following questions to give the class an opportunity to think about multiples and factors. Take time to discuss the answers.
- This is a thought-provoking exercise which will help learners to develop their mathematical reasoning skills.
- Some of the answers are simply true while others are more complex. In these cases the answer is “sometimes true” – which requires learners to understand that the particular statement is true in some instances and not in others.
- This activity involves higher level cognitive reasoning which will strengthen learners' ability to work on mathematical proofs.

Examples

1). If 9 is a factor of a number is 3 a factor as well? Explain.

Answer: This is always true.

Explanation

3 is a factor of 9 and 9 is a multiple of 3, so 3 will also be a factor of the number.

2). If 3 is a factor of a number is 9 a factor as well? Explain.

Answer: This is not always true.

Explanation

3 is a factor of 12, but 9 is not a factor of 12.

3). If 6 is a multiple of a number is 12 a multiple of the number as well? Explain.

Answer: This is always true

Explanation

6 is a factor of 12; 2 and 3 are factors of 6; and 12 is a multiple of 6: so 2 and 3 are also factors of 12.

Multiples of 2: 2; 4; **6**; 8; 10; **12**; 14; …

Multiples of 3: 3; **6**; 9; **12**; 15; …

4). If 16 is a multiple of a number is 12 a multiple of the number as well? Explain.

Answer: It is sometimes true.

Explanation

16 is a multiple of **8**: 8; **16**; 24; 32; …, but 12 is not a multiple of 8;

16 is a multiple of 4: 16 = 4 x 4 and 12 is a multiple of 4: 12 = 3 x 4.

5). If 12 is a multiple of a number is 6 a multiple of the number as well? Explain.

Answer: It is sometimes true.

Explanation

12 is a multiple of **4**: 4; 8; **12**; 16; 20; …, but 6 is not a multiple of 4;

12 is a multiple of 3: 12 = 3 x 4 and 6 is a multiple of 3: 6 = 3 x 2.

Factors, composite numbers and prime numbers

- We can use factors to categorise numbers.
- Some numbers have many factors, some only have a few. This is one thing that distinguishes types of numbers.
- A **prime number** is a number that has only two factors which are 1 and itself.
- A **composite number** is a number that has 3 or more factors.
- 1 is not a prime number because it has only one factor.
- The first 10 prime numbers are given below:

2; 3; 5; 7; 11; 13; 17; 19; 23; 29.

- These first 10 prime numbers are very important when it comes to prime factorisation.
- To find factors of a composite number we divide the number by prime numbers (prime factors) until there are no more factors.

Examples

1). Find the prime factors of 60

Prime factor	Number to be divided	
2	60	← 60 is an even number, so it can be divided by 2 to get 30.
2	30	← 30 is an even number, so it can be divided by 2 to get 15.
3	15	← 15 cannot be divided by 2 but can be divided by 3 to get 5.
5	5	← 5 cannot be divided by 2, or by 3. So we use the next prime number which is 5. 5 divided by 5 is 1.
	1	

Therefore, as a product of its prime factors 60 = 2 × 2 × 3 × 5

2). Find the prime factors of 340

Prime factor	Number to be divided	
2	340	← 340 is an even number, so it can be divided by 2 to get 170.
2	170	← 170 is an even number, so it can be divided by 2 to get 85
5	85	← 85 can not be divided by 2 or by 3, so we try 5. 85 ÷ 5 = 17
17	17	← 17 is a prime number. We divide by 17 to get 1
	1	

Therefore, as a product of its prime factors, 340 = 2 × 2 × 5 × 17

Activity: Factorising numbers into their prime factors

Factorise the following numbers into their prime factors.

a). 99

b). 15

c). 108

d). 63

e). 50

f). 600

g). 124

h). 525

Solutions

a). 99 = 3 × 3 × 11

b). 15 = 3 × 5

c). 108 = 2 × 2 × 3 × 3 × 3

d). 63 = 3 × 3 × 7

e). 50 = 2 × 5 × 5

f). 600 = 2 × 2 × 2 × 3 × 5 × 5

g). 124 = 2 × 2 × 31

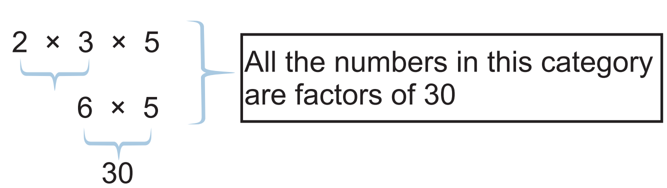
h). 525 = 3 × 5 × 5 × 7

Prime factors as building blocks

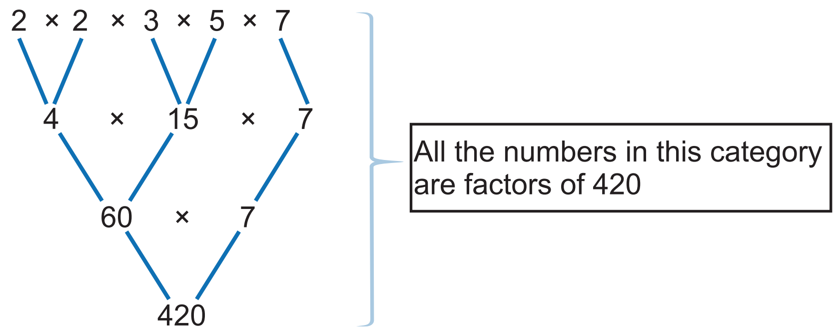
- Prime numbers are used to build composite numbers.
- Any number can be built from prime numbers.
- This is like the reverse operation of prime factorisation.

Examples

1). If, for instance 30 = 2 × 3 × 5 then 2 × 3 × 5 = 30. This can be written as



2). Now we can build another number from the following prime factors:
2, 3, 5, 5, 7. Thus:



- If we multiply any combination of the prime factors, we can get all the factors of 420.

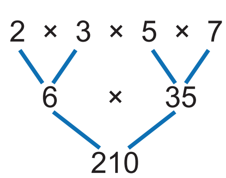
Building composite numbers from prime numbers

Examples

a). 2 × 3 × 5 × 7
b). 3 ×11 × 13
c). 2 × 3 × 2 × 2 × 17
d). 2 × 5 × 5 × 5 × 23
e). 2 × 2 × 2 × 2 × 5
f). 2 × 2 × 5 × 7 × 11
g). 11 × 11 × 13
h). 2 × 3 × 3 × 3 × 5 × 5
I). 2 × 2 × 2 × 2 × 3 × 3
j). 2 × 3 × 3 × 5 × 5 × 5 × 7
k). 2 × 3 × 7 × 11 × 13
l). 5 × 7 × 11 × 13 × 13 × 17

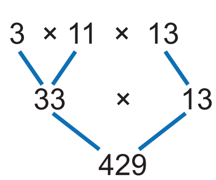
Solutions

a).



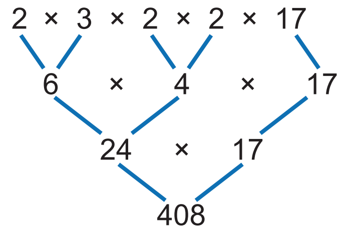
∴ The number is 210

b).



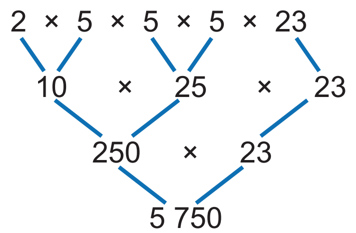
∴ The number is 429

c).



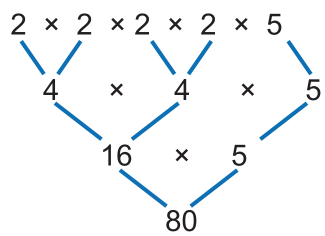
∴ The number is 408

d).



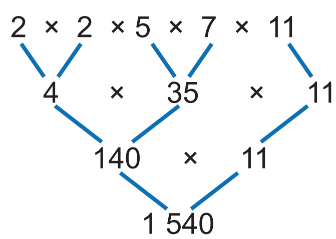
∴ The number is 5 750

e).



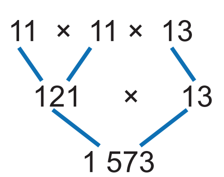
∴ The number is 80

f).



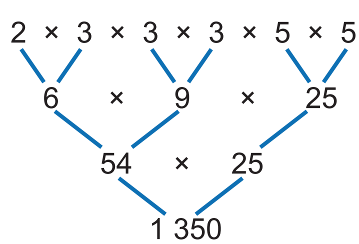
∴ The number is 1 540

g).



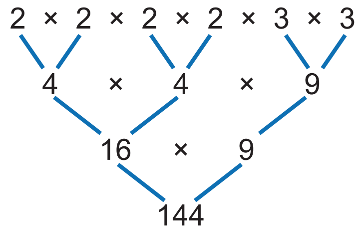
∴ The number is 1 573

h).



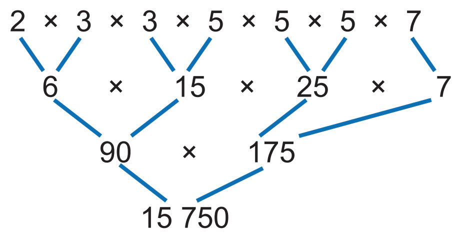
∴ The number is 1 350

i).



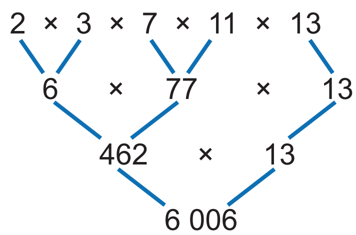
∴ The number is 144

j).



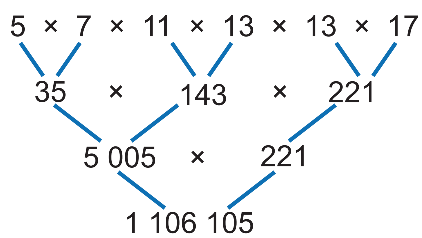
∴ The number is 15 750

k).



∴ The number is 6 006

l).



∴ The number is 1 106 105

Finding factors of a number

- In the two examples above we found a way to write composite numbers as prime factors and also how to build composite numbers from prime factors.
- In the following section we look at a different way to the find factors of a number.

Examples

1). Find the factors of 30

Step 1. Write 30 and its smallest factor, 1, at the opposite ends of a rectangular box or one-row table.

1							30

Step 2. After 1 we try out the next number, 2. 30 ÷ 2 = 15. Then we write these 2 factors into the table as shown.

1	2					15	30

Step 3. We then try out the next number, 3. 30 ÷ 3 = 10. We write 3 and 10 into the table.

1	2	3			10	15	30

Step 4. From 3 we go to 4. 4 cannot divide into 30 with no remainder. We try 5. 30 divided by 5 is 6. So we write 5 and 6 next in the boxes.

1	2	3	5	6	10	15	30

- The process stops here because there are no other numbers between 5 and 6.
- The factors of 30 then are:1; 2; 3; 5; 6; 10; 15 and 30

2). Find the factors of 50

Step 1. Write 50 and its smallest factor, 1, at the opposite ends of the table.

1							50

Step 2. Divide 50 by 2 to get 25. Enter 2 and 25 into the spaces as shown.

1	2					25	50

Step 3. The next number, 3, cannot divide into 50 with no remainder, so try 4. Four also does not work. Next is 5. If we divide 50 by 5 we get 10. We enter 5 and 10 into our 2 spaces as shown.

1	2	5			10	25	50

- There is no other number from 6 to 10 that can go into 50 without leaving a remainder.
- The factors of 50 then are: 1; 2; 5; 10; 25 and 50

3). Find the factors of 270

Step 1. Write 1 and 270 at opposite ends of a one-row table.

1																	270

Step 2. Divide 270 by 2 to get 135. Enter 2 and 135 into the table.

1	2															135	270

Step 3. Divide 270 by 3 to get 90. Enter 3 and 90 into the table.

1	2	3													90	135	270

Step 4. Four leaves a remainder when it divides into 270 so we consider 5. 270 ÷ 5 = 54. Enter 5 and 54 into the table.

1	2	3	5											54	90	135	270

Step 5. Consider 6. 270 ÷ 6 is 45. Enter 6 and 45 into the table.

1	2	3	5	6									45	54	90	135	270

Step 6. 7 and 8 both leave remainders, hence we go to 9. 270 ÷ 9 = 30. We enter 9 and 30 into the table.

1	2	3	5	6	9							30	45	54	90	135	270

Step 7. We consider the next number, 10. 270 ÷ 10 = 27. We enter 10 and 27 into the table.

1	2	3	5	6	9	10					27	30	45	54	90	135	270

Step 8. Numbers 11, 12, 13 and 14 all leave remainders when they divide into 270. We consider the next number, 15. 270 ÷ 15 = 18. We enter 15 and 18 into the table.

1	2	3	5	6	9	10	15			18	27	30	45	54	90	135	270

- There is no number between 15 and 18 that can divide into 270 without leaving a remainder.
- The factors of 27 are hence: 1; 2; 3; 5; 6; 9; 10; 15; 18; 27; 30; 45; 54; 90; 135 and 270.

Activity: Finding factors

Find the factors of the following numbers using the method described above.

a). 12

Factors of 12 are:

b). 35

Factors of 35 are:

c). 92

Factors of 92 are:

d). 174

Factors of 174 are:

e). 180

Factors of 180 are:

f). 267

Factors of 267 are:

g). 520

Factors of 520 are:

Solutions

a). 12

1	2	3					4	6	12

Factors of 12 are: 1; 2; 3; 4; 6 and 12.

b). 35

1	5							7	35

Factors of 35 are: 1; 5; 7 and 35

c). 92

1	2	4					23	46	92

Factors of 92 are: 1; 2; 4; 23; 46 and 92

d). 174

1	2	3	6			29	58	87	174

Factors of 174 are: 1; 2; 3; 6; 29; 58; 87 and 174

e). 180

1	2	3	4	5	6	9	10	12	15	18	20	30	36	45	60	90	180

Factors of 180 are: 1; 2; 3; 4; 5; 6; 9; 10; 12; 15; 18; 20; 30; 36; 45; 60; 90 and 180

f). 267

1	3							89	267

Factors of 267 are: 1; 3; 89 and 267

g). 520

1	2	4	5	8	10	13	20			26	40	52	65	104	130	260	520

Factors of 520 are: 1; 2; 4; 5; 8; 10; 13; 20; 26; 40; 52; 65; 104; 130; 260 and 520

Use prime factorisation to look for other factors: Checking for divisibility by using factors

If we know that a number is a factor of another number, then we will know that the second number is a multiple of the first.

Example

If we know that 16 is a factor of 104, then we will know that 104 is a multiple of 16.

Let us look at the ways in which we can check:

- If 2 is a factor of a number: the last digit must be even;
- If 4 is a factor of a number: 4 must divide into the last two digits;
- If 8 is a factor of a number: 8 must divide into the last three digits.

- So a quick way would have been to divide 104 by 8. If the quotient is even, then another 2 can divide into it and thus 16 would be a factor.
- So we can reason:

  104 ÷ 8 = 13, but 13 is not even, so 16 will not divide into 104.

Another way is to divide by 2 repeatedly:

2	104
2	52
2	26
13	13
	1

so, 104 = 2 × 2 × 2 × 13
= 8 × 13
So 16 is not a factor of 104

Activity: Using prime factorisation

Use prime factorisation to find out if:

a). 12 is a factor of 204

b). 18 is a factor of 234

Solutions

a).

$2 \times 2 \times 3 = 12$

2	204
2	202
3	51
17	17
	1

So, $204 = 2 \times 2 \times 3 \times 17$

Thus 12 is a factor of 204

b).

$2 \times 3 \times 3 = 18$

2	234
3	117
3	39
13	13
	1

So, $234 = 2 \times 3 \times 3 \times 13$

Thus 18 is a factor of 234

Other examples of how to test factors and multiples

ANA 2014 Grade 6 Mathematics Item 1.4

1.4 Which number is not a factor of 96?

A 32
B 16
C 48
D 36

[1]

ANA 2014 Grade 6 Mathematics Item 7

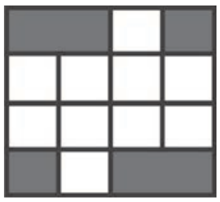
7. Write down the multiples of 7 between 21 and 56.

____ [1]

Fractions

ANA 2013 Grade 6 Mathematics Item 1.9

1.9 What fraction of the diagram is shaded?



A $\frac{1}{4}$

B $\frac{3}{8}$

C $\frac{1}{2}$

D $\frac{5}{8}$ [1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Identify fractions of a whole;
- Interpret the shaded parts of a whole;
- Count the number of subdivisions of the whole;
- Simplify fractions to the lowest form.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Numbers, Operations and Relationships.

Topic: Common fractions.

Concepts and skills:

- Fractions of whole numbers.

What would show evidence of full understanding?

- An answer of $\frac{6}{16}$ correctly reduced to $\frac{3}{8}$ (choice B).

What would show evidence of partial understanding?

- Incorrect conversion of the fraction to resulting in $\frac{1}{2}$, which is option C.

What would show evidence of no understanding?

- If the learner chose option A or D. However, it is difficult in the case of multiple choice questions to determine a learner's thought processes when the learner has not shown any working. Teachers can interview learners to find out what they were thinking when they worked out the answers and see if there was any partial understanding in the learners' reasoning.

What do the item statistics tell us?

32% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The learners may not know how to convert a fraction to its simplified form
- The dark shading in the diagram in the item makes it difficult for learners to identify how many individual blocks were shaded.
- The division of the shapes into parts and shading given in the question was unusual. This may have distracted learners who looked for something they expected to see – which in the case of eighths would be a shape divided into 8 parts (not 16 as in the item).

Teaching strategies

Fractions as part of a whole

- The whole, which can be in the form of a diagram or a real object, can be divided into smaller equal parts.
- We can use grids, circles, blocks or other illustrations to represent fractions.
- In the examples below, the fraction is represented as:

$$\frac{\text{Numerator}}{\text{Denominator}}$$

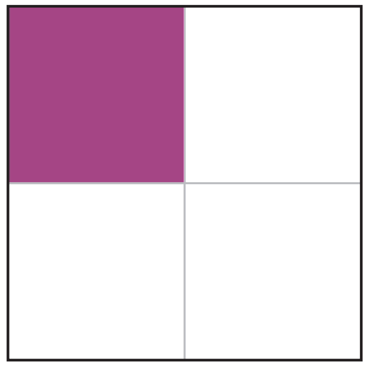
Numerator → The number of objects (or pieces) that are shaded or distinguished in some way.

Denominator → The total number of equal pieces which the whole was originally divided into.

Examples

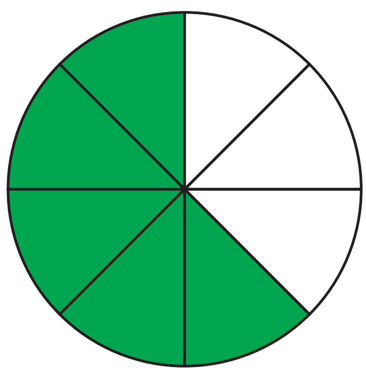
Consider the diagrams shown below.

1). What fraction of the whole diagram is shaded?



- The numerator is the number of parts shaded (= 1)
- The denominator is the total number of equal parts (= 4)
- Therefore the shaded fraction is $\frac{1}{4}$

2). What fraction of the diagram is shaded?



- We must find the numerator and the denominator.
- The numerator is represented by all the equal parts which are shaded.
- The number of parts shaded is 5.
- The denominator is all the parts which the circle is divided into, that is, the total number of parts that make up the whole circle.
- There are 8 parts in total.
- Therefore the fraction shaded is $\frac{\textit{number of parts shaded}}{\textit{total number of parts}} = \frac{5}{8}$.

Activity: Identifying fractions as part of a whole

Identify the fraction that is shaded in the diagrams below

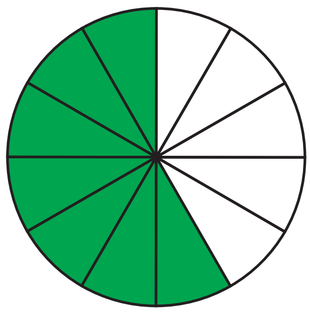
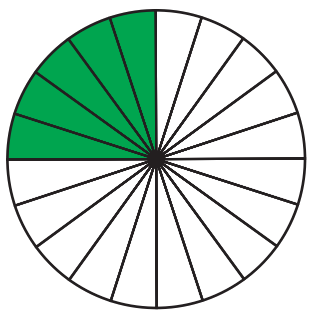
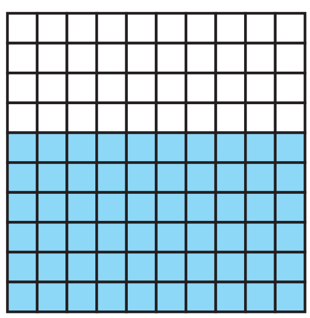
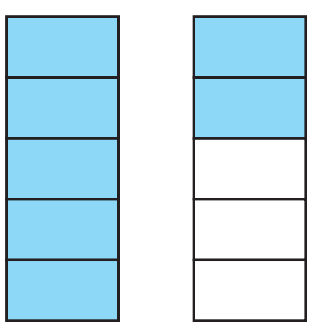
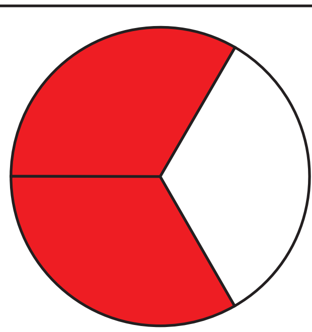
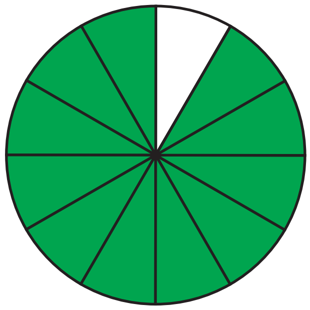
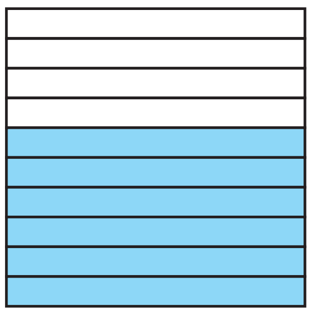
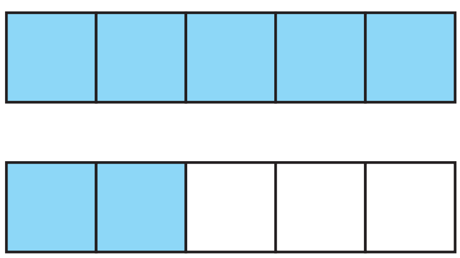
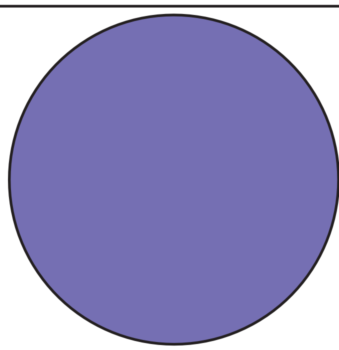
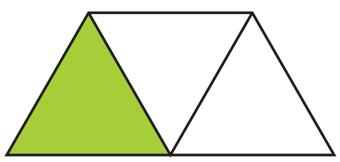
	Diagram	Fraction shaded
1).		
2).		
3).		
4).		
5).		

	Diagram	Fraction shaded
6).		
7).		
8).		
9).		
10).		

Solutions

1). $\frac{7}{12}$

2). $\frac{5}{20} = \frac{1}{4}$

3). $\frac{60}{100} = \frac{3}{5}$

4). $\frac{7}{5} = 1\frac{2}{5}$

5). $\frac{2}{3}$

6). $\frac{11}{12}$

7). $\frac{6}{10} = \frac{3}{5}$

8). $\frac{7}{5} = 1\frac{2}{5}$

9). 1

10). $\frac{1}{3}$

Reducing fractions to simplest form

- To reduce a fraction to its simplest form we have to divide both the numerator and the denominator by the highest common factor (HCF).
- Another way of saying simplest terms is to say "lowest form".
- The instruction to "simplify" a fraction means to put the fraction into the simplest form.
- For example: $\frac{3}{5}$ is in simplest form but $\frac{30}{50}$ is not.
- To simplify a fraction we find the HCF of the factors of the numbers making up the fraction (the numerator and the denominator).
- For example: $\frac{30}{50}$ has an HCF of 10 common to the numerator and the denominator. Using this HCF we can simplify $\frac{30}{50}$ to its simplest form, $\frac{3}{5}$.
- Simplification can also be done using any common factor and reducing the size of the fraction until the simplest form is reached.
- For example: $\frac{30}{50} = \frac{15}{25} = \frac{3}{5}$ (first divide by 2 which is common and then divide by 3 which is common.)

Examples

1). Consider the fraction $\frac{24}{32}$.
   - Factors of the numerator 24 are: 2; 3; 4; 6; 8; 12; 24.
   - Factors of the denominator 32 are: 2; 4; 8; 16; 32.

- From the listed factors we see that some of the factors are common: these are: 2; 4; 8.
- Of the common factors, 8 is the biggest. We call this biggest factor the Highest Common Factor (HCF).
- To reduce the fraction $\frac{24}{32}$ to the simplest form we divide both the numerator and the denominator by the HCF, 8.
- Thus $\frac{24 \div 8}{32 \div 8} = \frac{3}{4}$.
- Reduced to its simplest form $\frac{24}{32} = \frac{3}{4}$.

2). Consider the fraction $\frac{25}{75}$.
- Factors of the numerator 25 are: 5; 25.
- Factors of the denominator 75 are: 3; 5; 15; 25; 75.
- Common factors are: 5; 25.
- Highest Common Factor: 25.
- Thus $\frac{25 \div 25}{75 \div 25} = \frac{1}{3}$.
- When it is reduced to its simplest form $\frac{25}{75} = \frac{1}{3}$.

Activity: Simplifying fractions

Simplify the following fractions to the simplest form

	Fraction to be simplified	Factors	HCF	Simplified Fraction
a).	$\frac{16}{32}$			
b).	$\frac{240}{300}$			
c).	$\frac{32}{60}$			
d).	$\frac{10}{12}$			
e).	$\frac{50}{70}$			
f).	$\frac{63}{99}$			
g).	$\frac{33}{44}$			
h).	$\frac{35}{80}$			
i).	$\frac{75}{150}$			

Solutions

	Fraction to be simplified	Factors	HCF	Simplified Fraction
a).	$\frac{16}{32}$	**16**: 1; 2; 4; 8; 16 **32**: 1; 2; 4; 8; 16; 32	16	$\frac{1}{2}$
b).	$\frac{240}{300}$	**240**: 1; 2; 3; 4; 5; 6; 8; 10; 12; 15; 16; 20; 24; 30; 40; 48; 60; 80; 120;240 **300**: 1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 25; 30; 50; 60; 75; 100; 150; 300	60	$\frac{4}{5}$
c).	$\frac{32}{60}$	**32**: 1; 2; 4; 8; 16; 32 **60**: 1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 30; 60	4	$\frac{8}{15}$
d).	$\frac{10}{12}$	**10**: 1; 2; 5; 10 **12**: 1; 2; 3; 4; 6; 12	2	$\frac{5}{6}$
e).	$\frac{50}{70}$	**50**: 1; 2; 5; 10; 25;50 **70**: 1; 2; 5; 7; 10; 14; 35; 70	10	$\frac{5}{7}$
f).	$\frac{63}{99}$	**63**: 1; 3; 7; 9; 21; 63 **99**: 1; 3; 9; 11; 33; 99	9	$\frac{7}{11}$
g).	$\frac{33}{44}$	**33**: 1; 3; 11; 33 **44**: 1; 2; 4; 11; 22; 44	11	$\frac{3}{4}$
h).	$\frac{35}{80}$	**35**: 1; 5; 7; 35 **80**: 1; 2; 4; 5; 8; 10; 16; 20; 40; 80	5	$\frac{7}{35}$
i).	$\frac{75}{150}$	**75**: 1; 3; 5; 15; 25; 75 **150**: 1; 2; 3; 5; 15; 25; 75; 150	75	$\frac{1}{2}$

Addition of fractions

ANA 2013 Grade 6 Mathematics Item 5.5

5.5	Calculate the answer:	
	$4\frac{1}{8}+3\frac{3}{8}$	[2]

What should a learner know to answer this question correctly?

Learners should:

- Understand the meaning of a fraction;
- Know that only fractions "of the same kind" (with the same denominators) can be added;
- Be able to add mixed numbers.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Numbers, Operations and Relationships.

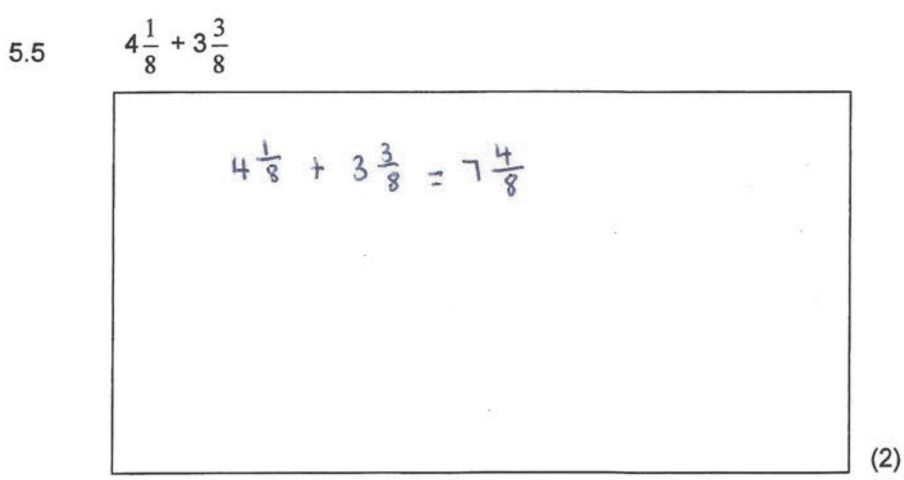
Topic: Common fractions.

Concepts and skills:

- Addition of fractions and mixed numbers.

What would show evidence of full understanding?

- The correct answer of $7\frac{4}{8}$ or $7\frac{1}{2}$ demonstrates full understanding. The answer could be obtained using a variety of strategies as shown in the examples that follow.


5.5 $4\frac{1}{8}+3\frac{3}{8}$

$4\frac{1}{8}+3\frac{3}{8}=7\frac{4}{8}$

(2)

5.5 $4\frac{1}{8} + 3\frac{3}{8}$

$4\frac{1}{8} + 3\frac{3}{8} = 7\frac{4}{8} = 7\frac{1}{2}$ ✓

(2)

- In the next example the learner wrote the mixed numbers as improper fractions, added the fractions correctly and left the answer as an improper fraction which he/she did not simplify.($\frac{60}{8} = 7\frac{4}{8}$)

5.5 $4\frac{1}{8} + 3\frac{3}{8}$

$4\frac{1}{8} + 3\frac{3}{8}$

$= \frac{4 \times 8 + 1}{8} + \frac{3 \times 8 + 3}{8}$

$= \frac{33}{8} + \frac{247}{8}$

$= \frac{60}{8}$

(2)

- The following example gives us insight into how this learner reasoned to get to the answer. He or she followed the procedure step by step and shows us each step of reasoning (for the whole numbers and the fractions) before giving the final correct answer.

5.5 $4\frac{1}{8} + 3\frac{3}{8}$

$4\frac{1}{8} + 3\frac{3}{8}$

$4 + 3 = 7$

$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

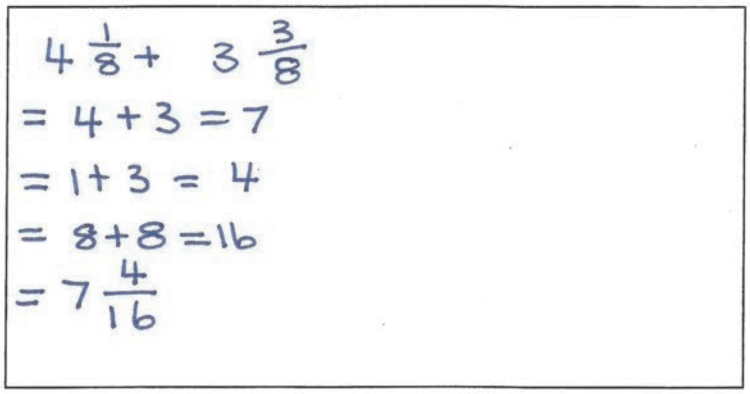
$= 7\frac{4}{8}$

(2)

What would show evidence of partial understanding?

- If a learner added the two whole numbers correctly, but not the fractions, the learner demonstrated partial understanding of the addition of mixed numbers;
- Many learners added the denominators as well as the numerators, a common error in the addition of fractions. This shows that learners have not yet fully understood how to deal with fractions when adding, although they do know how to deal with whole numbers.

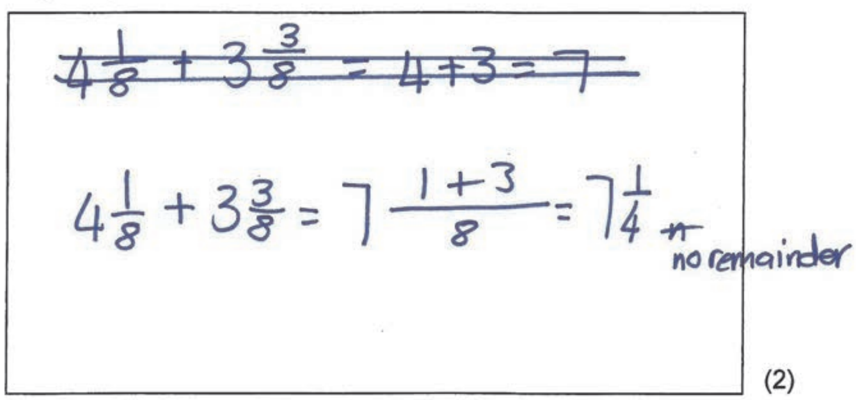
5.5 $4\frac{1}{8} + 3\frac{3}{8}$



(2)

- In the following example the learner used the correct algorithm, but made a mistake with the simplification of the fractions at the end.

5.5 $4\frac{1}{8} + 3\frac{3}{8}$

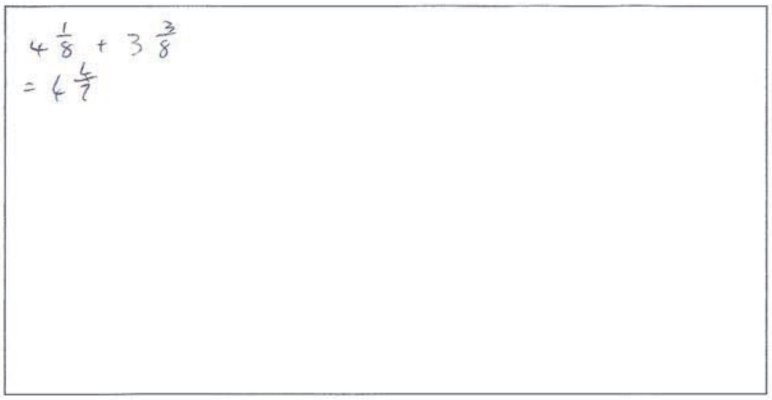


(2)

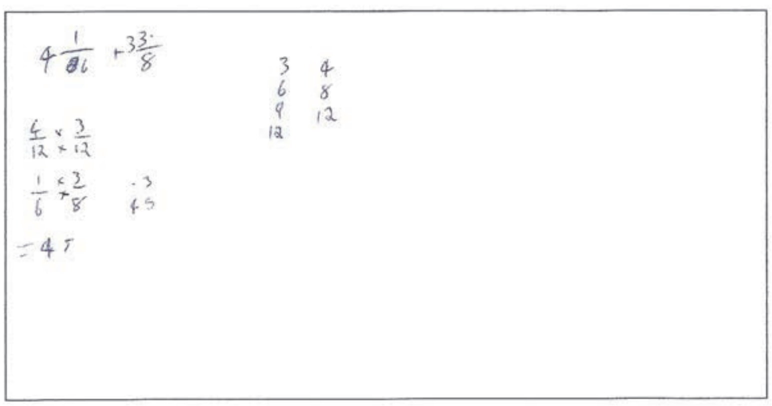
What would show evidence of no understanding?

- If the learner worked in a haphazard manner, without showing any method or pattern, this indicates no understanding. In the next two examples the learners demonstrated no understanding of the addition of fractions.

5.5 $4\frac{1}{8} + 3\frac{3}{8}$

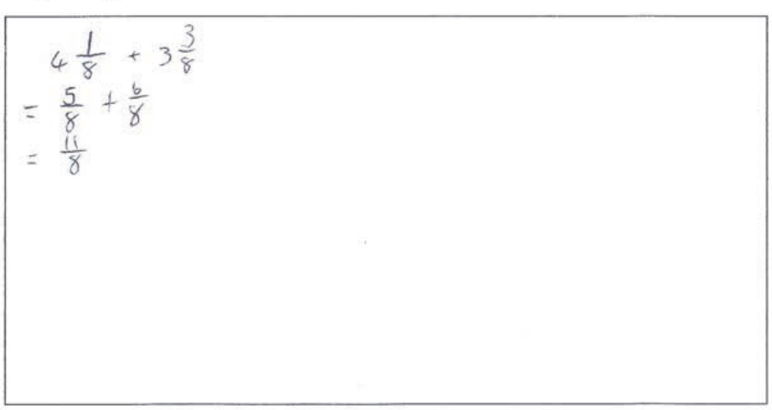


5.5 $4\frac{1}{8} + 3\frac{3}{8}$



- In the last example the learner added the whole number to the numerator in each case to get $\frac{5}{8}$ and $\frac{6}{8}$ respectively and then proceeded to add the numerators correctly, keeping the denominator. This learner may have partially remembered the rules for addition of fractions, but with little understanding.

5.5 $4\frac{1}{8} + 3\frac{3}{8}$



What do the item statistics tell us?

51% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may have the misconception that numerators as well as denominators must be added when adding fractions.
- Learners may have general difficulty working with fractions.

Teaching strategies

Probing learners' understanding of addition fractions

- You need to ask directed questions in order to find out what your learners know about how to add fractions.
- First you could ask broad questions so that the learners can explain their understanding of the question. For example, write on the board: $4\frac{1}{8}+3\frac{3}{8}$.
- Then ask: "Taki, tell us what you think about when you read this question"
- Possible answer: "When I read this question, I think about addition of fractions. I see that there are whole numbers and fractions. I also see that the two fractions are both eighths".
- Ask individual learners to explain how they would go about solving the problem. For example: "Zilla, tell us how you will find the answer"
- Possible answer: I know that I can add the whole numbers and then the fractions. I will add the whole numbers first, so I get 4 plus 3 which is seven. Both the fractions are eighths. And I can add one eighth to three eighths and get four eighths.

The language of fractions

- Learners often find it difficult to express the word "eighth" and may only say "eight". Teachers should stress the difference between these two words.
- Allow individual learners to say the words and explain what they understand them to mean.
- Ask questions such as:
  - If I divide a whole into 5 equal parts, each part is called a …..? (fifth)
  - If I divide a whole into 3 equal parts, each part is called a …..? (third)
  - If I divide a whole into 7 equal parts, each part is called a …..? (seventh)
  - If I divide a whole into 12 equal parts, each part is called a …..? (twelfth)
  - If I divide a whole into 8 equal parts, each part is called an …..? (eighth)
  - If I divide a whole into 18 equal parts, each part is called an …..? (eighteenth)
  - If I divide a whole into 9 equal parts, each part is called a …..? (ninth)
  - If I divide a whole into 2 equal parts, each part is called a …..? (half)
- Discuss the similarities and differences between the ways in which fraction words are written, for

example six**th**, fif**th**, third, half. Emphasise the pronunciation of the “th” which is at the end of many fraction names.

Estimation

- To help learners understand the addition of fractions, it is useful to let them do an estimation of the answer. Estimation is a powerful tool in helping the learners to focus on the task and on what should be done to get the answer.
- In the ANA item, $4\frac{1}{8} + 3\frac{3}{8}$, if learners had learned to estimate their answers before computing them, they would have been able to tell that the answer should be “seven and a bit”. This mental check would have made many of them check their written calculations and possibly arrive at the right answer.
- The mental check that learners need to carry out is to decide whether the fraction parts being added will come to more than, less than or be equal to 1.
- Learners can be asked the following questions to help them develop estimation skills when working with fractions.

Examples

1). When I add: $\frac{1}{8} + \frac{4}{8}$, will the answer be more than 1 or less than 1 or equal to one?

Illustrate the sum using fraction strips to help learners visualise the answer:

The illustration shows us we will have 5 small blocks coloured in in total – this is $\frac{5}{8}$ which is less than 1.

2). When I add: $\frac{3}{8} + \frac{5}{8}$ will the answer be more than 1 or less than 1 or equal to 1?

Illustrate:

The illustration shows us we will have 8 small blocks coloured in in total – this is $\frac{8}{8}$ which is equal to 1.

Activity: Will the following fraction sums be more than, less than or equal to 1?

a). $\frac{2}{8} + \frac{3}{8} =$

b). $\frac{3}{7} + \frac{5}{7} =$

c). $\frac{3}{6} + \frac{1}{6} =$

d). $\frac{4}{9} + \frac{5}{9} =$

e). $\frac{2}{5} + \frac{2}{5} =$

f). $\frac{3}{8}+\frac{5}{8}=$

g). $\frac{3}{4}+\frac{2}{4}=$

h). $\frac{4}{8}+\frac{6}{8}=$

Solutions

a). The answer will be less than 1. The answer is $\frac{5}{8}$

b). The answer will be more than 1. The answer is $\frac{8}{7}$

c). The answer will be less than 1. The answer is $\frac{4}{6}$

d). The answer will be equal to 1. The answer is $\frac{9}{9}$

e). The answer will be less than 1. The answer is $\frac{4}{5}$

f). The answer will be equal to 1. The answer is $\frac{8}{8}$

g). The answer will be more than 1. The answer is $\frac{5}{4}$

h). The answer will be more than 1. The answer is $\frac{10}{8}$

Using models to develop the fraction concept

- It is important for learners to experience a variety of models in fraction work because this concrete work will help them to develop and consolidate their understanding of the concepts.
- Learners in Grade 6 still rely on visual or concrete materials. You can use different models to demonstrate how the algorithm works, after which learners will be more ready to apply the algorithm abstractly.
- You should not use the algorithm for adding of fractions before learners understand how and why the algorithm works. The concrete models will help them to reach this understanding.

1). **Area model**

It is important to remember when using this model that the same size whole must be chosen. The whole must divided into parts of the same size.

Example

Draw the following diagrams on the board.

one-eighth three-eighths four-eighths

- Ask learners to explain to you what the diagram shows them. Discuss this as a class.
- Ask learners to write down the number sentence for the calculation illustrated in the area model.

Solution

$\frac{1}{8}+\frac{3}{8}=\frac{4}{8}$

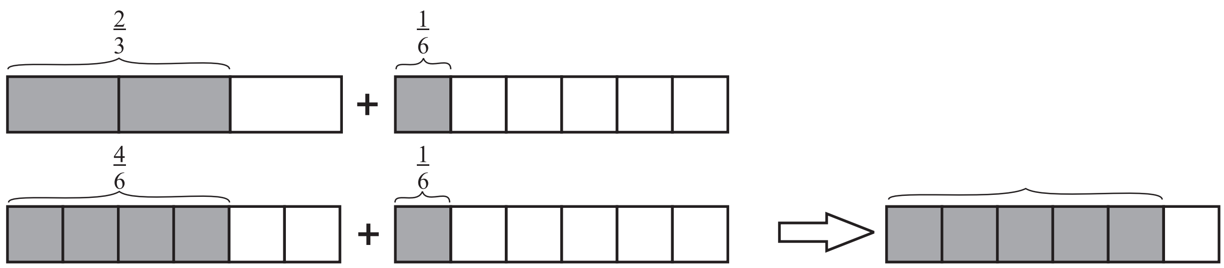
- Make sure that the learners write the fraction numerals correctly. Make sure that they pronounce the fraction names correctly.

Activity: Using the area model

Use fraction strips (which can be cut from a fraction wall) to work out other fraction additions, for example: $\frac{2}{3}+\frac{1}{6}=$

(Give learners a clue to convert thirds into sixths in order to illustrate and calculate this sum.)

Solution



In symbols: $=\frac{2}{3}+\frac{1}{6}$

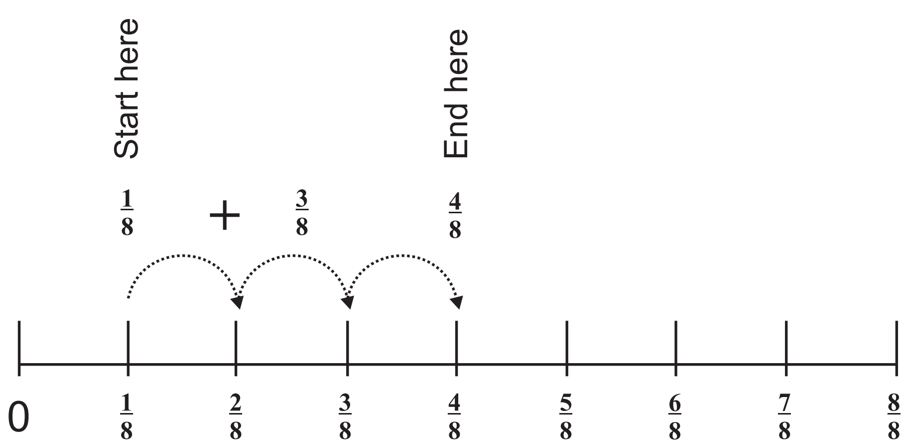
$=\frac{4}{6}+\frac{1}{6}$

$=\frac{5}{6}$

2). **Length model (working on a number line):**

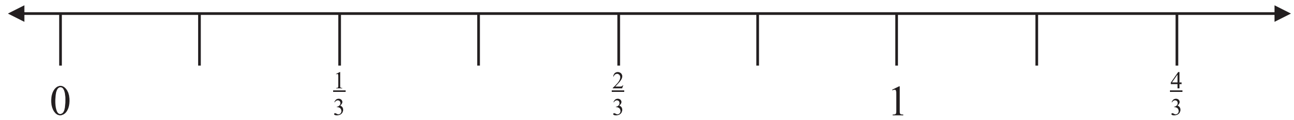
Illustrate on the number line $\frac{1}{8}+\frac{3}{8}=\frac{4}{8}$

- Show learners that you have to start on $\frac{1}{8}$ and then "count on" to three eighths.

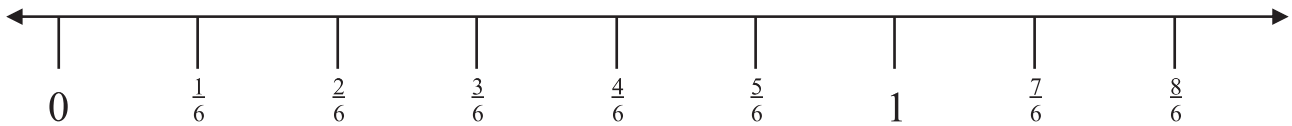


- It is very important that learners start seeing fractions on the number line.

Count in thirds

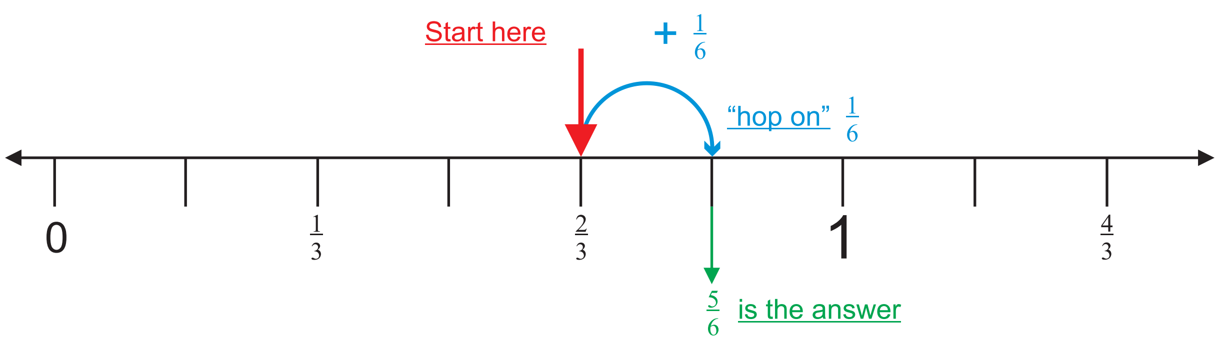


Count in sixths



Example

- Show learners how to add $\frac{2}{3}+\frac{1}{6}$ on the number line.
- Draw a number line demarcated in thirds. Ask learners to show where one would draw the sixths on the same number line.



Activity: Fractions on the number line

Show on the number line how the following fractions can be added:

a). $\frac{1}{2}+\frac{1}{4}$ (draw on number line marked in quarters)
b). $\frac{1}{3}+\frac{1}{6}$ (draw on number line marked in sixths)

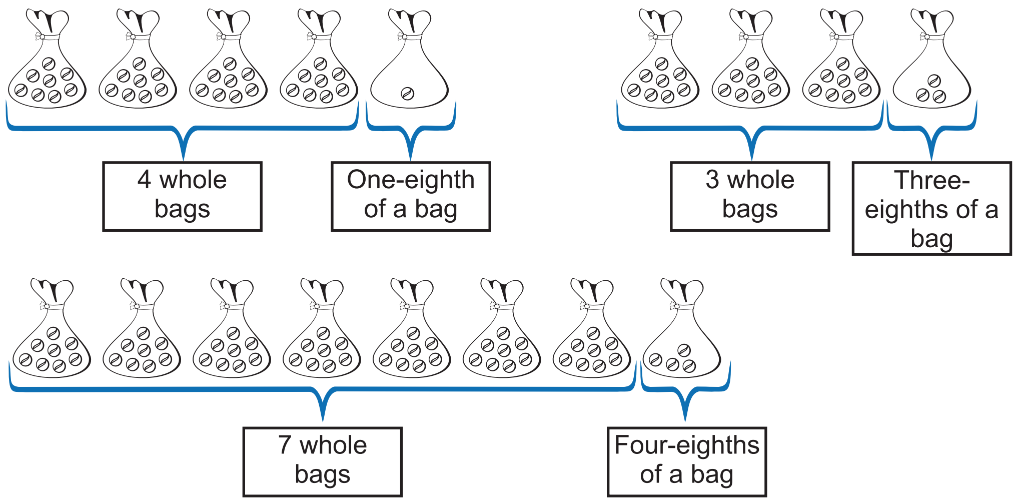
3). **Set model – working with counters.**

- A set model is used when we work with loose (separate) objects which form a whole.

Example

Add: $4\frac{1}{8} + 3\frac{3}{8}$

In this example we can use a bag of 8 marbles to form a whole.

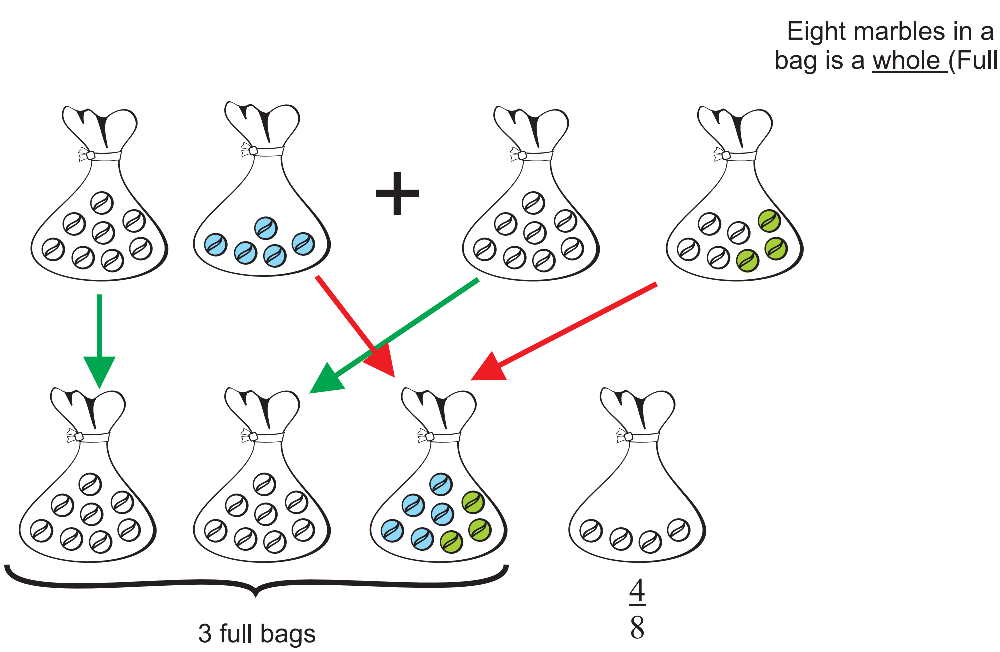


Activity: Using the set model

Consolidate this understanding by working through more examples.

Use the **set model** to show how you will add $1\frac{5}{8} + 1\frac{7}{8}$

Possible solution:



$1\frac{5}{8} + 1\frac{7}{8} = 3\frac{4}{8}$ or $3\frac{1}{2}$

In symbols: $1\frac{5}{8}+1\frac{7}{8}$

$=2+\frac{5}{8}+\frac{7}{8}$

$=2+\frac{12}{8}$

$=2+\frac{8}{8}+\frac{4}{8}$

$=3+\frac{4}{8}=3\frac{1}{2}$

Activity: Add the following fractions

a). $7\frac{7}{8}+1\frac{1}{8}$

b). $7\frac{7}{8}+1\frac{3}{8}$

c). $2\frac{2}{3}+1\frac{5}{6}$

d). $2\frac{2}{5}+3\frac{7}{10}$

Solutions

All of the steps are shown in the working below. If learners feel confident to leave out some of the steps this is fine. It is not necessary for all the steps to be shown, but enough steps must be shown to enable a reader to follow the learner's reasoning.

a). $7\frac{7}{8}+1\frac{1}{8}$

$=8+\frac{7}{8}+\frac{1}{8}$

$=8+\frac{8}{8}$

$=8+1$

$=9$

b). $7\frac{7}{8}+1\frac{3}{8}$

$=8\frac{7}{8}+\frac{3}{8}$

$=8\frac{10}{8}$

$=8+1+\frac{2}{8}$

$=9\frac{2}{8}=9\frac{1}{4}$

c). $2\frac{2}{3}+1\frac{5}{6}$

$=2+1+\frac{4}{6}+\frac{5}{6}$

$=3+\frac{9}{6}$

$=3+1+\frac{3}{6}$

$=4\frac{1}{2}$

d). $2\frac{2}{5}+3\frac{7}{10}$

$=5+\frac{4}{10}+\frac{7}{10}$

$=5+\frac{11}{10}$

$=5+\frac{10}{10}+\frac{1}{10}$

$=6\frac{1}{10}$

- Learners can use other methods to show their calculations. Some alternatives are provided below. Other methods may also be possible but not all of them can be illustrated here.

Alternative Method (using the Lowest Common Denominator (LCD) to add fractions)

a). $7\frac{7}{8}+1\frac{1}{8}$

$=\frac{7\times 8+7}{8}+\frac{1\times 8+1}{8}$

$=\frac{63}{8}+\frac{9}{8}$

$=\frac{72}{8}$

$=9$

b). $7\frac{7}{8}+1\frac{3}{8}$

$=7\frac{7}{8}+1\frac{3}{8}$

$=\frac{7\times 8+7}{8}+\frac{1\times 8+3}{8}$

$=\frac{63}{8}+\frac{11}{8}$

$=\frac{74}{8}$

$=9\frac{1}{4}$

c). $2\frac{2}{3}+1\frac{5}{6}$

$=\frac{2\times 3+2}{3}+\frac{1\times 6+5}{6}$

$=\frac{8}{3}+\frac{11}{6}$

$=\frac{16}{6}+\frac{11}{6}$

$=\frac{27}{6}$

$=4\frac{1}{2}$

d). $2\frac{2}{5}+3\frac{7}{10}$

$=\frac{2\times 5+2}{5}+\frac{3\times 10+7}{10}$

$=\frac{12}{5}+\frac{37}{10}$

$=\frac{61}{10}$

$=6\frac{1}{10}$

Another example of how to test addition of fractions

ANA 2014 Grade 6 Mathematics Item 5.5

5.5	$4\frac{3}{8} + 2\frac{1}{8}$	[2]

Notes:

Fractions and decimals

ANA 2013 Grade 6 Mathematics Items 5.6 and 12

5.6 Calculate the answer:

20% of 400 [2]

12 Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%		$\frac{1}{5}$
	0,75	$\frac{3}{4}$
5%	0,05	

[3]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Interpret the symbol %;
- Calculate a percentage of a given whole number;
- Understand the operation associated with the word "of";
- Convert percentages or common fractions to decimal fractions;
- Convert decimal fractions or common fractions to percentages;
- Convert percentages or decimal fractions to common fractions;
- Recognise equivalence amongst percentages, decimals and common fractions of the same number.

Where is this topic located in the curriculum? Grade 6 Term 4

Content area: Numbers, Operations and Relationships.

Topic: Common fractions.

Concepts and skills:

- Find percentages of whole numbers;
- Recognise and use equivalent forms of common fractions.

What would show evidence of full understanding?

Item 5.6

- The correct solution, 80, would show evidence of complete understanding, provided the learner performed the correct procedure.

5.6 20% of 400

$\frac{20}{100}$ of 400

$= (400 \div 100) \times 20$

$= 4 \times 20$

$= 80$

- In some instances learners obtained the answer of 80, but the operations the learner used are not clear, such as demonstrated in the following example. Teachers have to be aware of the process and not only the product.

5.6 20% of 400

20% of 400

$= (400 \div 20)$

$= 20 \times 4$

$= 80$

Item 12

- Correct conversions made, as shown in this answer.

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,20 ✓	$\frac{1}{5}$
75% ✓	0,75	$\frac{3}{4}$
5%	0,05	$\frac{1}{20}$ ✓

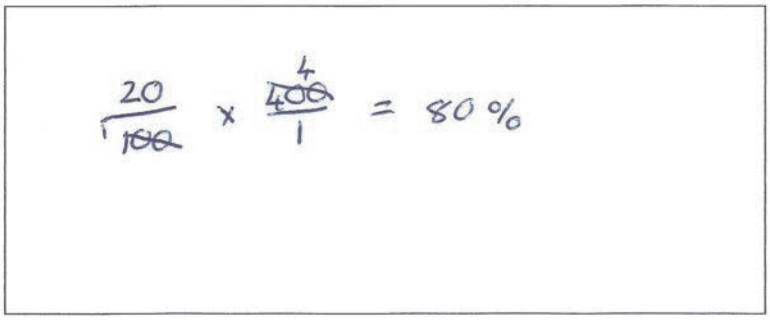
$\frac{3}{3}$ (3)

What would show evidence of partial understanding?

Item 5.6

- If a learner put a "%" sign in the solution, giving the answer as 80%: this shows partial understanding as the learner calculated correctly but does not understand that a percentage of a number is a fraction of that number.

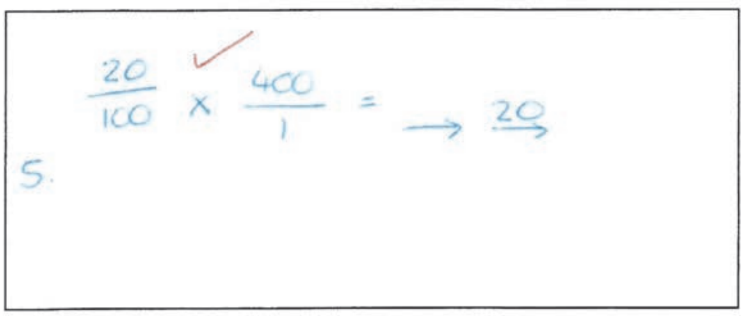
5.6 20% of 400



(2)

- If a learner wrote 20% as the fraction $\frac{20}{100}$ as shown in the next example, but did not proceed with the correct calculation, this also shows partial understanding.

5.6 20% of 400



Item 12

- In the next example the answers are correct, except that the common fraction $\frac{5}{100}$ is not in its simplest form.

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,20 ✓	$\frac{1}{5}$
75% ✓	0,75	$\frac{3}{4}$
5%	0,05	$\frac{5}{100}$ ✓

3 (3)

- In the following examples, only one or two of the three answers are correct.

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,12 ✗	$\frac{1}{5}$
75% ✓	0,75	$\frac{3}{4}$
5%	0,05	$\frac{1}{5}$ ✗

(3)

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,20 ✓	$\frac{1}{5}$
75% ✓	0,75	$\frac{3}{4}$
5%	0,05	$\frac{5}{10}$ $\frac{2}{5}$ ✗

$\frac{2}{3}$ (3)

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,25 ?	$\frac{1}{5}$
75% ✓	0,75	$\frac{3}{4}$
5%	0,05	$\frac{5}{100}$ ✓ $\frac{20}{100}$?

$\frac{2}{3}$ (3)

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,20 ✓	$\frac{1}{5}$
750% ✗	0,75	$\frac{3}{4}$
5%	0,05	$\frac{5}{2}$ ✗

(3)

What would show evidence of no understanding?

Item 5.6

- If a learner does not know that the percentage sign means "out of 100" then he or she does not understand the concept of percentage.

For example, this learner added the 20 to 400, to get 420.

5.6 20% of 400

~~20~~ 20 400 420

Item 12

- No conversions were done correctly, as in the three examples that follow.

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	$\frac{12}{20}$ ✗	$\frac{1}{5}$
50% ✗	0,75	$\frac{3}{4}$
5%	0,05	$\frac{3}{5}$ ✗

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	0,10 ✗	$\frac{1}{5}$
100% ✗	0,75	$\frac{3}{4}$
5%	0,05	$\frac{1}{5}$ ✗

12. Complete the table:

PERCENTAGE	DECIMAL FRACTION	COMMON FRACTION IN SIMPLEST FORM
20%	—	$\frac{1}{5}$
—	0,75	$\frac{3}{4}$
5%	0,05	—

(3)

What do the item statistics tell us?

Item 5.6

24% of learners answered the question correctly.

Item 12

50% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- The concept of percentage is new in Grade 6. It often does not get the attention it deserves;
- Learners do not know how to represent a percentage as a common fraction;
- Learners do not know the meaning of the word "of" in calculations;
- Learners do not understand how to find a part of the whole (for example $\frac{1}{5}$ of 400);
- The conversion from the decimal fraction 0,05 to the common fraction $\frac{1}{20}$ is not well understood by learners;
- Multiple conversions from percentages to decimals to common fractions in simplest terms were required.

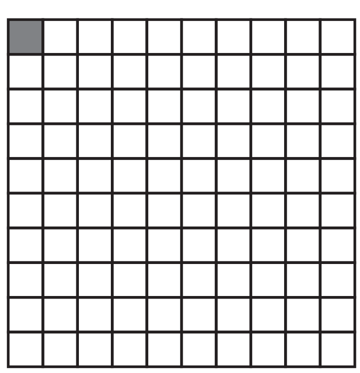
Teaching strategies

Understanding the concept of percentages

- The term per cent means "per hundred" or "out of a hundred". This is the definition of a percentage.
- How many cents are there in one rand? (There are 100 cents in a rand. One cent is one-hundredth of a rand.)
- What does the word "century" mean? (A century is one hundred years.)
- Percentages must be written with the correct percentage symbol, namely %.

- Ask the following questions:
- What does it mean if we say that 50% of the learners in the class are boys? It means that if there were 100 learners in the class, then 50 of them are boys.
- What does the symbol % mean? The symbol means "per hundred" or "out of 100".

- Show learners that a percentage is a part of a whole which can be divided into 100 equal parts.



- The whole is divided into 100 equal parts. Each block is one-hundredth of the whole.
- The shaded part is also called 1 percent and we write it as 1%.
- The shaded part in the diagram is 1% of the whole.

Example

Write the following percentages in fraction form and then write the fractions in their simplest form.

1). 10%
2). 50%
3). 25%
4). 20%
5). 75%

Solutions

Learners should be able to recall these without calculations.

1). 10% $= \frac{10}{100} = \frac{1}{10}$
2). 50% $= \frac{50}{100} = \frac{1}{2}$
3). 25% $= \frac{25}{100} = \frac{1}{4}$
4). 20% $= \frac{20}{100} = \frac{1}{5}$
5). 75% $= \frac{75}{100} = \frac{3}{4}$

A percentage as a fraction, a part of a whole

Example

Demonstrate the following calculation to learners.

20% of 400

- Calculating 20% of 400 can be done by first writing 20% as a common fraction: 20% $= \frac{20}{100} = \frac{1}{5}$
- To find 20% of 400 has the same meaning as to find $\frac{1}{5}$ of 400. This means one has to divide 400 into 5 equal parts. One part is then $\frac{1}{5}$ which is 20%.
  $\frac{1}{5}$ of 400 = 80
  20% of 400 = 80

Activity: Calculating percentages

Ask learners to calculate the following values.

1). 25% of 240
2). 30% of 300

Solutions

1). 25% $= \frac{25}{100} = \frac{1}{4}$
$\frac{1}{4}$ of 240 = 240 ÷ 4 = 60

2). $30\% = \frac{30}{100} = \frac{3}{10}$

$\frac{1}{10}$ of 300 $= 300 \div 10 = 30$

$\frac{3}{10}$ of 300 $= 30 \times 3 = 90$

Real-life contextual work with percentages

Ask the learners questions such as:

- If you are offered a 50% discount on a dress costing R100, what will you have to pay?
  - (Discount = 50% $= \frac{50}{100} = \frac{1}{2}$, half of R100 = R50. Price to be paid is R100 – R50 = R50)
  - If you are offered a 50% discount on a dress costing R200, what will you have to pay?
  - (Discount = 50% $= \frac{50}{100} = \frac{1}{2}$, half of R200 = R100. Price to be paid is R200 – R100 = R100)
  - What is 50% of 200?
  - (50% $= \frac{50}{100} = \frac{1}{2}$; half of 200 = 100; 50% of 200 = 100)
  - What is 50% of 400?
  - (50% $= \frac{50}{100} = \frac{1}{2}$; half of 400 = 200; 50% of 400 = 200)

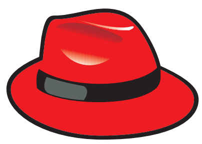
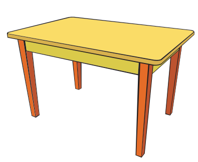
Once learners have understood the concept that 50% is the same as one half, move on to other percentages, such as 25% and 75%.

- If you are offered a 25% discount on a dress costing R300, what will you have to pay?
  - (Discount = 25% $= \frac{25}{100} = \frac{1}{4}$, a quarter of R300 = R300 ÷ 4 = R75. Price to be paid is R300 – R75 = R225)
  - If you are offered a 25% discount on a dress costing R400, what will you have to pay?
  - (Discount = 25% $= \frac{25}{100} = \frac{1}{4}$, a quarter of R400 = R400 ÷ 4 = R100. Price to be paid is R400 – R100 = R300)
  - What is 25% of 200?
  - (25% $= \frac{25}{100} = \frac{1}{4}$, a quarter of R200 = R200 ÷ 4 = R50)
  - What is 75% of 200?
  - (150)
  - What is 25% of 400?
  - (100)
  - What is 75% of 400?
  - (300)

Allow your learners the chance to work through examples like those just demonstrated by using an activity such as the one that follows.

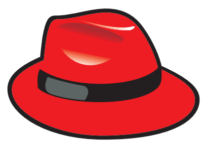
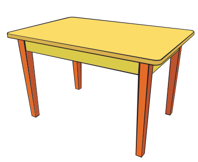
Activity: Percentage discount

A store has a sale. Calculate the prices of the following articles after the discount.

Price and discount	Solutions
Hats **R50**  Discount 10%	
Note books **R12**  Discount 50%	
Table **R400**  Discount 20%	
T-shirt **R80**  Discount 25%	

Solutions

These solutions show two different methods. Learners may use these or other methods. In one method more steps are shown, while in another less working is shown. Both are acceptable.

Price and discount	Solutions
Hats R50  Discount 10%	10% of R50 Use equivalent fractions to find the discount: $10\% = \frac{10}{100}$ Find the fraction (over 50) which is equivalent to $\frac{10}{100}$ ÷2 $\frac{10}{100} = \frac{[\ ]}{50}$ ⇨ [ ] = 5 ⇨ Discount R5 ÷2 The unknown number (making the equivalent fraction) is 5 So the discount is 10% of R50 = R5 New price: R50 – R5 = R45
Note books R12  Discount 50%	50% of R12 Ask: What is 50% of a number? 50% is the same as $\frac{1}{2}$ of the number? We need to find $\frac{1}{2}$ of R12 to find the discount. What is $\frac{1}{2}$ of R12 ⇨ Discount R6 New price: R6
Table R400  Discount 20%	20% of R400 Use equivalent fractions (as above): ×4 $\frac{20}{100} = \frac{[\ ]}{400}$ ⇨ [ ] = 80 ⇨ Discount R80 ×4 New price: R320
T-shirt R80  Discount 25%	25% of R80 Ask: What is 25% of a number? 25% is the same as $\frac{1}{4}$ of the number What is $\frac{1}{4}$ of R80 ⇨ Discount R20 New price: R80 – R20 = R60

Conversion of decimals to percentages

- When learners do this work they will build on their knowledge of the equivalence of decimals and percentages because of the relationship between decimals and percentages: percentages are hundredths.

  $0{,}01 = \frac{1}{100} = 1\%$

  Decimals are written using a decimal comma, while percentages are written using a % sign.

Examples

1). 3% = 3 hundredths $= \frac{3}{100} = 0{,}03$

- The easiest way to convert any given decimal to a percentage is to multiply by 100 (because percentages are hundredths) and then use the % sign.

2). Convert 0,4982 to a percentage.

- We have to multiply 0,4982 by 100, or simply move the decimal comma 2 places to the right (equivalent to multiplying by 100), thus:

0,4982 × 100 = 0 , 4 9 8 2 = 49,82%

The decimal place moves 2 places to the right, to between 9 and 8.

Notes:

Activity: Converting decimals to percentages

Convert the following decimals to percentages			
	Decimal	**Percentage**	**Solutions: Percentages**
1).	0,115	____________	11,5%
2).	0,025	____________	2,5%
3).	0,56	____________	56%
4).	0,34	____________	34%
5).	0,06	____________	6%
6).	0,25	____________	25%
7).	0,99	____________	99%
8).	0,51	____________	51%
9).	0,50	____________	50%
10).	0,01	____________	1%
11).	1,00	____________	100%
12).	0,10	____________	10%

Converting percentages to decimals

- To convert from percentages to decimals we divide the percentage given by 100 and we remove the percentage sign.
- This is the reverse of what was done in the first set of examples above and reinforces the relationship between percentages and decimals.

Example

Convert 25% to a decimal.

- Percent means per hundred – this means write the percentage as a fraction over a hundred. 25% means $\frac{25}{100}$.
- As for division by 100, we move the decimal place to the left by 2 places.

25% = 2 5 , 0 = 0,25

The decimal place moves 2 places to the left, to come before 2 so that the decimal number becomes 0,25.

Activity: Converting percentages to decimals

Convert the following percentages to decimals			
	Percentage	Decimal	Solutions: Decimal form
1).	32%	______________	0,32
2).	2,5%	______________	0,025
3).	89%	______________	0,89
4).	55%	______________	0,55
5).	67%	______________	0,67
6).	11%	______________	0,11
7).	49%	______________	0,49
8).	7%	______________	0,07
9).	13%	______________	0,13
10).	90%	______________	0,90
11).	76%	______________	0,76

Converting fractions to decimals

- Decimal fractions can be found by converting a given fraction into tenths or hundredths, as needed.
- You need to work though many examples with your learners to show them how this is done. Here are a few for you to use. Be sure to find more examples if your learners need more practice before they do the work on their own.
- Remember that at this level learners should be given examples that convert to tenths or hundredths using equivalent fractions. They should not be required to use their calculators to do the conversions.

Examples

1). Convert $\frac{3}{5}$ to a decimal.
Answer: $\frac{3}{5}=\frac{6}{10}=0{,}6$

2). Convert $\frac{20}{50}$ to a decimal.
Answer: $\frac{20}{50}=\frac{40}{100}=\frac{4}{10}=0{,}4$

3). Convert $\frac{9}{18}$ to a decimal.
Answer: $\frac{9}{18}=\frac{1}{2}=\frac{5}{10}=0{,}5$

- Another method that can be used is to divide the numerator by the denominator. Long division or short division may be used.

4). Convert $\frac{3}{5}$ to a decimal.

Answer: $\frac{3}{5} = 3 \div 5 = 0{,}6$

Activity: Converting fractions to decimals

Convert the following fractions to decimals			
	Fraction	**Decimal**	**Solutions: Decimal**
1).	$\frac{3}{6}$	____________	0,5
2).	$\frac{5}{20}$	____________	0,25
3).	$\frac{2}{25}$	____________	0,08
4).	$\frac{7}{56}$	____________	0,125
5).	$\frac{15}{30}$	____________	0,5
6).	$\frac{7}{28}$	____________	0,25
7).	$\frac{3}{15}$	____________	0,2
8).	$\frac{1}{4}$	____________	0,25
9).	$\frac{9}{10}$	____________	0,9
10).	$\frac{15}{100}$	____________	0,15
11).	$\frac{1}{8}$	____________	0,125
12).	$\frac{7}{2}$	____________	3,5
13).	$\frac{1}{5}$	____________	0,2
14).	$\frac{2}{20}$	____________	0,1

Converting from decimals to fractions in the simplest form

- The following method is used to convert from decimals to fractions in the simplest form.

Example

- Convert 0,35 to a fraction in its simplest form

$\frac{0{,}35}{1}$ — Re-write the decimal as a fraction by writing it over 1.

$\frac{35}{100}$ ← For every digit after the decimal (comma), multiply by 10. 0,35 has two digits after the comma, therefore we multiply by 100, both the numerator and the denominator

Factors of 35 = 1, 5, 7, 35
Factors of 100 = 1, 2, 4, 5, 10, 20, 25, 50, 100

HCF = 5

Find the Highest Common Factor (HCF) of the fraction by first finding the factors of both the numerator and the denominator then selecting the highest common factor.

$\frac{35 \div 5}{100 \div 5}$ — To simplify the fraction to its lowest terms, divide both the numerator and the denominator by the highest common factor.

$\frac{7}{20}$ ← This is the answer in simplified terms.

Activity: Convert the following decimals to fractions in their simplest form.

	Decimal	Fraction	Solutions: Fractions
1).	0,001	$\frac{1}{1000}$	$\frac{1}{1000}$
2).	0,25	______________	$\frac{25}{100}=\frac{1}{4}$
3).	0,7	______________	$\frac{7}{10}$
4).	0,08	______________	$\frac{8}{100}=\frac{2}{25}$
5).	0,6	______________	$\frac{6}{10}=\frac{3}{5}$
6).	0,46	______________	$\frac{46}{100}=\frac{23}{50}$
7).	0,35	______________	$\frac{35}{100}=\frac{7}{20}$
8).	1,35	______________	$\frac{135}{100}=\frac{27}{20}$
9).	2,5	______________	$\frac{25}{10}=\frac{5}{2}$

Other examples of how fractions and decimals can be tested

ANA 2014 Grade 6 Mathematics Item 4.6

4.6 $\frac{2}{5}$ of 300 [2]

ANA 2014 Grade 6 Mathematics Item 10

10 $\frac{1}{4}$ 75% 0,5

Match each of the three numbers given below with a number in the above frame.

10.1 $\frac{3}{4}$ = __________ [1]

10.2 50% = __________ [1]

10.3 0,25 = __________ [1]

Decimals and operations on decimals

ANA 2013 Grade 6 Mathematics Items 5.7 and 13

5.7.	Calculate the answer:	
	7,83 + 5,39 - 4,86	[3]
13.	Write the following decimal numbers from the smallest to the biggest:	
	4,24 ; 42,4 ; 0,42	[1]

What should a learner know to answer these questions correctly?

Learners should be able to

- Understand the concept of place value (in particular for decimal numbers) and be able to determine the units, tenths and hundredths of a given number;
- Interpret the values of the digits in a number (e.g. 0,4 is 4 tenths);
- Arrange decimals in ascending order;
- Add and subtract decimal fractions;
- Use the vertical algorithm when working with decimal numbers;
- Line up decimal numbers so that the place values coincide.

Where is this topic located in the curriculum? Grade 6 Term 2

Content area: Numbers, Operations and Relationships.

Topic: Whole numbers.

Concepts and skills:

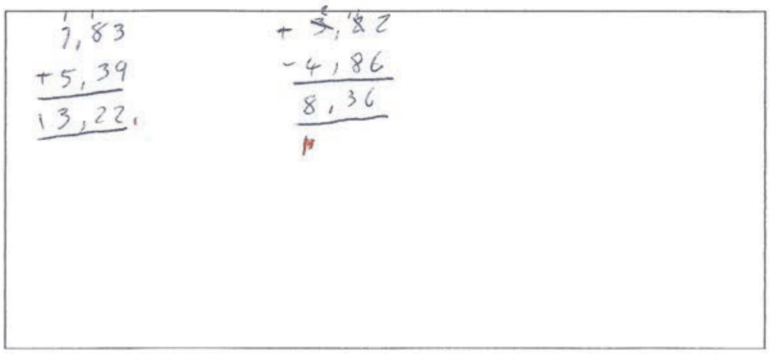
- Recognising, ordering and place value of decimal fractions;
- Addition and subtraction of decimal fractions of at least two decimal places.

What would show evidence of full understanding?

Item 5.7

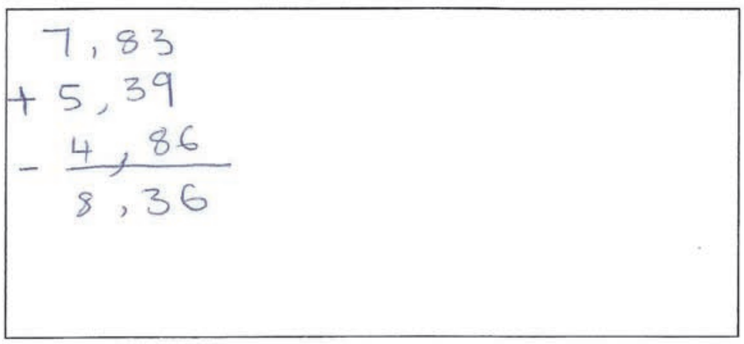
- The correct solution, 8,36 would show evidence of complete understanding, provided the learner performed the correct procedure.
- Learners have to understand that when addition and subtraction are the only operations, they have to follow a left to right order to calculate the answer. In this instance they had to add 7,83 to 5,39, and then subtract 4,86.

5.7 7,83 + 5,39 – 4,86



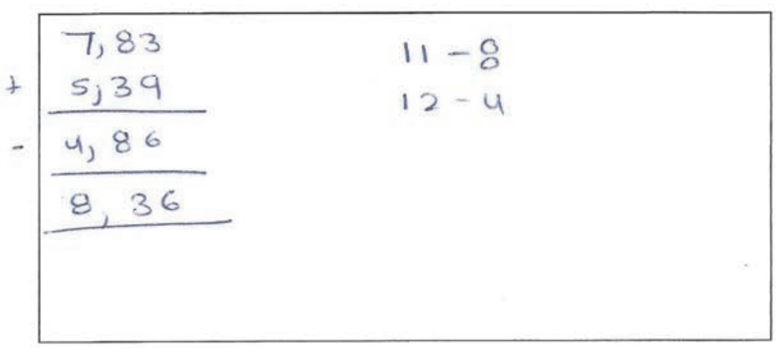
- In the examples below, these learners could do the addition and subtraction in one step and did not have to “borrow” or “carry over”, which made the calculations easier.

5.7 7,83 + 5,39 – 4,86



(3)

5.7 7,83 + 5,39 – 4,86



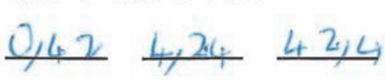
(3)

Item 13

- Learners who could arrange the decimal numbers from the smallest to the biggest displayed full understanding.

13. Write down the following decimal numbers from the smallest to the biggest:

4,24 ; 42,4 ; 0,42



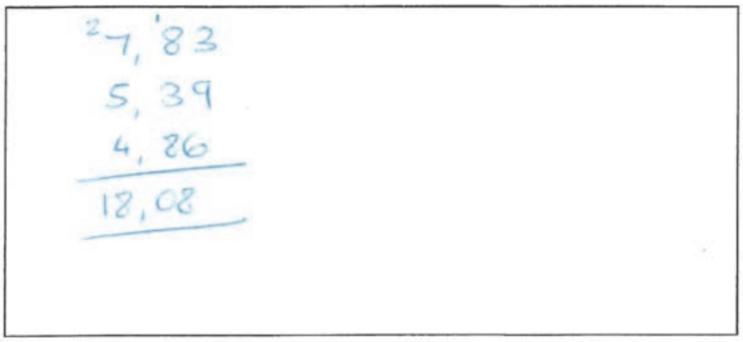
(1)

What would show evidence of partial understanding?

Item 5.7

- Many learners made calculation errors with this item. These errors varied from incorrect addition or subtraction to incorrect “borrowing” or “carrying'. Some learners ignored or did not pay careful attention to the operations, + and – and thus made errors as seen in the examples given. .

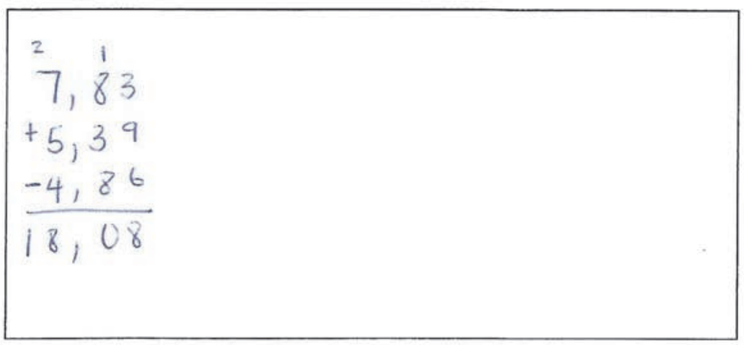
5.7 7,83 + 5,39 – 4,86



(3)

- In the following example, although the learner placed the - sign in the correct position (possibly intending to subtract), the learner then added all the numbers.

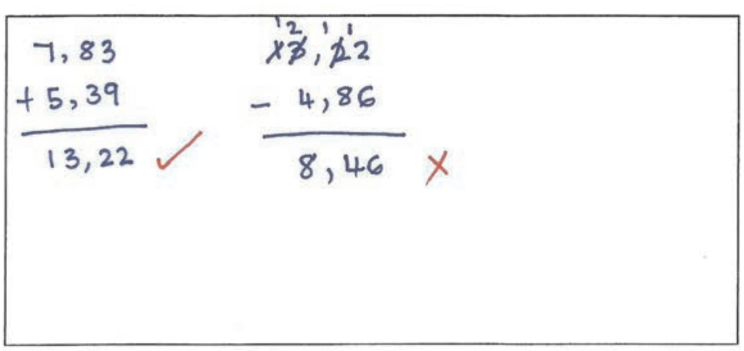
5.7 7,83 + 5,39 – 4,86



(3)

- In the next example the learner placed brackets around the first two terms and added them correctly, but made a calculation error in the subtraction.

5.7 7,83 + 5,39 – 4,86



(3)

- In the example that follows the learner also used brackets, but placed them around the last two terms. In this case, the learner did the subtraction first. An interesting observation is that the learner "borrowed" from the 4 instead of the 5 to get 2,53 instead of 0,53. Then the learner added 2,53 to 7,83 to get the answer 10,36.

5.7 7,83 + 5,39 – 4,86

2,53

7,83 + (5,39 – 4,86) = 10,36. (3)

- In the next example the learner subtracted instead of adding the first two numbers. The learner then realised that 2,44 is smaller than 4,86 and simply switched the numbers around for the subtraction.

5.7 (7,83 + 5,39) – 4,86

```
  7,83          4,86
+ 5,39        – 2,44
  2,44          2,42
```

(3)

Item 13

- In this example the learner arranged the numbers from biggest to smallest, perhaps misreading the question.

13. Write down the following decimal numbers from the smallest to the biggest:

4,24 ; 42,4 ; 0,42

42,4 4,24 0,42 (1)

What would show evidence of no understanding?

Item 5.7

- In the example given here an interesting observation can be made. In the first part the learner added correctly, which may indicate partial understanding. The last part of the working, however, shows no understanding. The learner showed evidence of the procedure for addition using the vertical algorithm, then subtracted the whole 4 from the whole number 13 to get 9 and then attempted to subtract 86 from 9,22, completely ignoring the place values of the digits in what should have been 0,86. This confusion shows evidence of procedural knowledge, but not of conceptual understanding.

5.7 7,83 + 5,39 – 4,86

7,83 + 5,39 13,22	13,22 – 4,86 13 – 4 = 9 9,22 – 86 = 1,16 1.16 answer

(3)

Item 13

- The next example shows little evidence of understanding.

13. Write down the following decimal numbers from the smallest to the biggest:

4,24 ; 42,4 ; 0,42

0,24 0,12 05,2

(1)

- It seems as if this learner tried to halve the fractions, but did not really succeed.

13. Write down the following decimal numbers from the smallest to the biggest:

4,24 ; 42,4 ; 0,42

3,21 0,10 0,5

- This learner misunderstood the question and subtracted 0,01 from each term to form a number sequence starting from 0,41.

13. Write down the following decimal numbers from the smallest to the biggest:

4,24 ; 42,4 ; 0,42

0,41 0,40 0,39

(1)

What do the item statistics tell us?

Item 5.7

40% of learners answered the question correctly.

Item 13

51% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- Learners may have a poor understanding of place value.
- Learners may not know the order of operations.
- Learners may have no understanding of the concept of borrowing when using the vertical algorithm.
- Learners may not be able to work with brackets.

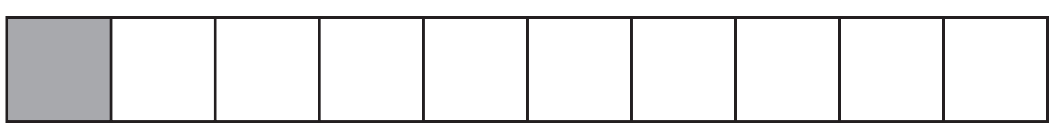
Teaching strategies

Understanding decimal fractions and place value

- Learners encounter decimal fractions for the first time in Grade 6.
- The conversion from common fractions to decimal fractions can be difficult for learners.
- The way in which we write decimal fractions is very different to the way we write common fractions.
- Whereas there are no place values in common fractions, place value gives meaning to the digits in a decimal fraction.
- A good way to introduce the concept of decimals to learners a good way is to make the link between proper fractions and decimal fractions and to use concrete aids that will enable learners to develop the abstract concept of decimal numbers.

Examples using decimal fraction strips

Prepare the rectangular strip shown below divided into tenths, with one part shaded.



Ask learners:

- Into how many parts is the whole divided? (Ten)
- What part of the whole is shaded? (One – one tenth is shaded)
- How do we write this in fraction form? ($\frac{1}{10}$)

- Now explain to learners how to write $\frac{1}{10}$ in a decimal form.
- Discuss the decimal comma. The decimal comma separates the whole number part of the number and the fraction part of a number.

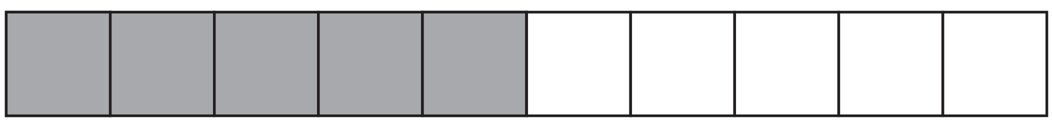
$\frac{1}{10}$ in decimal form will be written as:

T	U	,	t
	0	,	1

Discuss:

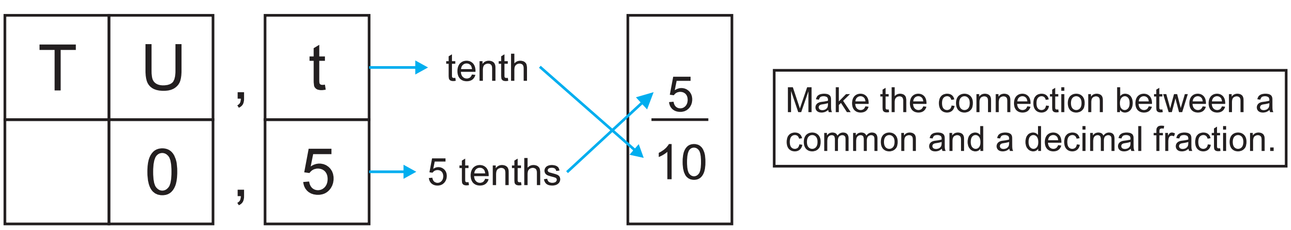
- Why do we put a zero in the place of the unit box? (There are no units in the fraction $\frac{1}{10}$)
- Why do we put a one in the place of the tenth box? (There is one tenth in the fraction $\frac{1}{10}$)

- Now draw the following on the board:



- Ask the learners: What part of the whole is shaded? (Five tenths is shaded)
- How do we write this in fraction form?

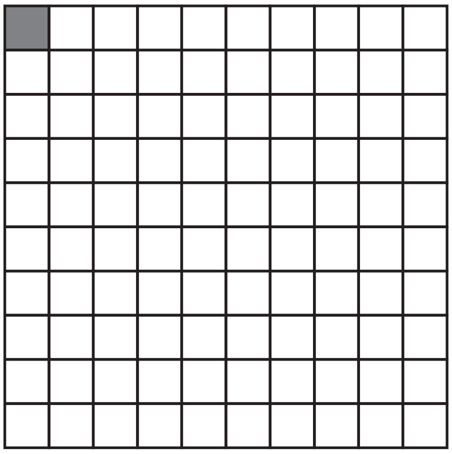
Common fraction: $\frac{5}{10}$
Decimal fraction: 0,5



- Write $\frac{5}{10}$ as a fraction in the simplest form
  $\frac{5}{10} = \frac{1 \times 5}{2 \times 5} = \frac{1}{2}$
- Explain the reason why $\frac{1}{2}$ and 0,5 have the same value. (They are different number symbol representations of the same amount.)
- Discuss the idea of using different number symbols to represent the same number values.

Understanding hundredths

- To move on to hundredths, prepare a grid divided into hundredths, with one part shaded



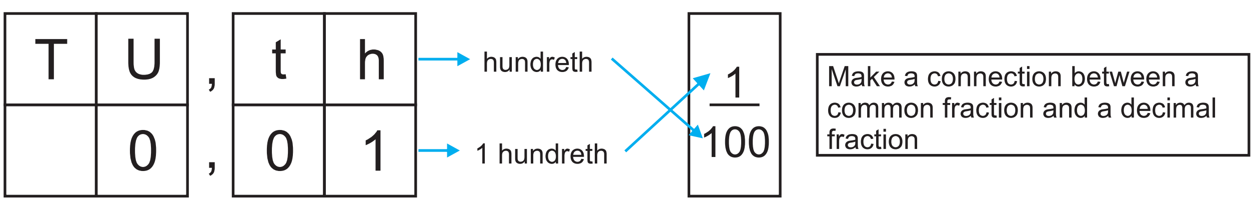
> There are 100 of these blocks in the whole. Each block is one hundredth of the whole.

Examples

Ask the learners:

- What part of the whole is shaded? (One hundredth is shaded)
- How do we write this in fraction form? ($\frac{1}{100}$)
- Now ask learners how to write $\frac{1}{100}$ in a decimal form. How should we extend the place value table to show hundredths?

Solution



Discuss:

- Refer to your hundred-grid.
- Shade different numbers of blocks and talk about the amounts shaded, the fraction represented and the decimal representation of these amounts.

Using base ten blocks to work with decimal fractions

- Most learners are taught the skill of dividing whole numbers up into their constituent parts of hundreds, tens and units (ones).
- Learners may have no problems in saying that, for example, the number 378 consists of 3 hundreds, 7 tens and 8 units (or 8 ones).
- This is looking at numbers in an abstract way. This knowledge of hundreds, tens and units might simply be based on the position of the digits and might not demonstrate conceptual understanding of place value.
- Dienes blocks give children a practical (concrete) example of what a number looks like. For

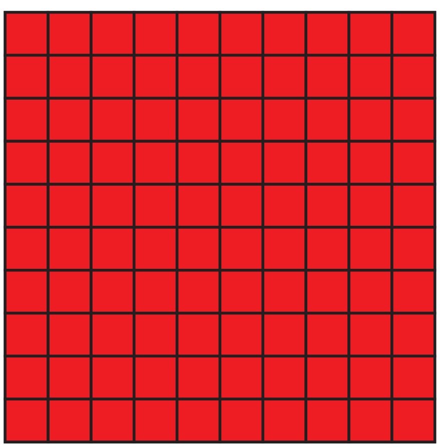
example, one may place 3 hundred blocks, 7 tens blocks and 8 units blocks in front of the learner to illustrate the number 378. However, the same number can be represented as 37 tens and 8 units or as 378 units or as 2 hundreds, 17 tens and 8 units. This can also be demonstrated using Dienes blocks.

- Presenting numbers in a variety of ways gives children a deep and conceptual understanding of the number system, which is essential if children are to become proficient in performing reliable calculations.
- Conceptual understanding is developed through the use of concrete demonstrations.
- Base ten blocks (Dienes blocks) can be used to develop learners' ability to work with a hundred-grid and answer abstract questions about decimal number values. This builds on the whole number value understanding we have just discussed.
- The following base ten blocks can be used to develop conceptual understanding of decimal numbers based on concrete demonstrations and examples:

Example

Lay out:

A flat block for ten tenths, or one unit

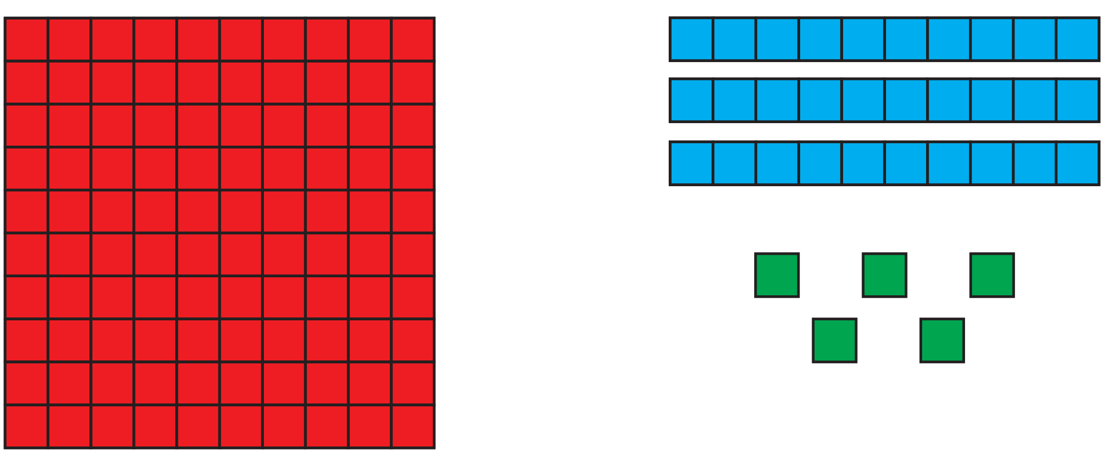


A long block for ten hundredths, or one tenth

A small block for one-hundredth

Lay out:

One flat, three longs and 5 small blocks



Ask:

- What number is represented in the display? (Answer: 1,35)
- How do you know this? (The flat block represents one unit, the long block is one tenth of the flat block and so 3 tenths are displayed; the small block is one hundredth of the flat block and so 5 hundredths are displayed. This gives us the number 1,35).

Ask learners the following questions to help probe/consolidate their understanding of place value. Discuss the answers each time and refer to base ten blocks if necessary to help learners understand.

- How many units are there in one ten? (Answer: 10)
- How many tenths are there in a unit? (Answer: 10)
- How many tenths are there in one ten? (Answer: 100)
- How many hundredths are there in one unit? (Answer: 100)
- How many hundredths are there in one tenth? (Answer: 10)
- How many hundredths are there in one ten? (Answer: 1 000)

- Finally, use the place value grid/chart to help learners to find the number of tens, units, tenths and hundredths in a given number.

Comparing decimal fractions

- Identification of the value of a digit according to place value is an essential skill learners need in order to work with decimals.
- The number concept development activities in the first strategy should enable learners to do this.
- You can consolidate and confirm this knowledge using the following activity.

Example

Write down the value of the underlined digit in the numbers:

a). 42,04
b). 24,42
c). 4,24
d). 4,02

Solutions

a). 42,04	Answer: 4 hundredths
b). 24,42	Answer: 4 tenths
c). 4,24	Answer: 4 units
d). 4,02	Answer: 0 tenths

Ordering decimal numbers

- Ordering decimal numbers is based on knowledge of the values of those numbers and this is linked to being able to identify the place value of each digit in any number.
- Recap place values and the place value grid. Draw the following blank grid on the board and explain all of the elements in the grid:

T	U	,	t	h
		,		

- Write a number into the place value grid, explaining again how we place the numbers:

56,17

T	U	,	t	h
5	6	,	1	7

- The value of the number in the tens place is 5 tens.
- The value of the number in the units place is 6 units.
- The value of the number in the tenths place is 1 tenth.
- The value of the number in the hundredths place is 7 hundredths.

Using a place value chart will help learners to order decimal numbers:

Example

Place these numbers in order from smallest to biggest: 4; 24; 2; 42; 4; 24

Solution

- Write the numbers in a place value grid and use the gird to examine the values of the digits according to place value in each of the numbers.
- The explanations and answer are shown here:

T	U	,	t	h
	4	,	2	4
	2	,	4	2
	4	,	4	2

1). Start looking at the unit digits. 2 is the smallest, so 2,42 will be the smallest number.
2). The other two numbers have the same units digit, so look at the tenths digit.
3). The numbers are 4,24 and 4,42. The 4,24 has a smaller tenths digit than 4,42, so 4,24 is smaller.

Order from small to big:
2,42 ; 4,24 ; 4,42

Activity: Arranging decimal numbers

Use place value to arrange the following decimal numbers in ascending order (from smallest to biggest).

a). 14,01; 14,11; 14,10
b). 10,09; 10,01; 10,10
c). 2,23; 2,33; 2,32
d). 0,11; 0,10; 0,01

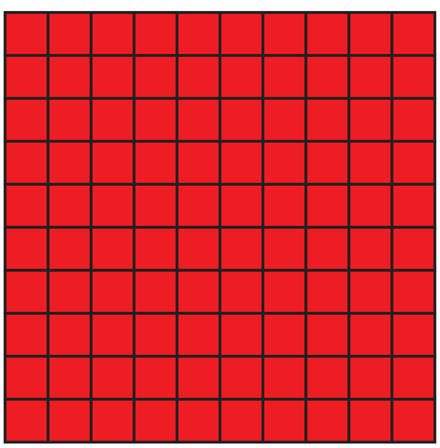
Solutions

a). 14,01; 14,10; 14,11
b). 10,01; 10,09; 10,10
c). 2,23; 2,32; 2,33
d). 0,01; 0,10; 0,11

What does it mean to borrow and carry over?

- You can use base 10 blocks to explain borrowing and carrying.
- In this example, the following base ten blocks can be used (as shown above).

A flat block for ten tenths, or one unit



A long block for ten hundredths, or one tenth

A small block for one-hundredth

Addition of decimal fraction using base 10 blocks (carrying)

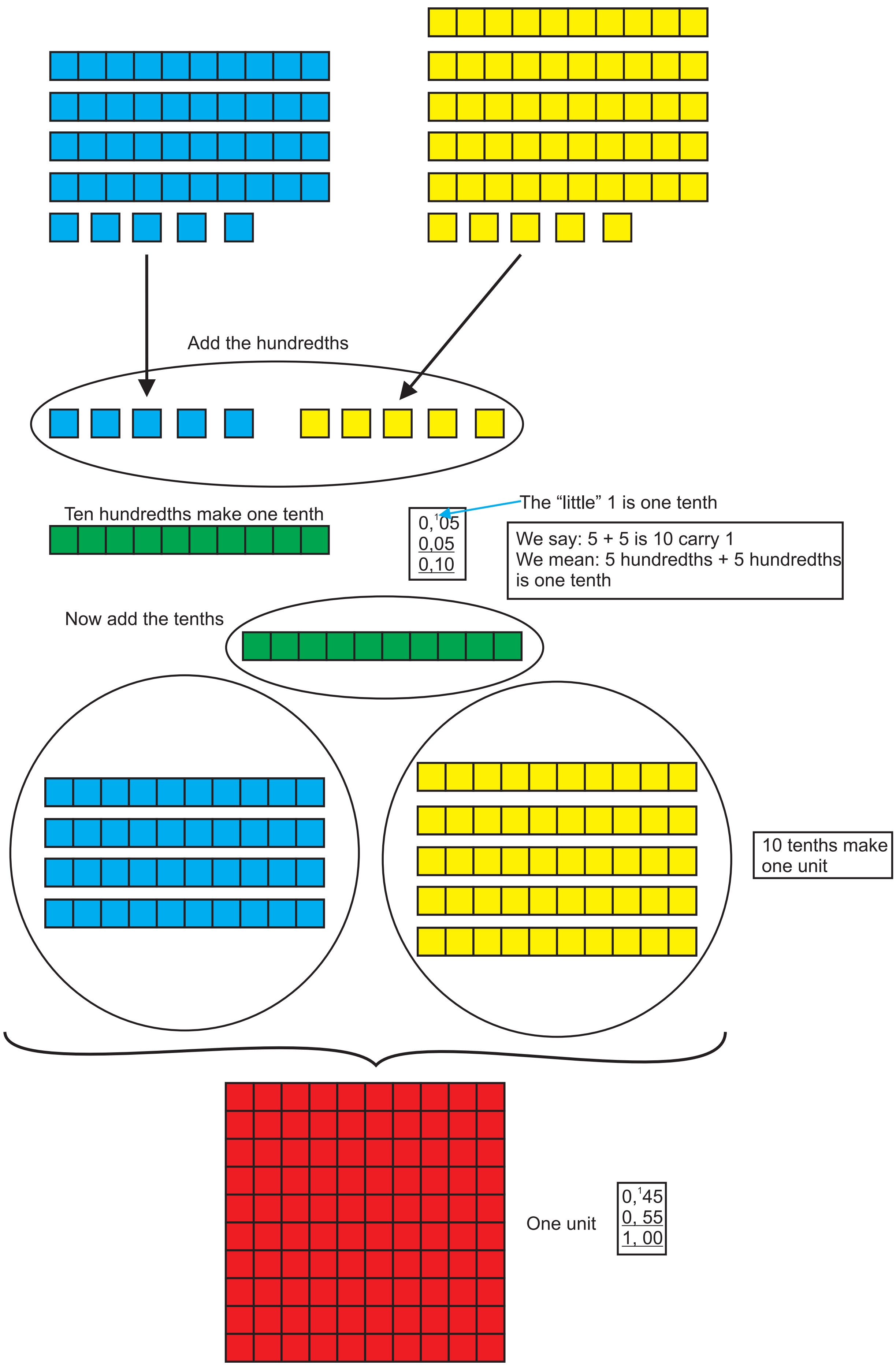
Example

Add the following: 0,45 + 0,55 = 1,00

0,45 ⇨ four tenths and 5 hundredths

0,55 ⇨ five tenths and 5 hundredths

Set out 4 long blocks and 5 small ones on the one side and 5 long blocks and 5 small ones next to that.



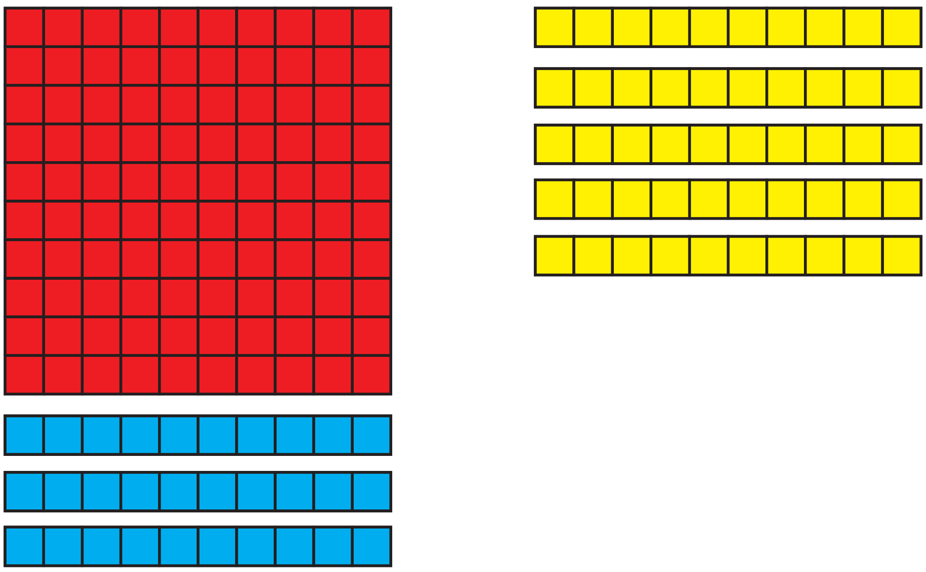
Subtraction of decimal fraction using base 10 blocks (borrowing)

Example

1,3 – 0,5 =

Solution

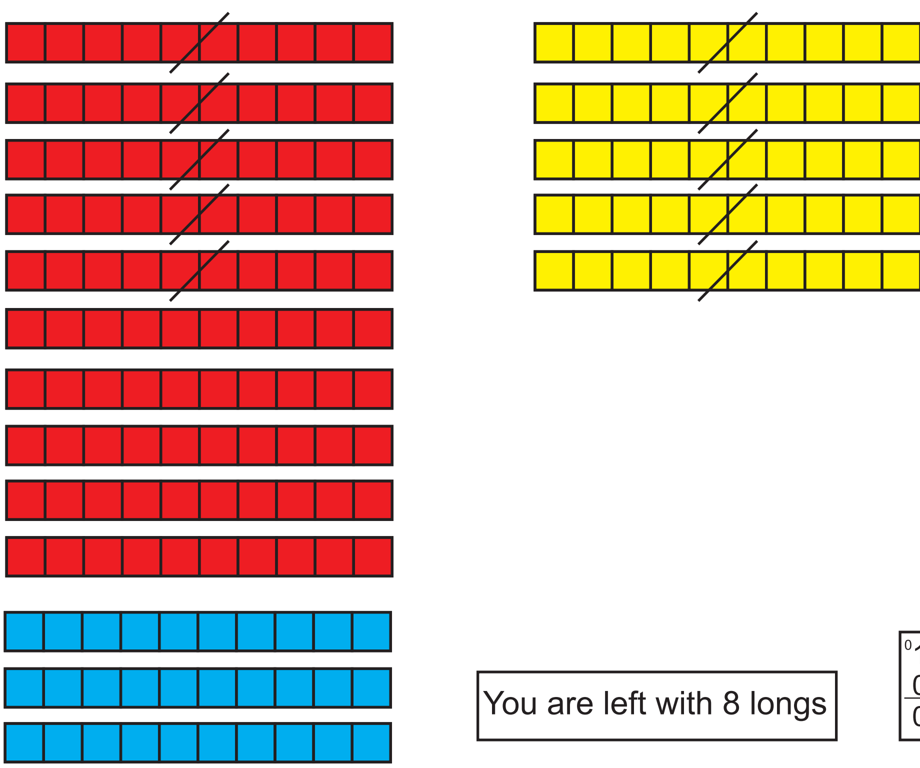
Set out 1 flat and 3 longs on the one side and 5 longs next to that.



You need to take 5 longs away from one flat and three longs.

Break up the flat into 10 longs. ⇨ This is the same as “borrowing”

$$\begin{array}{r} ^{0}\not{1},\ ^{1}3 \\ \underline{0,\ 5} \end{array}$$



You can now take 5 longs from 13 longs, to be left with 8 longs.

You are left with 8 longs

$$\begin{array}{r} ^{0}1,\ ^{1}3 \\ \underline{0,\ 5} \\ 0,\ 8 \end{array}$$

Activity: Add and subtract

a). 3,56 – 2,64 =

b). 9,01 + 8,59 =

c). 2,23 – 1,76 =

d). 6,84 + 4,57 =

Solutions

a). 3,56 – 2,64 = 0,92

b). 9,01 + 8,59 = 17,6

c). 2,23 – 1,76 = 0,47

d). 6,84 + 4,57 = 11,41

Using brackets in number strings

- It is useful to use brackets to help us when we work with strings of numbers. We usually do this to show the order in which we will do the operations.
- Inserting brackets must be done correctly, according to the laws of numbers and operations.

Let us look at addition and subtraction of decimal numbers.

- When only addition is involved: 3,45 + 4,56 + 5,67 =
- We know from addition of whole numbers that the associative property for addition can be used.
- The associative property means that we can add numbers in any order if several numbers are being added.
  - Thus you may put brackets around the first two numbers:
    (3,45 + 4,56) + 5,67
    = 8,01 + 5,67
    = 13,68
  - You may also put brackets around the last two numbers:
    3,45 + (4,56 + 5,67)
    = 3,45 + 10,23
    = 13,68
- The associative property holds for addition.

- When subtraction is involved: 8,45 – 4,56 – 1,67 =
- In a case like this, you cannot put brackets around the last two numbers because both the numbers 4,56 and 1,67 must be subtracted.
  - (8,45 – 4,56) – 1,67 = 3,89 – 1,67 = 2,22 is not the same as
    8,45 – (4,56 – 1,67) = 8,45 – 2,89 = 5,56
- The associative property does not hold for subtraction.

- In subtraction you **<u>must</u>** work from left to right.
  - 8,45 – 4,56 – 1,67 = 3,89 – 1,67 = 2,22 is the correct answer.

It is always a good idea to consolidate numerical processes by giving learners an activity to work on independently. You can use the one below.

Activity: Add and subtract:

a). 8,45 + 4,67 – 3,46

b). 8,45 – 4,67 – 3,46

c). 8,45 – 4,67 + 3,46

Solutions

a). 9,66

b). 0,32

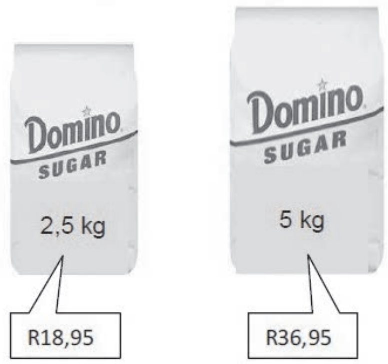
c). 7,24

Notes:

Ratio

ANA 2013 Grade 6 Mathematics Item 24

24 Examine the packets of sugar and their prices. Which is the best buy?



[2]

What should a learner know to answer this question correctly?

Learners should be able to:

- See the relationship between the mass of the bags of sugar and the prices (ratio);
- Add and subtract decimal numbers.

Where is this topic located in the curriculum? Grade 6 Term 3

Content area: Measurement.

Topic: Mass.

Concepts and skills:

- Solve problems relating to mass, including price per kilogram and ratio problems.

What would show evidence of full understanding?

- If, as in the example given, the learner answered "the 5 kg bag" and arrived at the answer by doubling (multiplying by 2) the price of the 2,5 kg bag and finding that its price was higher than the price of the 5 kg bag. The learner made the correct deduction on this basis.

24. Examine the packets of sugar and their prices. Which is the best buy?

R18,95 x 2 = R37,90

So the better buy is 5kg on for R 36,95

- In the following example the learner also indicated how much one would save if one bought the 5 kg packet.

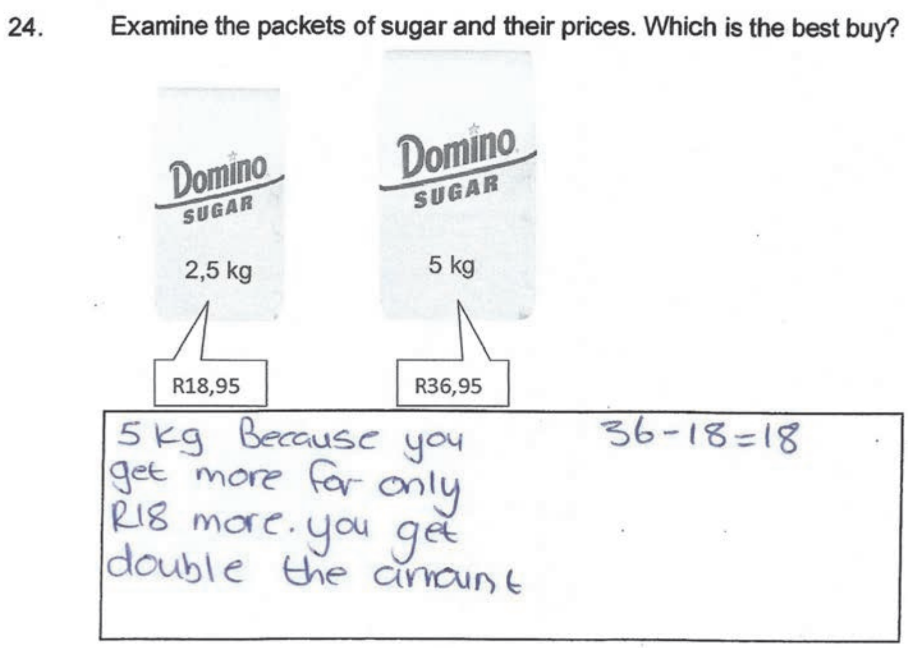
24. Examine the packets of sugar and their prices. Which is the best buy?

R 18,95
R 18,95
R 37,90

R 37,90
36,95
00,95

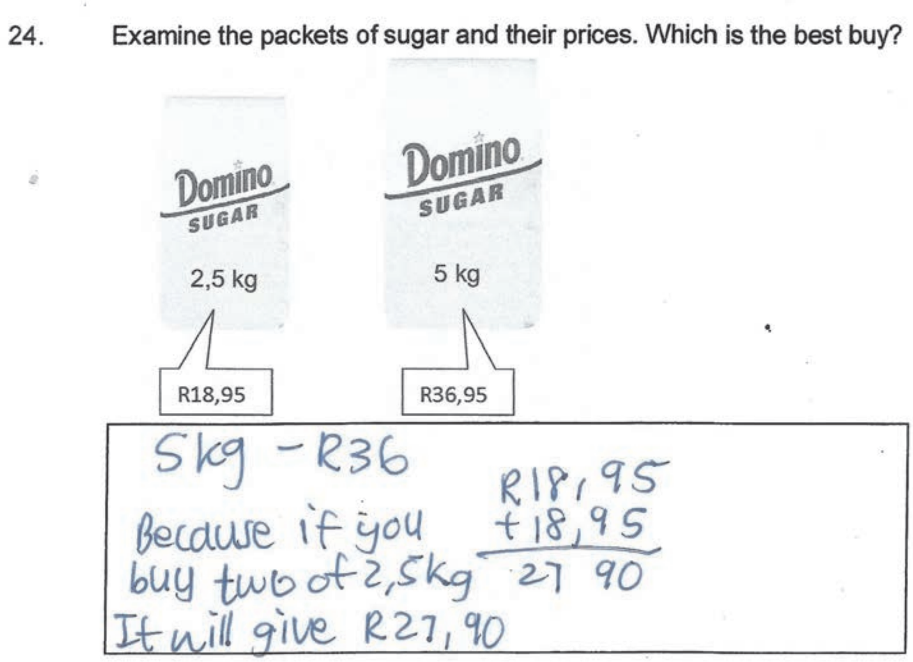
5 kg is the best by 95c

- In the next example the learner gave a good explanation for his/her choice, adding that for just R18 more instead of R18,95 more you can get anotther 2,5 kg of sugar.

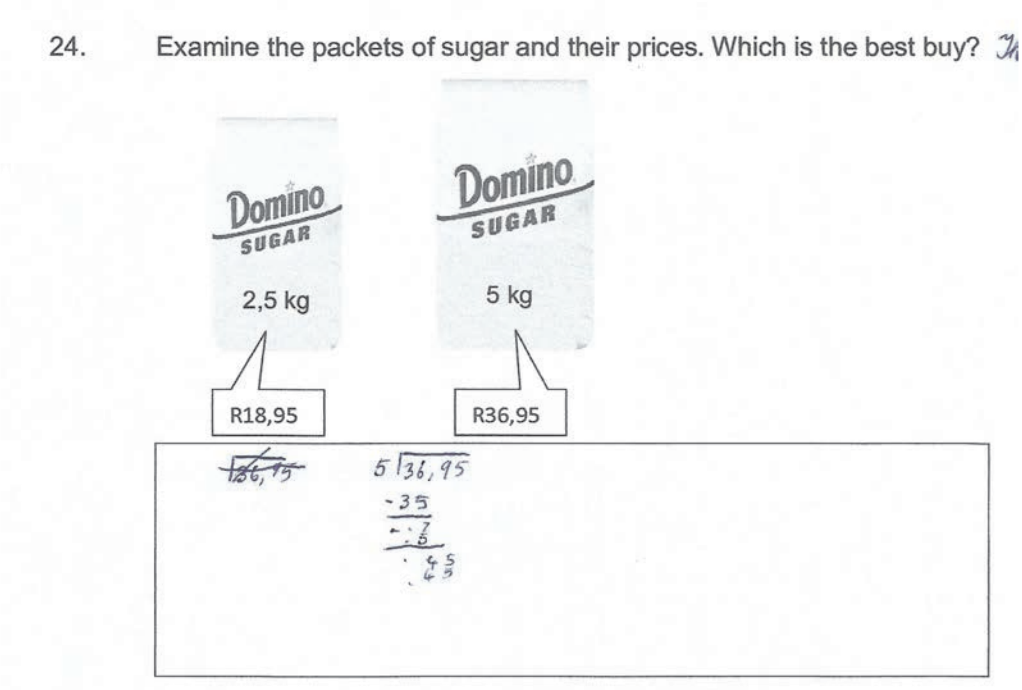


What would show evidence of partial understanding?

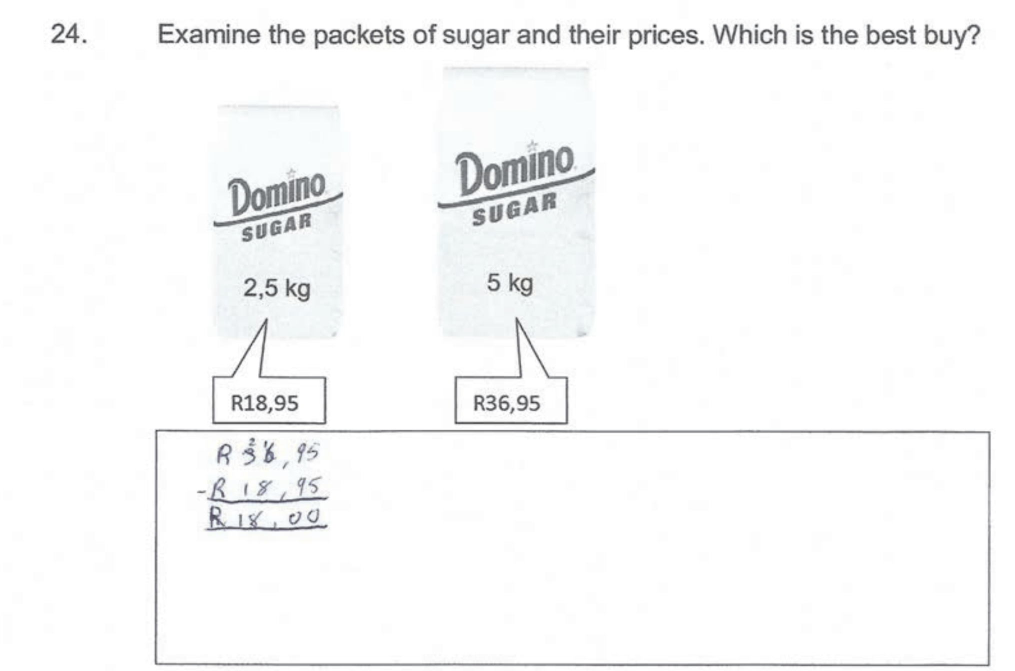
- In the following example the learner compared the prices using an appropriate strategy, but made a calculation error (added incorrectly).



- In the next example the learner calculated the price of 1 kg of sugar using the unitary method, but then failed to take this method further to calcualte the price of 2,5 kg.
- The unit price of the 5 kg packet would be R7,39. To find out if the 2,5 kg packet would be cheaper the learner had to multiply 7,39 by 2,5. At Grade 6 level learners have not yet encountered multiplication of decimal numbers. However, the learner deduced correctly that 7,39 × 2,5 is less than R18,95.

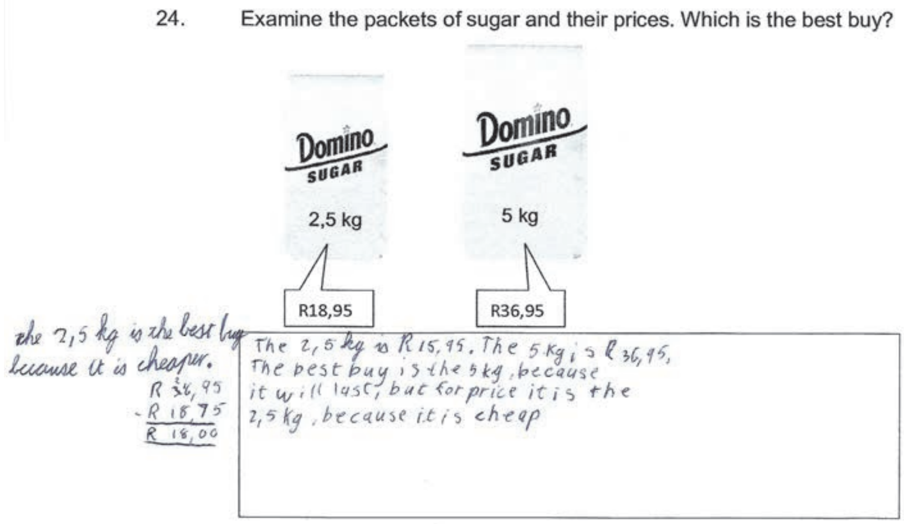


- In the following example the learner followed the same reasoning as that described above, but could not follow up on the argument that you will only need to add R18,00 to get the price of the 5 kg packet and not another R18,95.

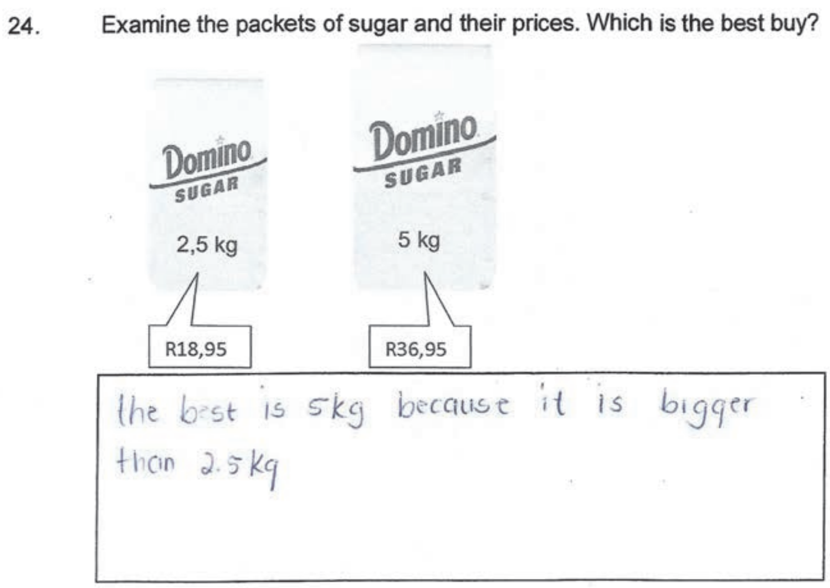


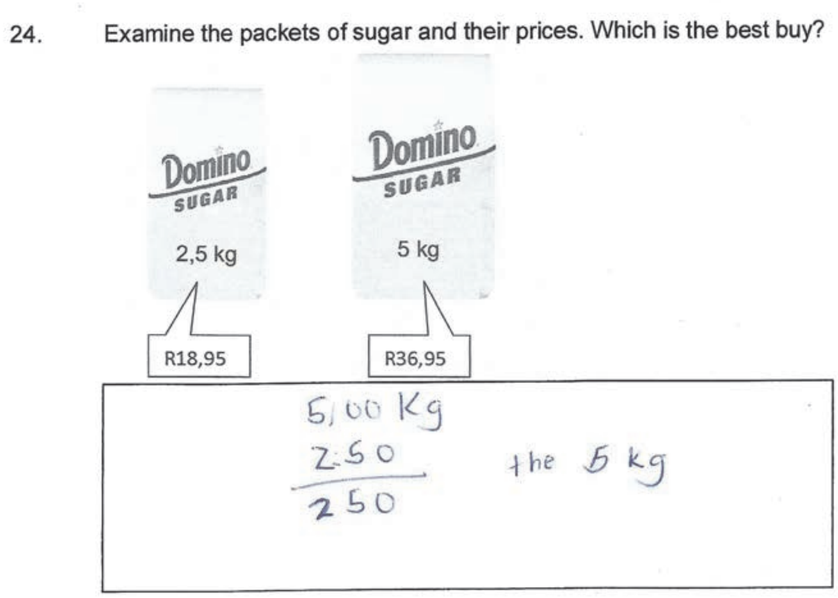
What would show evidence of no understanding?

- Some learners found the difference between the prices, but not in terms of cost-for-mass. These learners looked at the cost of each of the articles and concluded that the 2,5 kg packet is the better buy because it costs less than the 5 kg packet.


24. Examine the packets of sugar and their prices. Which is the best buy?

- In the next two examples the learners did not display any understanding of the problem, but simply looked at the size or mass of the packets and decided that the 5 kg would be a better buy because it is larger. The price was not considered at all.


24. Examine the packets of sugar and their prices. Which is the best buy?


24. Examine the packets of sugar and their prices. Which is the best buy?

What do the item statistics tell us?

34% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not have understood the term 'best buy'.
- The numbers used in the problem contributed to the degree of difficulty.
- Learners are not yet fluent with calculations involving decimal numbers, therefore most of the mistakes they made in this item were due to incorrect calculations.
- Some learners may not understand the use of ratio to compare prices and may not have had a method to use to make the comparison.

Teaching strategies

Ratio

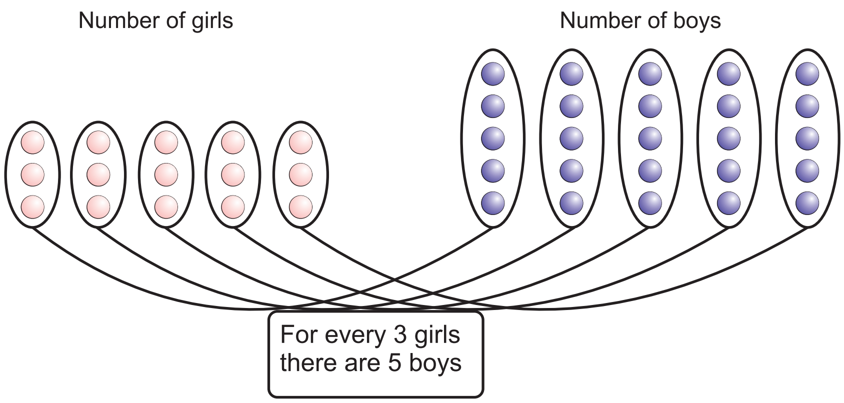
- Ratio refers to the comparison of two quantities of the same kind, whereas rate refers to the comparison of two quantities of a different kind. To strengthen learners' understanding of proportional reasoning, manipulation of concrete objects is necessary. You can discuss ratio using real life examples and counters.
- Remind learners that the order in which a ratio is written has a meaning. You cannot just write ratios in any order. The ratio of boys : girls is not the same as the ratio of girls : boys. When learners are asked to find or simplify a given ratio they must remember to keep the order correct!
- When you work through the example show the learners how the counters represent people (as in the example that follows) or other things, according to the examples you may choose.

Examples

1). If we say that there are 15 girls and 25 boys in the classroom, what is the ratio of girls to boys in the class?

Solution

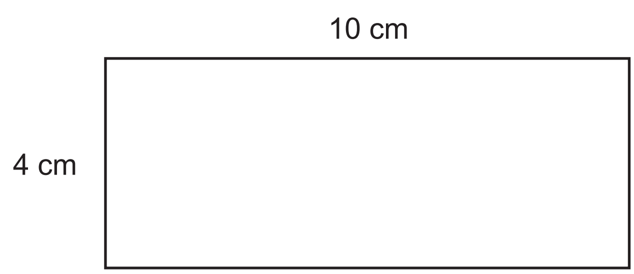
- First write the ratio in the right order using the numbers that you are given.
- We can say that the number of girls : the number of boys = 15 : 25.
- We also say: for every 15 girls there are 25 boys.
- We can use counters to represent the number of girls and boys and then group the counters to simplify the ratio:



The ratio
number of girls : number of boys
= 15 : 25
= 3 : 5

- We can group the counters on each side of the ratio into 5 groups.
- On the left hand side there are 3 counters in each group.
- On the right hand side there are 5 counters in each group.
- This shows us that for every 3 girls there are 5 boys.
- It also shows us that we can simplify the ratio 15 : 25 into the ratio 3:5.

2). A rectangle with dimensions indicated is given:



a). What is the ratio of the width to the length?
b). What is the ratio of the length to the width?
c). How many times is the rectangle longer than what it is wide?
d). What fraction is the width of the length?

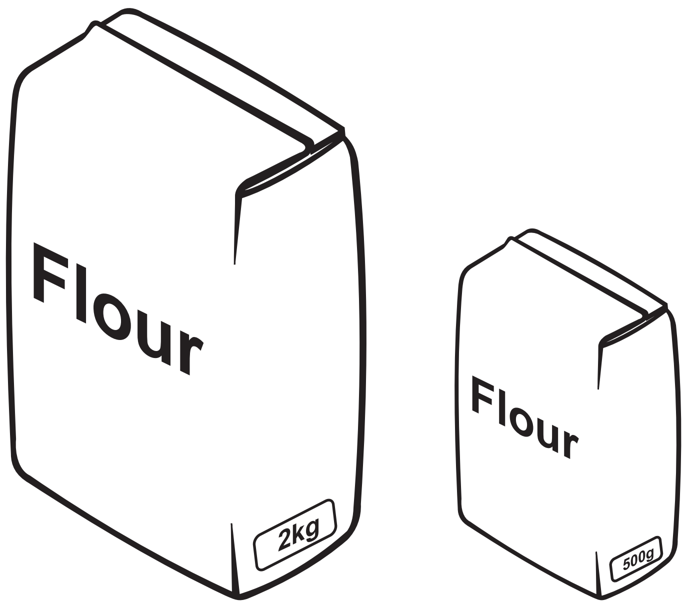
Solution

a). width : length = 4 : 10 = 2 : 5
b). length : width = 5 : 2
c). $10 \div 4 = \frac{5}{2}$, so $4 \times \frac{5}{2} = 10$. The length is $\frac{5}{2}$ times the width
d). $\frac{4}{10} = \frac{2}{5}$

Activity: Working with ratio

1). 250 ml of energy concentrate makes 1 litre of energy drink. How much concentrate is needed for 1,5 litres of energy drink?

2). Which is the best buy? Two kg of flour for R34,50 or 500 g of the same flour for R8,50?



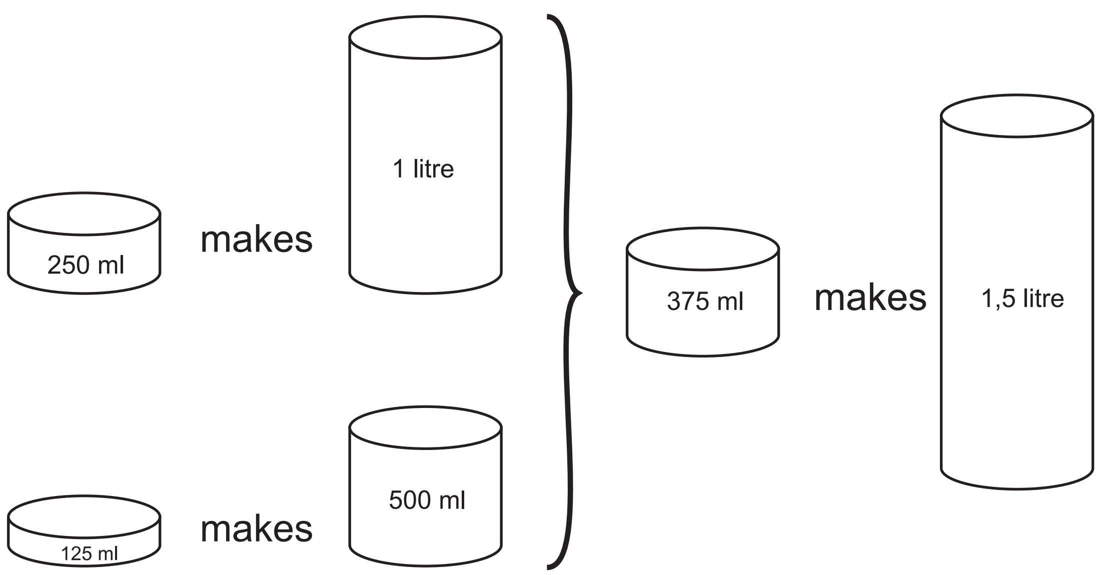
Solutions

1). The ratio of concentrate to drink is 250 : 1 000

1,5 litres of drink requires 1,5 times concentrate

1,5 times concentrate is the same as 250 ml + 125 ml = 375 ml

375 ml of concentrate is needed for 1,5 litres of drink.



2). The ratio of flour given in the two quantities is 2 kg : 500 g

Make the units the same: 2 000 g : 500 g = 4 : 1. This means that four 500 g packets will make 2 kg flour.

Now see that the price is also in the same ratio:

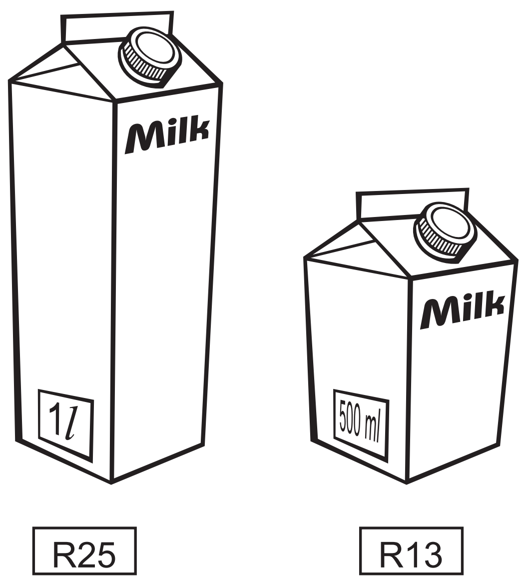
Ratio of price: R34,50 : R8,50

4 × R8,50 = 2 × R17 = R34,00

So it is 50c cheaper to buy 4 packets of 500 g flour, making the 500 g packet of flour the better buy.

Rate

- When we compare, for example, the mass of an object to the price, we refer to rate.
- Rate is similar to ratio and works in a similar way, but it expresses comparisons between different kinds of things.



The rate for the one litre box of milk
R25 per litre

The rate for the 500 ml litre box of milk
R26 per litre

Best buy

- If an item is a 'best buy' it means that it is the cheapest in terms of price per unit.

- Consider a 2,5 kg packet of sugar costing R20,00 and a 5 kg packet costing R35,00.
- This means that for the 2,5 kg packet, the cost per kg is R8 and for the 5 kg is R7,00 per kg.
- It therefore means that the 5 kg packet is the 'best buy' because it is the cheapest per kilogram.

Activity: Identifying the best buy

For every question below, identify which item represents the best buy:

1). 2 kg of rice costing R23,50 or 5 kg of rice costing R63,25.

2). 1 litre of purified water costing R3 or 2 litres of the same purified water costing R5,90.

3). 3 single bath soaps costing R27 or a pack of 4 bath soaps of the same type costing R32,00.

4). 2 litres of cooking oil costing R21,50 or 5 litres of the same type of cooking oil costing R59,99.

5). 1 shirt costs R65 whilst a pair of the same type of shirts costs R125.

Solutions

1) 2 kg of rice. (2 kg costs R11,75/kg. The 5 kg bag is more expensive at R12,65/kg).

2). 2 litresof water. (2 l costs R2,95/l. The 1 l bottle is more expensive at R3/l).

3). A pack of 4 bath soaps. (1 of the 4-pack costs R8/soap while 1 of the 3-pack. costs R9/soap).

4). 2 litres of cooking oil. (2 l costs R10,75/l. The 5 l bottle is more expensive at R11.99/l).

5). The pair of shirts. (1 of the pair costs R62,50/shirt while the single shirt costs R65/shirt).

Notes:

Number Patterns

ANA 2013 Grade 6 Mathematics Item 1.1

1.1 What is the next number in the number sequence?
13, 17, 21, 25, _____

A. 28

B. 29

C. 30

D. 46 [1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Count on by adding in 4s;
- Perform repetitive addition of 4 starting from 13.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Patterns, Functions and Algebra.

Topic: Numeric patterns.

Concepts and skills:

- Extend a given sequence using a constant difference.

What would show evidence of full understanding?

- If the learner added 4 to 25 to get 29 (option B), as shown.

1. Circle the letter of the correct answer.

1.1 What is the next number in the number sequence?
13 ; 17 ; 21 ; 25 ; _____

A 28
B 29
C 30
D 46

What would show evidence of partial understanding?

- If the learner failed to add 4 to 25, but added 3 instead. The majority of the learners who failed to get 29 chose 28 (option A).

- In another example the learner added 5 to get 30 (option C). This again shows partial understanding since the learner recognised that a number less that ten should be added to get to the next number in the sequence.

What would show evidence of no understanding?

- A choice of 46 (option D): this answer bears no relation at all to the given sequence.

What do the item statistics tell us?

78% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not understand how to identify the constant difference.
- The number sequence starts at a number that is not 0 or 1.

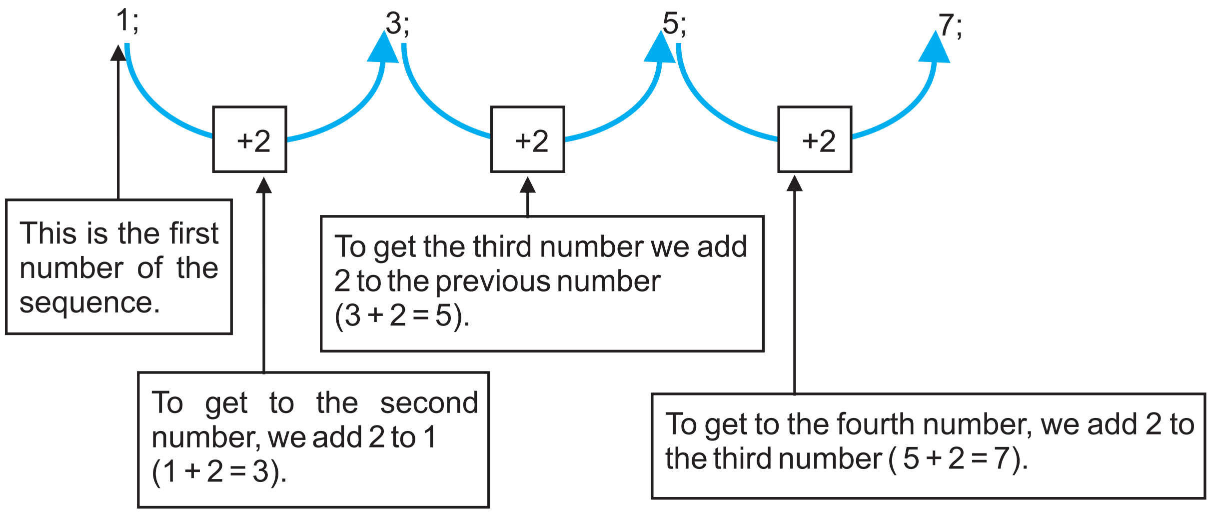
Teaching strategies

Definition of a number pattern

- A number pattern is a sequence of numbers that follow a particular order.
- In some patterns there is a certain number that must be added or subtracted to get the next number.
- This number is called a “common difference”.
- A common difference can be a negative or a positive number.
- In some more difficult number patterns the next term (or number) in the sequence is found by multiplying the previous number by a specific term. This would provide the answer for the next number in the sequence.

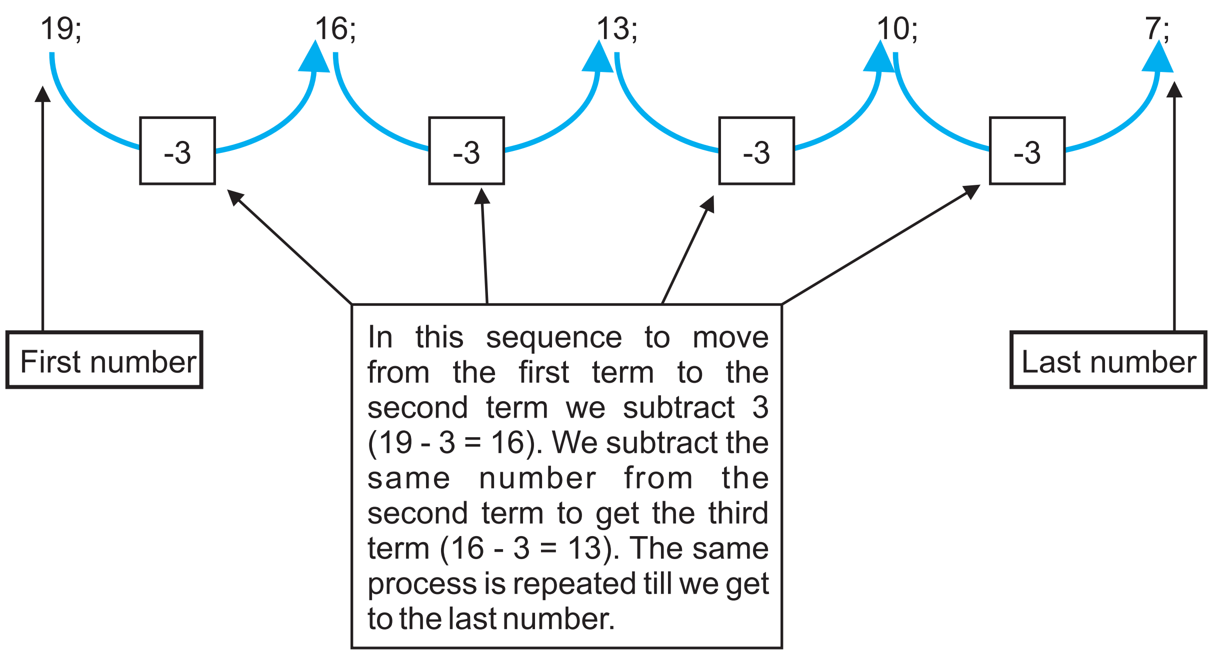
Examples

1). Consider the sequence that follows:



- The + 2 that we keep adding is called the **common difference.**

2). Consider the next sequence:



- In the example shown, the common difference is – 3.

Now use the method applied in the given examples to find the constant difference in each of the following cases.

Activity: Finding the common difference in number patterns

Find the common difference in each of the following number patterns. The first one has been done for you.

a). 100; 200; 300; 400; 500; …
   To get to the next term you have to add 100. The common difference is +100.

b). 1; 2; 3; 4; 5; …

c). 500; 400; 300; 200; 100; …

d). 10; 15; 20; 25; 30; …

e). 9; 18; 27; 36; 45; …

f). 2; 13; 24; 35; 46; …

g). 50; 44; 38; 32; 26; …

h). 97; 90; 83; 76; 69; …

i). 300; 317; 334; 351; 368; …

j). 2 000; 2 140; 2 280; 2 420; 2 560; …

k). 15 000; 11 500; 8 000; 4 500; 1 000; …

Solutions

	Number pattern	Common difference
a).	100; 200; 300; 400; 500; …	+100
b).	1; 2; 3; 4; 5; …	+1
c).	500; 400; 300; 200; 100; …	-100
d).	10; 15; 20; 25; 30; …	+5
e).	9; 18; 27; 36; 45; …	+9
f).	2; 13; 24; 35; 46; …	+11
g).	50; 44; 38; 32; 26; …	-6
h).	97; 90; 83; 76; 69; …	-7
i).	300; 317; 334; 351; 368; …	+17
j).	2 000; 2 140; 2 280; 2 420; 2 560; …	+140
k).	15 000; 11 500; 8 000; 4 500; 1 000; …	-3 500

Extending number patterns

When extending number patterns we add the common difference to the last term in the given pattern to find the term that comes next.

Example

What is the next number in the number sequence given below?

4; 8; 12; 16, …

- To get the correct answer we must first find the common difference.
- From 4 to get 8 we add 4. From 8 to get 12 we again add 4. From 12 to get 16 we add 4.
- Therefore the common difference is +4.
- To find the next number in the sequence we must add 4 to the last given number.
- The next number therefore is 16 + 4 = 20

Activity: Extending number patterns

Find the common difference and the next term for each of the following sequences. The first one is done for you.

a). 2; 1; 0; –1; …
   The common difference is –1 and so the next term is –2.
b). 15; 22; 29; 36; …
c). 17; 21; 25; 29; …
d). 9; 3; -3; -9; …
e). 250; 300; 350; 400; …
f). 2 000; 1 700; 1 400; 1 100; …
g). 630; 760; 890; 1 020; …
h). 2 500; 7 500; 12 500; 17 500; …
i). 105; -95; -295; -495; …
j). 20; 200; 380; 560; …

Solutions

	Number pattern	Common difference	Next Term
a).	2; 1; 0; –1; …	-1	-2
b).	15; 22; 29; 36; …	+7	43
c).	17; 21; 25; 29; …	+4	33
d).	9; 3; -3; -9; …	-6	-15
e).	250; 300; 350; 400; …	+50	450
f).	2 000; 1 700; 1 400; 1 100; …	-300	800
g).	630; 760; 890; 1 020; …	130	1 150
h).	2 500; 7 500; 12 500; 17 500; …	+5 000	22 500
i).	105; -95; -295; -495; …	-200	-695
j).	20; 200; 380; 560; …	+180	740

Numeric and geometric patterns

ANA 2013 Grade 6 Mathematics Items 1.2, 14, 15 and 18

1.2 Which number is missing in this number pattern?

9,17 ; 9,15 ; 9,13 ; ______ ; 9,09

A 9
B 9,10
C 9,11
D 9,12

[1]

---

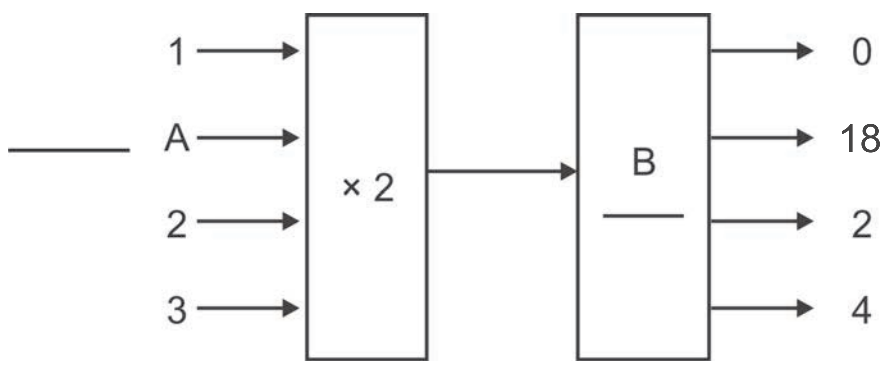
14 Complete the table:

x	y
1	3
2	7
3	11
8	

[1]

---

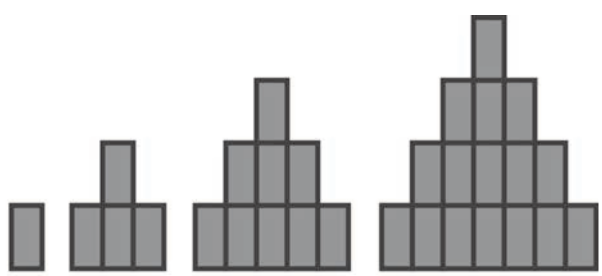
15 Replace A with a number and B with a rule in the flow diagram below.



[2]

---

18 Carefully look at the stacks of cans and then complete the table.



Stack number: 1 2 3 4

Stack number	1	2	3	4		6		
Number of cans	1	4	9	16				64

[2]

What should a learner know to answer these questions correctly

Learners should be able to:

- Recognise number patterns and rules that generate these patterns;
- Find the relationships between the terms in the sequence;
- Identify the relationship between the number of the term and the value of the term;
- Recognise square numbers;
- Calculate input and output values in a flow diagram;
- Relate geometric patterns to numeric values.

Where is this topic located in the curriculum? Grade 6 Terms 2 and 3

Content area: Patterns, Functions and Algebra.

Topic: Numeric patterns.

Concepts and skills:

- Determine input values, output values and rules for patterns and relationships using flow diagrams.

What would show evidence of full understanding?

Item 1.2

- If the learner chose the answer C (9,11) this displays full understanding.

Item 14

- If the learner correctly calculated the y-value to be 31 this displays full understanding. The second example shpwn includes the learner's reasoning.

14. Complete the table:

x	y
1	3
2	7
3	11
8	31

14. Complete the table:

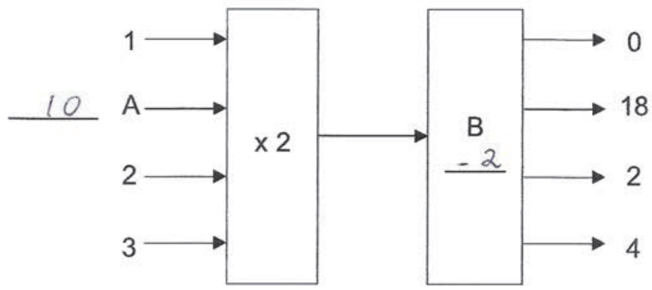
x	y
1	3
2	7
3	11
8	31

4 15
5 19
6 23
7 27
8 31

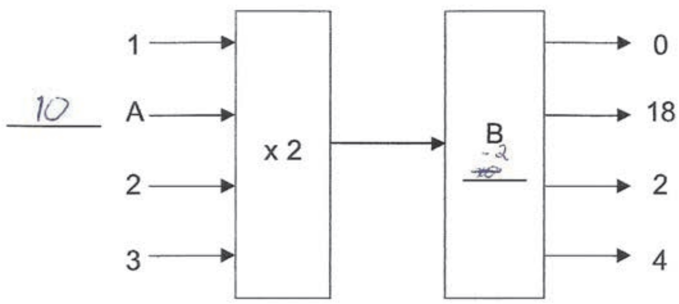
Item15

- If the learner gave "10" for the value of A and "–2" for the operation or rule for B this indicates full understanding as shown in the examples that follow.

15. Replace A with a number and B with a rule in the flow diagram below.



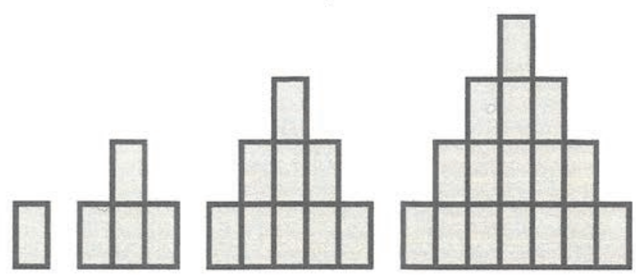
15. Replace A with a number and B with a rule in the flow diagram below.



Item 18

- If the learner filled in both missing numbers correctly this displays full understanding.
- In the example below the learner indicated that he or she recognised the pattern to be squares of the numbers in the top row, although it seems that the learner did not make use of the geometric pattern given in the diagram.

18. Carefully look at the stacks of cans and then complete the table.



Stack number: 1 2 3 4

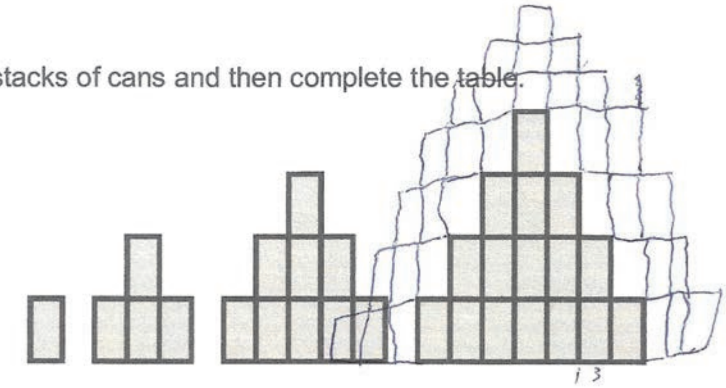
Stack number	1	2	3	4	5	6	7	8
Number of cans	1	4	9	16	25	36	49	64

1 × 1
2 × 2
3 × 3
4 × 4
5 × 5
6 × 6
7 × 7
8 × 8

(2)

- Another way of calculating the answer would be to build on to the geometric structure and count the number of cans. This learner recognised that the difference between the numbers is the set of odd numbers 3; 5; 7; etc.

18. Carefully look at the stacks of cans and then complete the table.



Stack number: 1 2 3 4

Stack number	1	2	3	4		6		8
Number of cans	1	4	9	16		36		64

(2)

What would show evidence of partial understanding?

Item 1.2

- If the learner chose B (9,10) or D (9,12): this could indicate a calculation error and either of these answers can thus be considered as demonstrating partial understanding.

Item 14

- If the learner answered 15 for the y-value when $x = 8$: this indicates that the learner obtained the answer by simply calculating the difference (4) of the y-values and adding 4 to the last y-value in the column (11), without considering the x-value.
- This shows that the learner realised that the constant difference was the key to generating the sequence, but failed to notice that the next x-value was 8 and not 4. This was a common error.
- We can assume that those learners calculated the constant difference (4) correctly, but did not use it to generate the y-value when $x = 8$.

14. Complete the table:

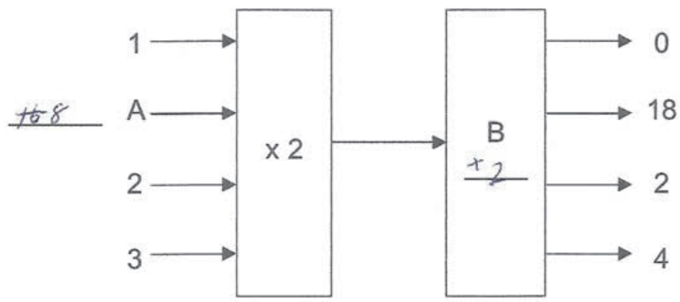
x	y
1	3
2	7
3	11
8	4

Item 15

- If the learner did not see the flow diagram as an entity, but considered only one value, this shows partial understanding.
- This means that the learner only considered the value of A and the operation needed to obtain an answer of 18. The learner realised that the value of A in the input table had to relate to 18 in the output table. However, the learner ignored the rest of the information given in the flow chart.

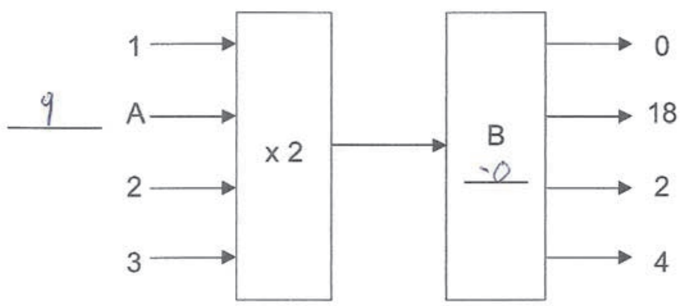
- There are, of course, an unlimited number of values that could have been chosen, as long as the operation in the second box resulted in an answer of 18. Thus in the example below the learner reasoned that $8 \times 2 + 2 = 18$.

15. Replace A with a number and B with a rule in the flow diagram below.



- In the following example the learner reasoned that $9 \times 2 - 0 = 18$

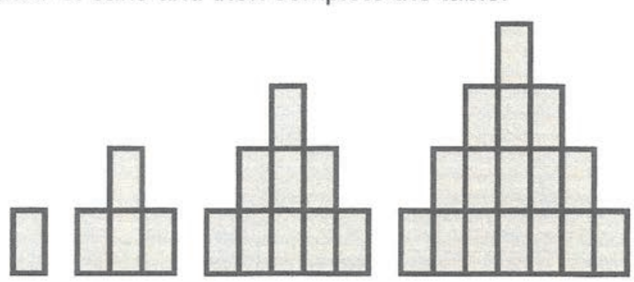
15. Replace A with a number and B with a rule in the flow diagram below.



Item 18

- If the learner calculated the squares of some, but not all the numbers, this shows partial understanding.
- The learner was supposed to complete the open blocks (not the coloured blocks) and hence received no marks as in the example given. This learner calculated the squares of 5 and 7, but failed to find the square of 6 and the square root of 64.

18. Carefully look at the stacks of cans and then complete the table.



Stack number: 1 2 3 4

Stack number	1	2	3	4	5	6	7	10
Number of cans	1	4	9	16	25	54	49	64

0

(2)

What would show evidence of no understanding?

Item 1.2

- If the learner answered 9 (choice A): this indicates that the learner considered neither the number pattern nor the decimal fractions in the question.

Item 14

- If the leaner's answers lack any form of logical reasoning: this indicates that the learner has no understanding of the concepts being tested.

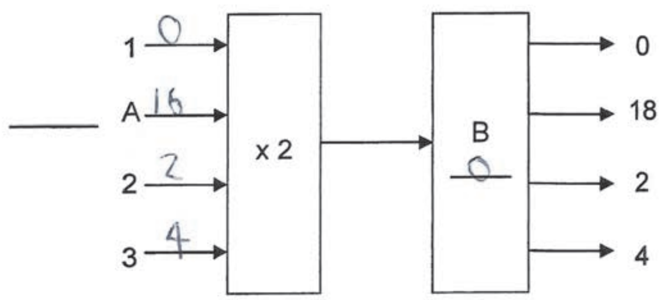
14. Complete the table:

x	y
1	3
2	7
3	11
8	14

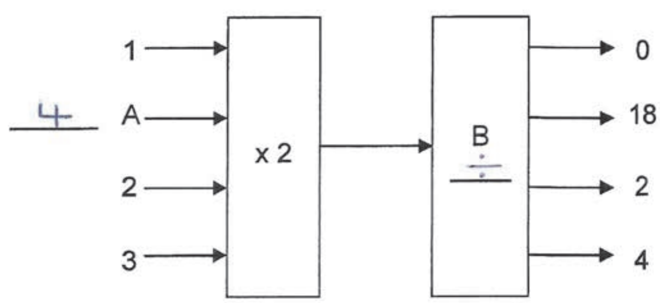
Item 15

- If the learner was not able to associate a rule with a number and an operation: this shows the learner does not understand flow diagrams, as illustrated in the examples that follow.

15. Replace A with a number and B with a rule in the flow diagram below.



15. Replace A with a number and B with a rule in the flow diagram below.

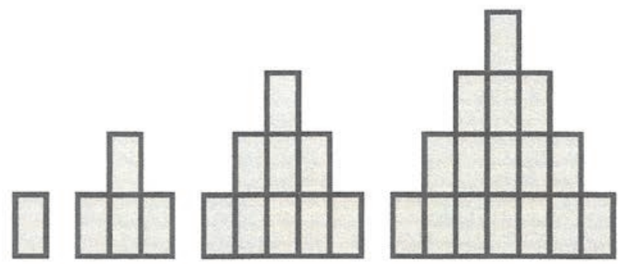


Item18

- If the learner did not consider all the numbers in a row to find the pattern: this shows the learner has no understanding of the rationale behind number patterns.

- The following examples show that the learners completed the top row with the natural numbers 5, 7 and 8 to match with the other given numbers.
- The learners then only considered the numbers from 16 onwards to find the missing number and were not concerned with the last term, 64, which did not fit into the sequence.

18. Carefully look at the stacks of cans and then complete the table.

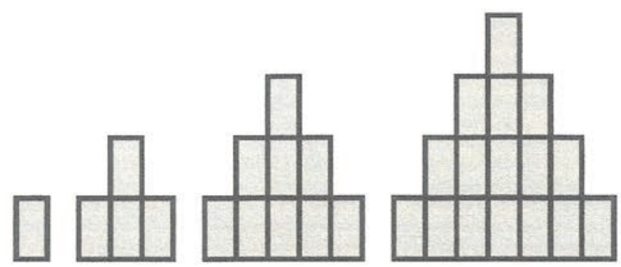


Stack number: 1 2 3 4

Stack number	1	2	3	4	5	6	7	8
Number of cans	1	4	9	16	32	48 ~~64~~	64	64

(2)

18. Carefully look at the stacks of cans and then complete the table.



Stack number: 1 2 3 4

Stack number	1	2	3	4	5	6	7	8
Number of cans	1	4	9	16	20	24	28	64

(2)

What do the item statistics tell us?

Item 1.2

71% of learners answered the question correctly.

Item 14

11% of learners answered the question correctly.

Item 15

30% of learners answered the question correctly.

Item 18

42% of learners answered the question correctly.

1Factors contributing to the difficulty of the items

- Counting “backwards” using decimals could have made the item more difficult for some learners (Item 1.2).
- There was a 'jump' in the x–values from 4 to 8. The last term in the x-column was 8 and not 4, as many learners might have expected (Item 14).
- The value of A was placed between the 1 and 2 of the input values which made the calculation more difficult (Item 15).
- Learners may not understand that a rule must be associated with a number and an operation (Item 15).
- Some learners could not identify the relationship between the numbers in the top row (stack number) and the numbers in the bottom row (number of cans).
- Some learners failed to see that the growing pattern could be obtained by adding on the next odd number in the bottom row (Item 18).
- Some learners could not relate the geometric pattern in the diagram to the information given in the table (Item 18).

Teaching strategies

Group work about forming geometric patterns

- Numeric and geometric patterns are closely linked because a geometric pattern usually grows in ways that can be counted.
- For example:
  - The number of sticks in a display could increase according to a pattern and the number of sticks in the pattern could then be counted;
  - The number of blocks in a shape could be increased according to a pattern and the number of blocks could then be counted.
- Practical work will enable learners to feel comfortable working with geometric patterns and using counts to find out the rules for how the patterns are generated.

You could use the following work-station activity in your class for this purpose.

Activity: Forming geometric patterns

- Collect the following concrete materials (manipulatives):
  - Squares **(see printables)**
  - Triangles **(see printables)**
  - Bottle tops
  - Matches
- Place desks together to make "stations" for learners to move around to in the classroom.
- Put about 30 of each of the manipulatives at each station. Make as many stations you will need for learners to work in groups of 4 to 6.

- Each station should have blank paper on which learners can draw their patterns and tables of values to be completed.
- Prepare tables of values for learners to complete at each station. Make sure that there are enough clean sheets and tables of values at each station.
- You can prepare the following tables to place at the work stations.

Step number/pattern number	1	2	3	4	5	6
Number of squares/triangles/bottle tops/matches						

- Explain to learners what they have to do:
  - At each station learners must build a pattern using the manipulatives given. They must build at least the first, second and third patterns (call them steps).
  - Learners must draw the patterns on an empty sheet of paper provided.
  - Learners must write down a description of how they are building their patterns.
  - While they are building the patterns, or after they have finished, the learners must complete the table that you have prepared.
  - After the learners have completed the task they must put their sheets up on the blackboard or a place you have set aside for this purpose. Put each groups' drawings and completed tables next to each other.
- If there is time, allow learners to move to the next station to work with other manipulatives.

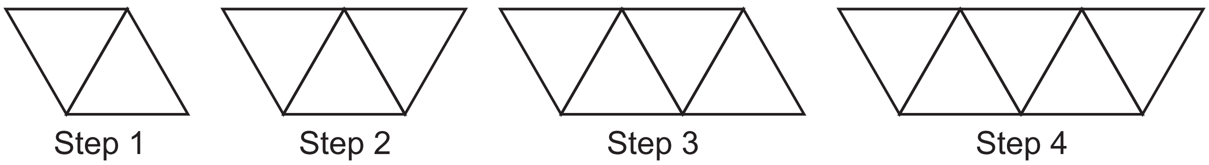
- Pictures of manipulatives that could be used:

Wooden cubes, bottle tops, squares, triangles and matches

Examples

Examples of possible patterns that learners could build:

Using triangles



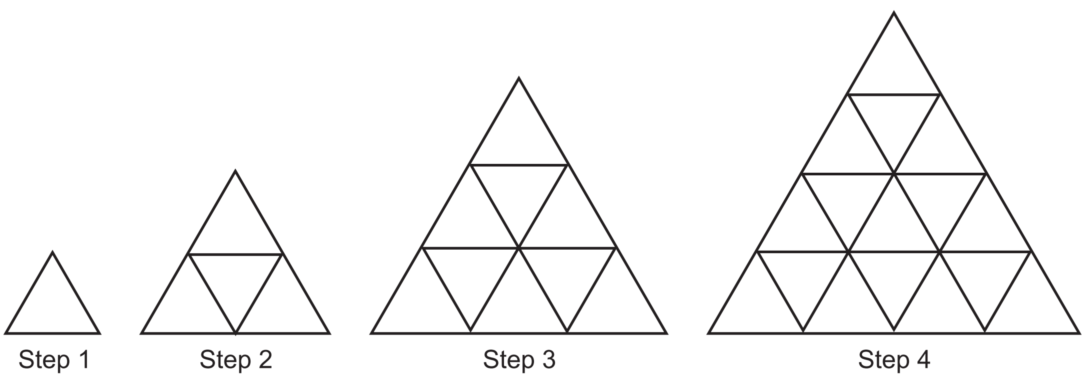
Description of the pattern

- There are two triangles in the first shape.
- The shape grows by placing one triangle to the right every time.

Table

Step number	1	2	3	4	5
Number of triangles	2	3	4	5	6

Another example using triangles



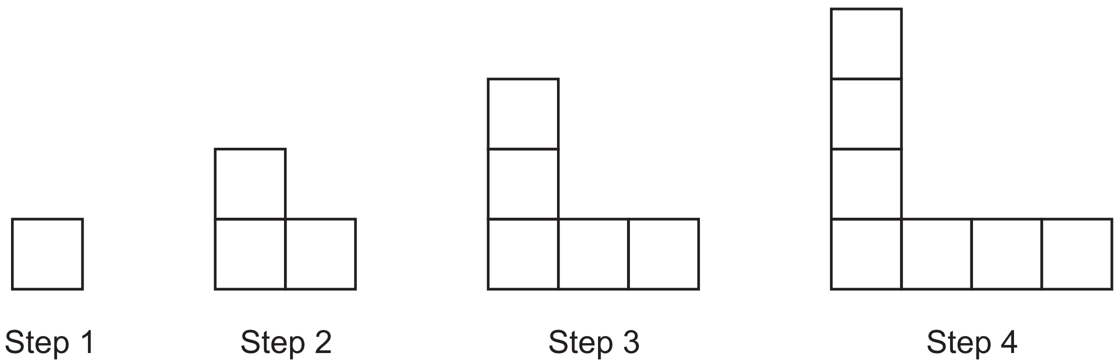
Description of the pattern

- The pattern is built by adding a row of triangles at the bottom to make a new triangle every time.

Table

Step number	1	2	3	4	5
Number of triangles	1	4	9	16	25

Using squares



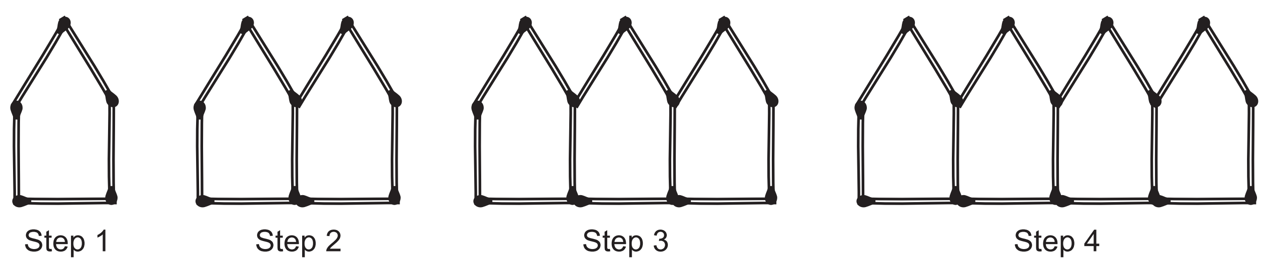
Description of the pattern

- The pattern is built by placing a square on the right and at the top every time to make a new shape.

Table

Step number	1	2	3	4	5
Number of squares	1	3	5	7	9

Using match sticks



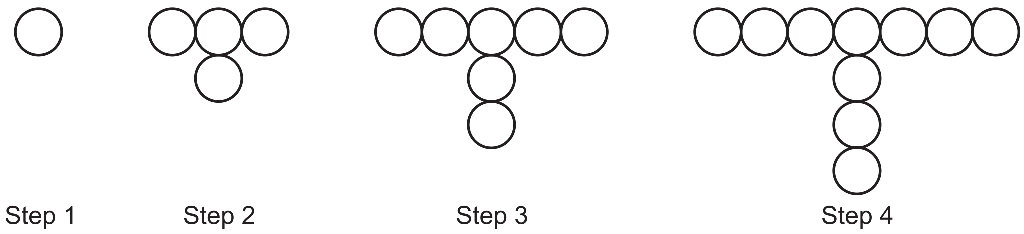
Description of the pattern

- The pattern starts off with a house (pentagon) and then another house is added on the right every time.

Table

Step number	1	2	3	4	5
Number of match sticks	5	9	13	17	21

Using bottle tops



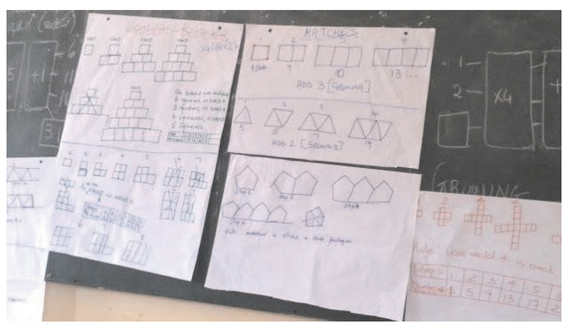
Description of the pattern

- The pattern starts with one bottle top and then grows by adding 3 bottle tops (left side, right side and at the bottom) every time.

Table

Step number	1	2	3	4	5
Number of bottle tops	1	4	7	10	13

Illustration of learners' work



- At the end of the lesson or in the following lesson you should discuss some of the patterns that the learners drew and wrote about.
- Make sure that the tables match the drawings and that the learners are able to talk about the way in which each pattern was generated.
- This will help learners to generalise the rules for pattern growth.
- The next strategy develops learners' "rule generating" knowledge, building on work done in the preceding activity.

Finding the rule when a pattern is given

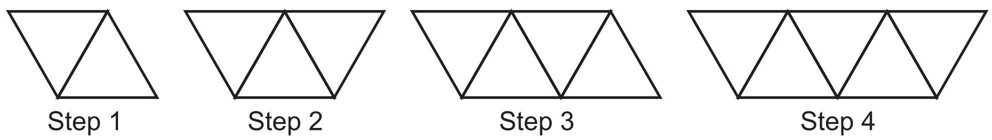
- After learners are able to draw patterns and write the information relating to the patterns in table form, they can be guided to place the information in flow diagrams and to determine the rules of the patterns.

- In this activity we will use the learners' own work to derive the rules which connect the step number with the number of objects used.

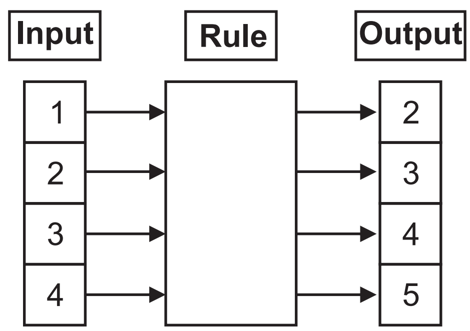
Examples

Using triangles

- Put the learners' drawing and table up on the board where you are going to work.
- Draw a flow chart using the information in the table.

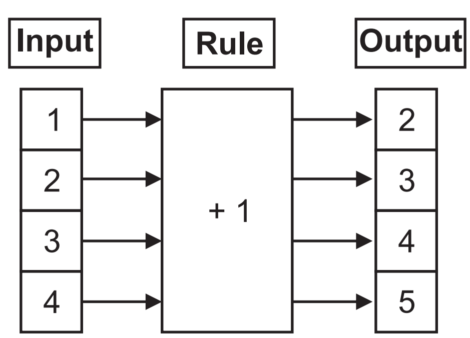


Step number (input)	1	2	3	4	5
Number of triangles (output)	2	3	4	5	6



Discussion

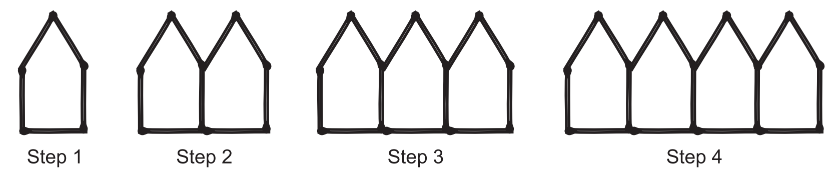
- Ask: What do we have to do with the input value to get the output value?
  - Say: Input 1, then output 2; input 2, then output 3.
- Ask: Can you see what happens?
- Answer: You have to add 1 to the input every time to get the output.
- Ask: How can we write "add one" mathematically?
- Answer: You write: "+ 1" and that is our rule.
- Ask: Can you see that the rule has to have an operation and a number?
- We can now fill that into the rule box.
- Remember to check that the rule works for all the input and output values.



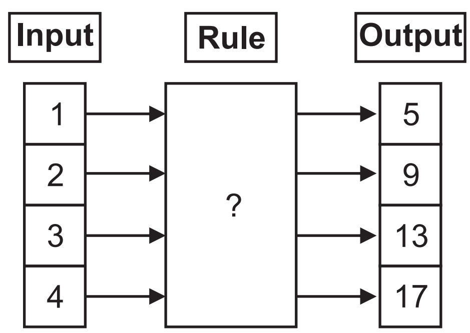
- Discuss with learners using their drawings to make sure they can see how the rule works.

Using matches

- Put the learners' drawing and table up on the board where you are going to work.
- Draw a flow chart using the information in table.

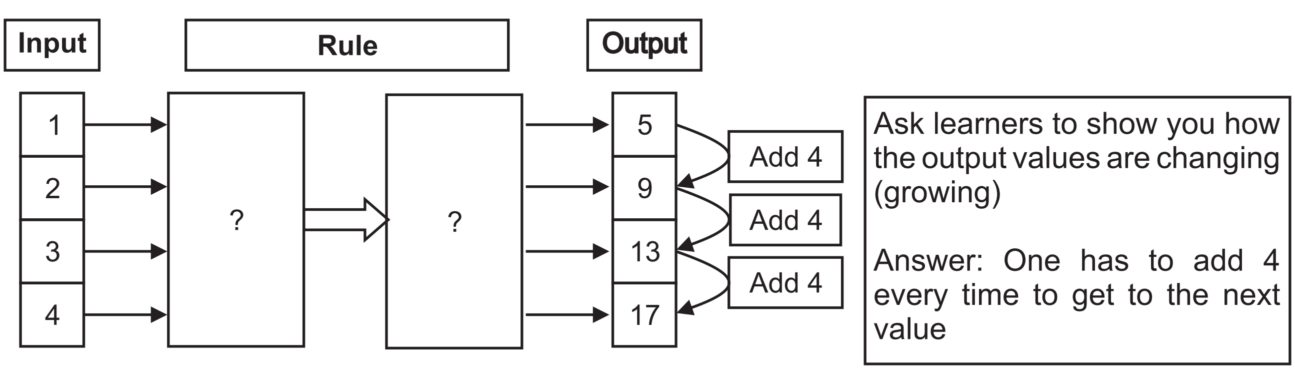


Step number	1	2	3	4	5
Number of match sticks	5	9	13	17	21



Discussion

- This example is not as straightforward as the previous one. The following questions and explanations can be used to carry out this activity in your class.
- Ask: Can you see that this question needs a little more than just adding a number to the input to get the output?
- Give the learners a hint: For this example, we need TWO operations in our rule.
- The secret lies in the input and output values. Let us look at them carefully.
  - The input values grow by 1 every time (1, 2, 3, 4…)
  - The output values grow by 4 every time (5, 9, 13, 17…)



- Ask: Can the rule be: We add 4 to the input values to get the output values?

Let us check: $1 + 4 = 5$
$2 + 4 = 6$
$3 + 4 = 7$

No, it does not work for all the input and output values.

- It is the **difference** that gives you the number that you must **multiply** the input value by.
- Can the rule be: We multiply the input values by 4 to get the output values?

No, it does not work. Our output values are.

Let us check
$1 \times 4 = 4 \longrightarrow 5$
$2 \times 4 = 8 \longrightarrow 9$
$3 \times 4 = 12 \longrightarrow 13$
$4 \times 4 = 16 \longrightarrow 17$

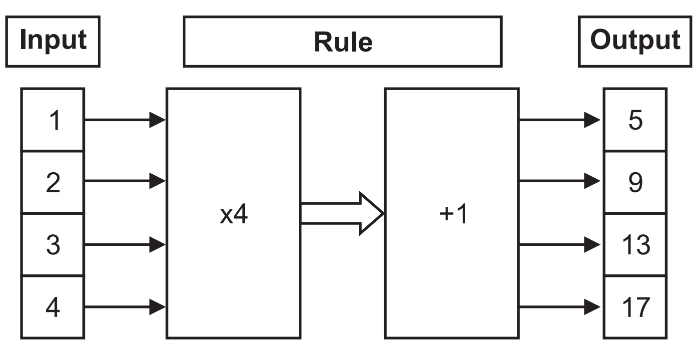
- Can the rule be: We multiply the input values by 4 and then add 1 to get the output values?

Let us check:

Input			Output
1	×4	+1	-5
2	×4	+1	-9
3	×4	+1	-13

We just have to add 1 to obtain the output values.

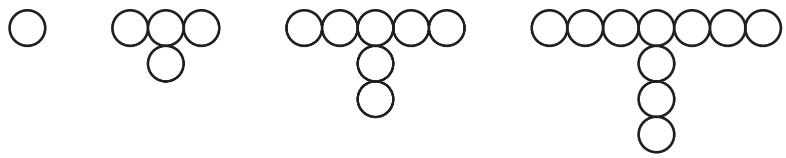
- So now we have our rule and we can complete the flow diagram.



Using bottle tops

Discussion

- Follow the same line of questioning as above.
- Here we will work directly from the table.



Step number	1	2	3	4	5
Number of bottle tops	1	4	7	10	13
	+3	+3	+3	+3	

- But we still have to do another operation:

Step number	1 ×3	2 ×3	3 ×3	4 ×3	5 ×3
Multiply by 3	3 -2	6 -2	9 -2	12 -2	15 -2
Number of bottle tops: subtract 2	1	4	7	10	13

- The rule is: Multiply the input by 3 and then subtract 2 to get the output.

Activity: Consolidating number patterns

1). What is the next number in this pattern?

302 400; 50 400; 7 200; 900; 100; ...

2). What is the next number in this pattern?

0,13; 0,12; 0,11; 0,10; …

A. 0,01
B 0,1
C. 0,09
D. 0,9
E. 0,10

3). What is the first number in this pattern?

______; 2100; 300; 60; 20

A. 3 900
B. 18 900
C. 2 900
D. 14 700
E. 4 580

Solutions

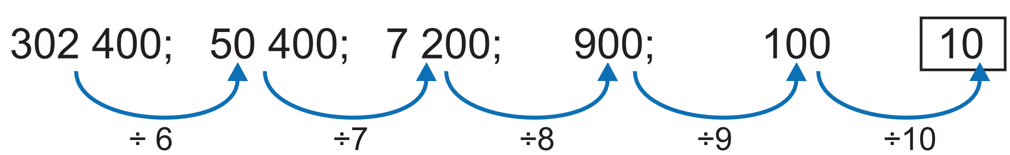
1). **10**

The difference between the numbers in the pattern is not constant. Therefore we look at a constant factor (a number that you must multiply or divide by to get the next number in the row).

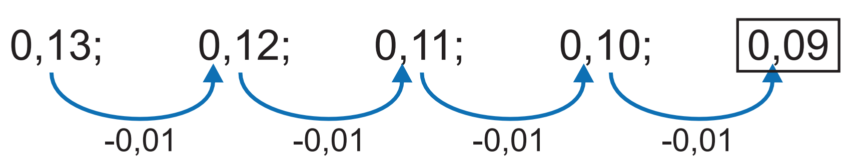
Look at the row from the smallest number on the right

- 100 × **9** = 900
- But 900 × 9 = 8 100 ≠ 7 200
- 900 × **8** = 7 200
- 7 200 × **7** = 50 400
- 50 400 × **6** = 302 400

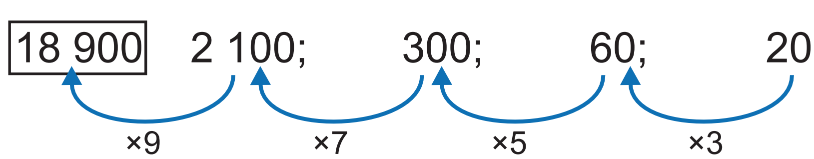
The pattern in the row is as shown:



2). **C. 0,09**



3). **B. 18 900**



Other examples of how to test number and geometric patterns

ANA 2014 Grade 6 Mathematics Item 1.5

1.5 What are the missing numbers in the number sequence?

0,9 ; 0,7 ; 0,5 ; ______ ; ______ .

A 0,4 ; 0,3
B 0,03 ; 0,1
C 0,3 ; 0,01
D 0,3 ; 0,1 [1]

ANA 2014 Grade 6 Mathematics Item 14

14 Look at the input and output numbers and complete the table.

Input	2	3	4	5	10	
output	5	8	11	14		44

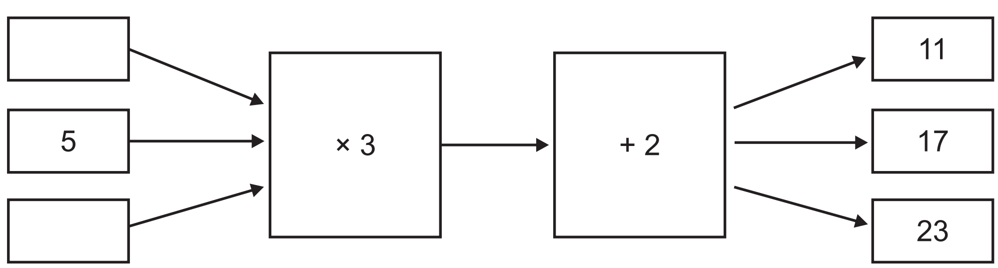
[2]

ANA 2014 Grade 6 Mathematics Items 15.1 & 15.2

15 Complete the flow diagram below.

15.1

15.2



[2]

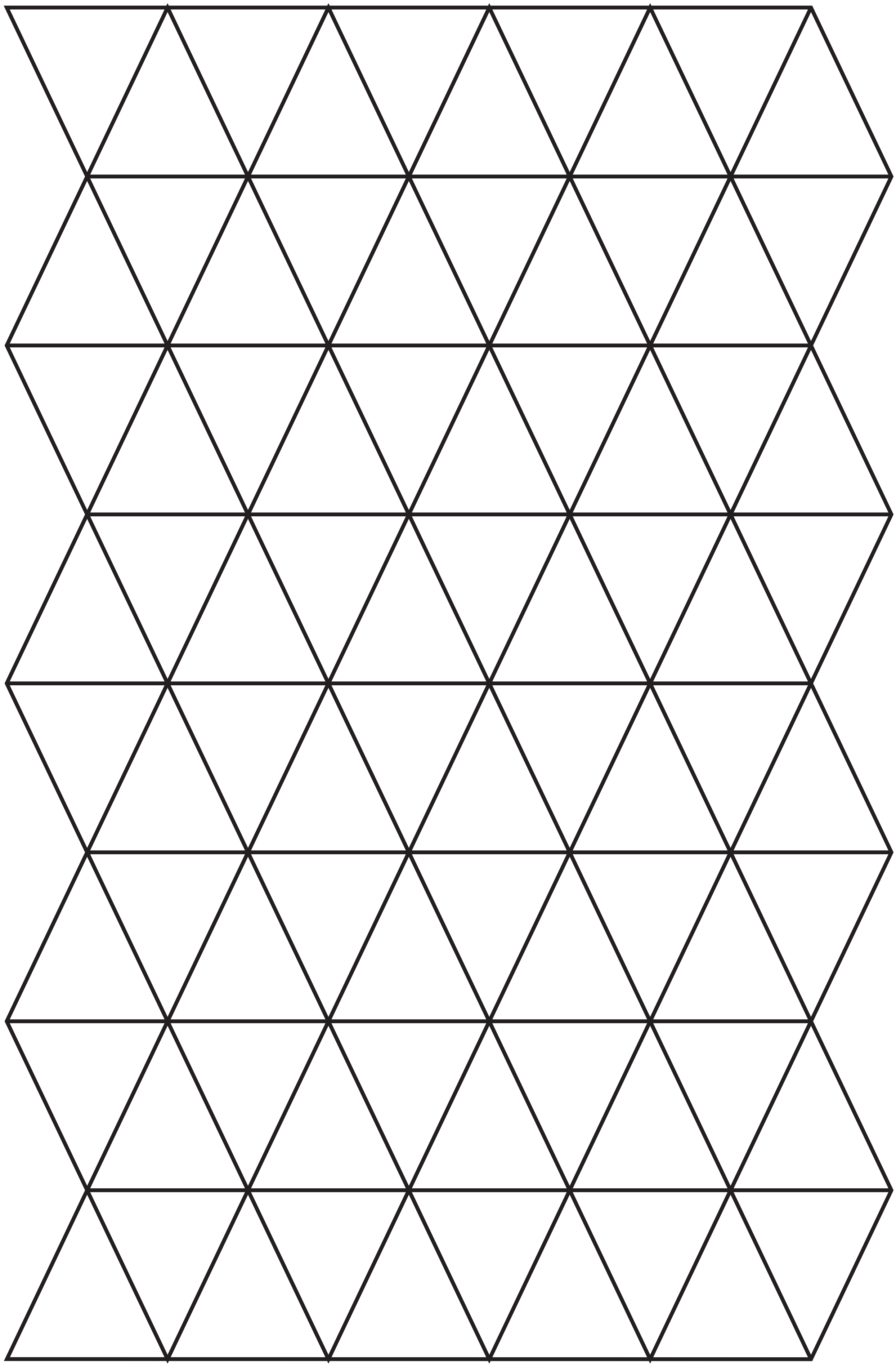
ANA 2014 Grade 6 Mathematics Item 16

16 How many matches will there be in the next figure if the diagram pattern is continued?

__________________ [1]

Printables: Squares grid

Printable: Triangles grid



Number sentences

ANA 2013 Grade 6 Mathematics Items 1.4, 6 and 7

1.4	Circle the letter with the correct answer: $22 + 35 = 63 - \Delta$ means that $\Delta =$ A 6 B 28 C 41 D 57	[1]
6	Complete: $0 \times (18 - 3) + (10 \div 2) - 2 =$ ________	[1]
7	Complete: $2 \times (4 + 5) = (2 \times$ ____ $) + (2 \times 5)$	[1]

What should a learner know to answer these questions correctly?

Learners should:

- Know the order of operations when performing calculations in a number sentence;
- Know that any number multiplied by 0 will give an answer of zero;
- Be able to recognise the distributive property of multiplication before addition;
- Understand the meaning of the equal sign;
- Know that the expression on the left hand side (LHS) of an equation is equal to the expression on the right hand side (RHS).

Where is this topic located in the curriculum? Grade 6 Term 1 and Term 2

Content area: Patterns, Functions, Algebra and Numbers, Operations and Relationships.

Topic: Number sentences: Whole numbers.

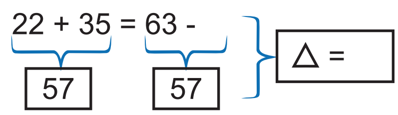
Concepts and skills:

- Solve and complete number sentences by
  - Inspection;
  - Trial and improvement.
- Recognise and use the commutative, associative and distributive properties of whole numbers.

What would show evidence of full understanding?

Item 1.4

- If the learner obtained the correct answer 6 (A): this shows full understanding as the learner realised that the sum of the numbers on the LHS must be the same as the difference of the numbers on the RHS.



Item 6

If the learner calculated and gave 3 as answer this displays full understanding.

- In the following example the learner calculated the values of the brackets first and then did the multiplication and addition in the correct order to get the answer of 3.

6. Complete: 0 x (18 – 3) + (10 ÷ 2) – 2 = 3

Item 7

- If the learner recognised this number sentence as the distributive property and could find the answer without any calculation.

- In the example shown the learner gave the value of the unknown number without showing any extra calculations. This indicates that the learner knows the distributive property of multiplication before addition.

7. Complete: 2 x (4 + 5) = (2 x 4) + (2 x 5) (1)

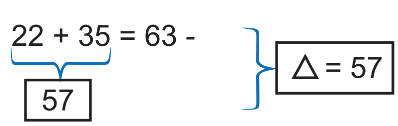
- Some learners calculated the LHS and RHS of the number sentence correctly to get the correct value, 4, of the unknown number as in the next example. The learner calculated the LHS and then found the value of the unknown on the RHS.

7. Complete: 2 x (4 + 5) = (2 x 4) + (2 x 5) (1)

What would show evidence of partial understanding?

Item 1.4

- If the learner chose 57 (D) as the answer this shows partial understanding based on the following reasoning:



- The learner ignored the other part of the RHS and gave the sum of the two numbers on the LHS as the solution. Learners saw the Δ as a placeholder for the "answer", demonstrating partial understanding of the role of the equal sign.

Item 6

- If the learner multiplied the entire left hand side by zero to get zero this shows that the leaner knows that any number multiplied by 0 will give an answer of zero, but the learner does not understand the distributive property.

6. Complete: 0 x (18 – 3) + (10 ÷ 2) – 2 = 0

- In the example illustrated the learner interpreted the number sentence incorrectly as:

$0 \times [(18 - 3) + (10 \div 2) - 2] =$ ________

- In other examples of partial understanding the learners ignored the zero on the left hand side and simply did the calculation as follows:

$(18 - 3) + (10 \div 2) - 2$ to get $15 + 5 - 2 = 18$

6. Complete: 0 x (18 – 3) + (10 ÷ 2) – 2 = 18

Item 7

- In this example the learner mistakenly saw the 4 + 5 as multiplication, but then calculated the rest of the number sentence correctly.

7. Complete: 2 x (4 + 5) = (2 x 15) + (2 x 5) (1)

- The following example demonstrates the common misconception that the equal sign means "find the answer". This learner simply calculated 4 + 5 and gave the answer as 9 in the open space, ignoring the rest of the numbers on the RHS.

7. Complete: 2 x (4 + 5) = (2 x 9) + (2 x 5) (1)

- Similarly, in the next example the learner calculated the value of the LHS correctly and simply filled in that as the value in the empty space, ignoring the rest of the numbers on the RHS.

7. Complete: 2 x (4 + 5) = (2 x 18) + (2 x 5) (1)

What would show evidence of no understanding?

Item 1.4

- If the learner arrived at 41 (C) as the answer by substituting 22 for Δ, did the subtraction and then gave 41 as the solution, this shows no understanding of the concepts being tested.

$$\left.\begin{array}{l} 22 + 35 = \underbrace{63 -}_{\boxed{41}} \end{array}\right\} \boxed{\Delta = 41}$$

- Similarly, in the next example the learner arrived at the incorrect answer of 28 (B) by substituting 35 for Δ, subtracting and then giving 28 as the solution. This shows that the learner does not understand the distributive property or that the expression on the LHS of an equation is equal to the expression on the RHS.

$$\left.\begin{array}{l} 22 + 35 = \underbrace{63 -}_{\boxed{28}} \end{array}\right\} \boxed{\Delta = 28}$$

Item 6

- If a learner gave any number other than 3 (full understanding) or zero (partial understanding) as the answer, this may show that learner either miscalculated as in the examples that follow or did not understand the processes required.

6. Complete: 0 x (18 – 3) + (10 ÷ 2) – 2 = 10

6. Complete: 0 x (18 – 3) + (10 ÷ 2) – 2 = 8

6. Complete: 0 x (18 – 3) + (10 ÷ 2) – 2 = 14

Item 7

- If the learner gave the incorrect answer as in the next example this shows no understanding of the purpose of brackets, nor of the distributive property. The learner multiplied 2 × (4 + 5) as follows:

2 × 4 = 8 + 5 = 13

7. Complete: 2 x (4 + 5) = (2 x 13) + (2 x 5) 2 x 4 = 4 + 5 = 13 - 2 4 (1)

What do the item statistics tell us?

Item 1.4

40% of learners answered the question correctly.

Item 6

17% of learners answered the question correctly.

Item 7

37% of learners answered the question correctly.

Factors contributing to the difficulty of the items

- Learners may be unable to follow the correct procedure for calculations.
- Learners may know the rule "brackets always come first", but may have ignored the multiplication sign before the bracket.
- The question followed directly after a similar question where learners had to "calculate the answer" for a number sentence (Item 7).
- Learners may not recognise the number sentence as an example of the distributive property (Item 7).
- Learners may not understand the relational meaning of the equal sign in a number sentence, .i.e. they do not know that the equal sign means "balance an equation", but think the equal sign means "find the answer".

Teaching strategies

Reading number sentences with understanding

- Learners need to be able to read number sentences and interpret their meaning correctly.
- Discuss with learners what a number sentence is.
    - It is a sentence made up of numbers and symbols.
    - Just like a sentence in English, a number sentence must also have a "verb". The verb in an equation is the equal sign.

Example 1

Write the following number sentences on the board and ask learners to explain (in words) how the answer or the unknown number in the sentence should be calculated:

$12 \div (4 + 2) \times 5 =$

Solution

I notice there are brackets in the number sentence

- I have to add the 4 and 2 to get 6.
- Then I have to divide 12 by 6 and multiply by 5.
- So I will say $12 \div 6$ is 2, multiplied by 5 is 10.

So my answer is 10.

Example 2

12 ÷ 4 + 2 × 5

Solution

There are three operations in this number sentence

- I will do the division and multiplication first, and then I will do the addition
- I can also put brackets around the numbers like this to help me to see the order in which I will deal with the operations:
- (12 ÷ 4) + (2 × 5) = 12 ÷ 4 is 3 and 2 times 5 is 10
- Then I will do the addition, 3 + 10 to get 13

So my answer is 13

The properties of zero

The activity below involves working with the properties of zero.

- When you multiply a number by zero the answer is zero.
  - For example, 2 x 0 = 0.
- When you add zero to a number the answer is the number itself.
  - For example, 2 + 0 = 2.

- Write the following questions on the board.
- Give your learners time to do the activity and then discuss the answers with them.

Examples

1). 7 + 5 × 0 – 4 =

2). 0 × 3 + 12 ÷ 3 =

3). 27 ÷ 3 + 5 + 0 =

4). 27 + 3 – 5 × 0 =

- Hints to learners:
  - When you see the × or ÷ operation sign use brackets to remind you that these operations have first priority.
  - Look out for zeros in a number sentence and be sure to work with zero correctly.

Solutions

- In the above examples the brackets could be inserted as follows to show how to work with the correct order of operations.

- Discuss the solutions, making sure that learners are able to work correctly with zero in the calculations.

1). $7 + (5 \times 0) - 4 = 7 + 0 - 4 = 3$
2). $(0 \times 3) + (12 \div 3) = 0 + 4 = 4$
3). $(27 \div 3) + 5 + 0 = 9 + 5 + 0 = 14$
4). $27 + 3 - (5 \times 0) = 30 - 0 = 30$

Understanding the role of the equal sign in number sentences

Learners sometimes think that the equal sign means "find the answer".

- This is because when they are introduced to the equal sign that is how the equal sign is used.
- In ANA Item 7 learners had to find the answer by calculating the left hand side of the number sentence to balance the equation, not simply "find the answer". This caused problems for some learners.
- You need to work through different examples with your class where the equal sign is not always between the working (on the left) and the answer (on the right).
- Here are some examples of number sentences that can be used to help your learners with the order of operations and the position of the equal sign in the number sentence.

Examples

When can the equal sign be interpreted as meaning "find the answer"?

- When a number sentence is given and you have to calculate only the value of the one side, then the equal sign means the "right hand side is equal to the solution when you perform the operations indicated on the left hand side". The RHS will then be "the answer".

For example:

- $14 \div 7 + 2 \times (9 + 0) =$ ___ (the answer of the calculation is on the RHS).
- The right hand side will contain the value that will be obtained after performing all the operations on the left hand side.
- The "answer" to the calculation is 20, which is the value of the "left hand side" of the number sentence.
- The equal sign then means that the left hand side is equal to the right hand side and the right hand side is also the "answer".

When does the equal sign NOT mean "find the answer of one side"?

- When the equal sign takes on the function of separating the left hand and the right hand side of the number sentence, the value of the unknown in the number sentence must be found in order to make the LHS equal to the RHS.
- The activity is sometimes called "balancing an equation".

For example:

Write the number sentence on the board: 2 × ___ = 13 – 7

- Ask learners to explain how they will find the number that is missing.
- You could use some of the following questions to prompt the class for responses.

- Show me which is the LHS and which is the RHS of the number sentence?

  Answer:

  $$\underbrace{2 \times \_\_\_}_{\text{LHS}} = \underbrace{13 - 7}_{\text{RHS}}$$

- On which side is the unknown value? On the LHS

- Which sides contains more information? The RHS.

- What is the value of the RHS? 13 – 7 = 6

- How does that help us to find the unknown number on the LHS? We know that 2 times the number is equal to 6

- So what is the unknown number? The unknown number is 3

- Therefore the completed number sentence will be: 2 × 3 = 13 – 7

Your learners need to do plenty of similar activities to consolidate this use of the equal sign. A short activity that you could use follows.

Activity: Balancing equations

Fill in the missing numbers to make true sentences.

1). 15 × ____ + 11 = 41
2). 11 – (30 ÷ 6) = 2 × ______
3). 48 – (14 + 13) = 15 + ______
4). 30 – (12 ÷ ___) = 28
5). 48 – 14 + 13 = 15 + ______
6). (30 – 12) ÷ 6 = 27 ÷ ___

Solutions

1). 15 × ____ + 11 = 41 Missing number: 2

2). 11 – (30 ÷ 6) = 2 × ______ Missing number: 3

3). 48 – (14 + 13) = 15 + ______ Missing number: 6

4). 30 – (12 ÷ ___) = 28 Missing number: 6

5). 48 – 14 + 13 = 15 + ______ Missing number: 32

6). (30 – 12) ÷ 6 = 27 ÷ ______ Missing number: 9

Recognising and using the commutative, associative and distributive properties of whole numbers

- The properties of numbers are theoretical and learners need to learn how the properties work, although they do not need to know the names of these properties.
- The properties of numbers help us to simplify calculations and so they are useful.
- Give learners examples that will help them to understand the way in which number calculations work.

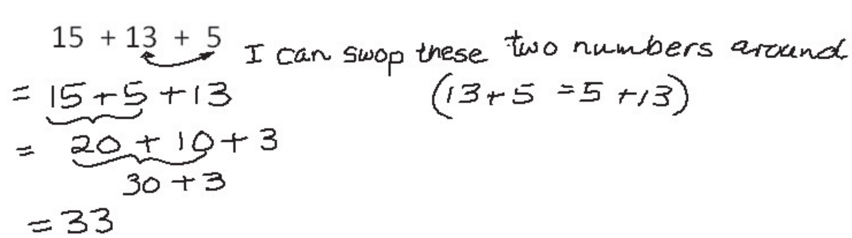
Examples

The following examples of learners' work illustrate learners' understanding of the properties of numbers and their explanations given for their answers.

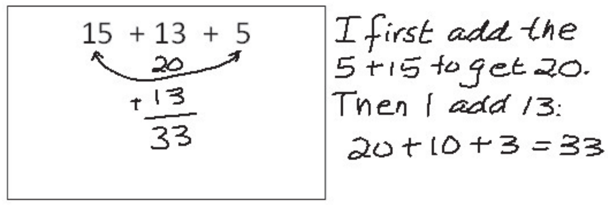
- Work through these examples yourself with your class.
- Some of the examples show working. Write this working on the board and discuss it with your learners: this will give them an opportunity to talk about the properties of numbers and how to use them in calculations.
- REMEMBER: You need to allow learners to express their understanding, but they don't have to say “this is an example of the distributive property” – they just need to show that they understand that they should apply the distributive property.

1). 15 + 13 + 5

- The first explanation given shows that the learner understands the commutative law of whole numbers.



- The next explanation shows that the learner first simplified the calculation by applying the commutative law and then was able to do a mental calculation. This learner also shows understanding.



2). 20 × 13

20 x 13 can be calculated by using the distributive property:

20 × 13

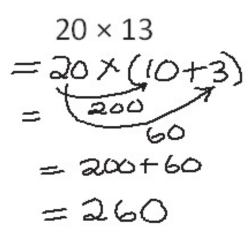
= 20 × (10 + 3)

= 20 × 10 + 20 × 3

= 200 + 60

= 260

- A worked example demonstrating the learner's understanding of the use of the distributive property to simplify the numbers so that mental calculations can be done is shown.



- Encourage learners to break up numbers so that they can use mental calculations to simplify calculations and to show the processes they use.

Activities: Calculations using grouping of numbers

1). In the four examples below the equal sign is between the calculation and the answer, but sometimes the “answer” will be on the right, sometimes on the left. This starts to alert learners to the idea that equal signs are not always in the same place in a number sentence.

a). 15 × 3 – 24 ÷ 4 = ________ (Answer: 39)

b). 5 + (120 ÷ 10) – 17 = _______ (Answer: 0)

c). _______ = 13 – (12 ÷ 4) (Answer: 10)

d). _______ = 15 × 4 + 15 × 2 (Answer: 90)

2). This activity involves grouping numbers or splitting numbers up to simplify your calculations.

NOTE: Try to do the calculations mentally as far as possible.

a). $3 + 4 + 7 + 6$

b). $12 \times 3 \times 5$

c). 3×15

d). 13×13

e). $12 + 15 + 17$

Solutions

The solutions suggest ways in which the numbers can be grouped and broken up in order to calculate the answers. Learners may have used other ways of doing this. Make sure you discuss alternative methods and look for other ways of doing the same calculation correctly.

a). $3 + 4 + 7 + 6$

10 + 10

$= 20$

b). $12 \times 3 \times 5$

×3

180

$= 180$

c). 3×15

$= 3 \times 3 \times 5$

$= 9 \times 5$

$= 45$

OR

3×15

$= 3 \times (10 + 5)$

30

+15

$= 45$

d). 13×13

$= 13 \times (10 + 3)$

130

+39

$= 169$

e). $12 + 15 + 17$

2 + 5 + 7 = 14

10 + 10 + 10 = 30

44

$= 44$

Other examples of how to test number sentences

ANA 2014 Grade 6 Mathematics Items 1.3

1.3	Fill in the missing number in 6 + 3 + 5 = _____ + 5. A 9 B 11 C 8 D 15	[1]

ANA 2014 Grade 6 Mathematics Items 5

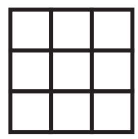
5.	Complete: (14 ÷ 2) + (51 – 48) = ___________	[1]

Notes:

2-D Shapes: Squares

ANA 2013 Grade 6 Mathematics Item 16

16 How many squares are there in the diagram below?



__________ [2]

What should a learner know to answer this question correctly?

Learners should be able to:

- Recognise squares;
- Identify patterns or arrangements which result in a square.

Where is this topic located in the curriculum? Grade 6 Term 3

Content area: Space and Shape.

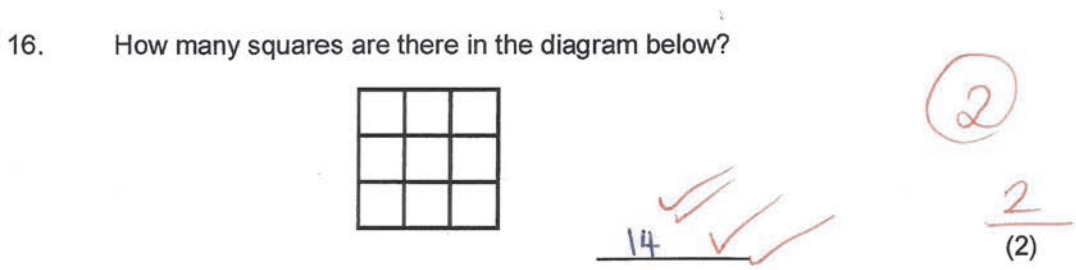
Topic: Properties of 2-D shapes.

Concepts and skills:

- Recognise squares.

What would show evidence of full understanding?

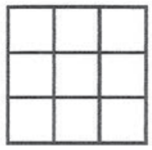
- Number of squares correctly identified as 14.



What would show evidence of partial understanding?

- If the learner identified the number of squares as 9 as seen in the example which follows: it is likely that this learner counted only the small squares, not any of the bigger squares which are made up of combinations of the smaller squares.

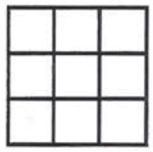
16. How many squares are there in the diagram below?



9

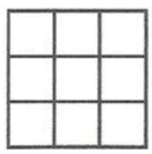
- In the following examples the learners did not manage to identify all the squares, but realised that there are more than 9: this means the learners did count some of the larger squares to differing extents.

16. How many squares are there in the diagram below?



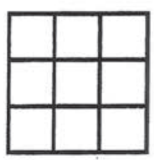
10 (2)

16. How many squares are there in the diagram below?



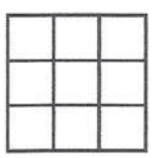
11

16. How many squares are there in the diagram below?



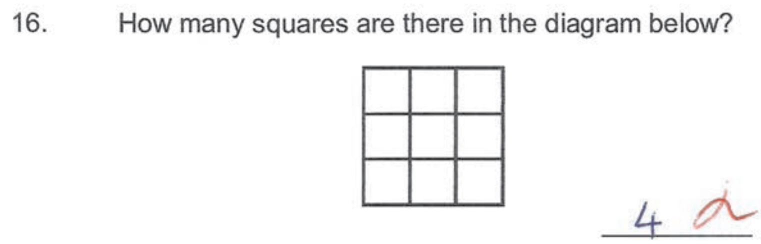
12 (2)

16. How many squares are there in the diagram below?



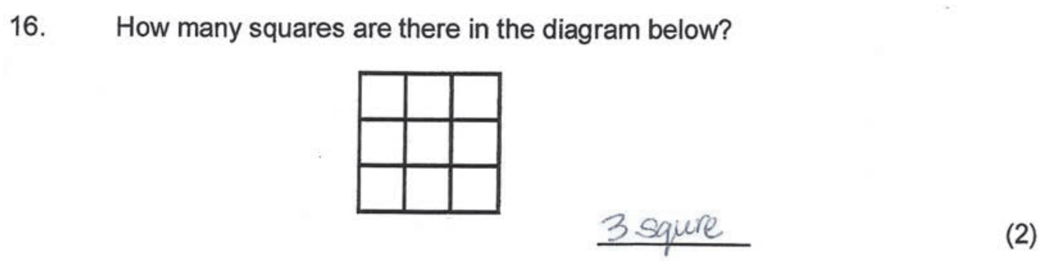
13

- If the learner identified the number of squares as being less than 9: in these examples the learners identified some squares, but did not count all of them.



16. How many squares are there in the diagram below?

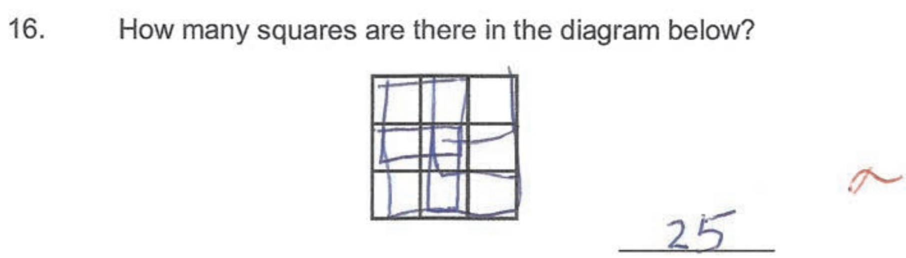
4



16. How many squares are there in the diagram below?

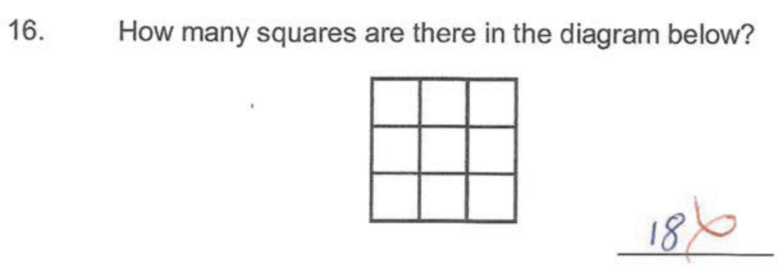
3 squre (2)

- If the learner identified the number of squares as being more than 14: these learners identified a number of squares, but must have re-counted some squares or counted some shapes that were not squares as well.



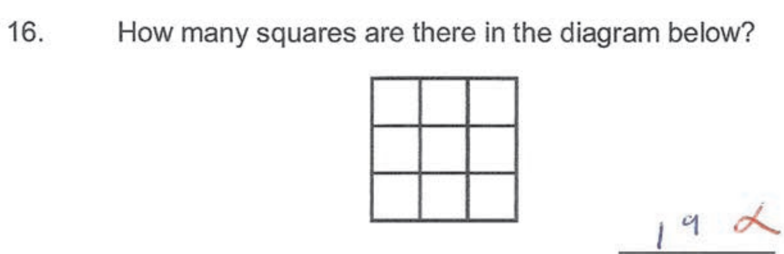
16. How many squares are there in the diagram below?

25



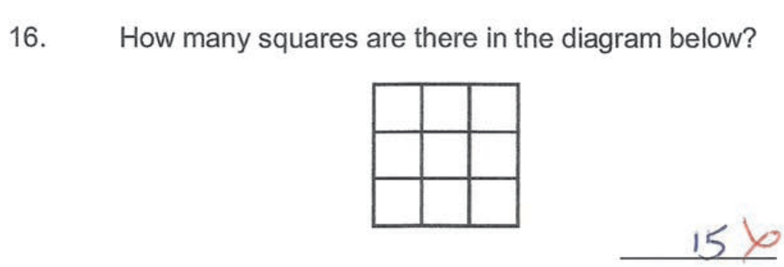
16. How many squares are there in the diagram below?

18



16. How many squares are there in the diagram below?

19



16. How many squares are there in the diagram below?

15

What would show evidence of no understanding?

- Not answering the question at all;
- Not counting the squares but just writing the word "squares" in the place of the answer.

16. How many squares are there in the diagram below?

What do the item statistics tell us?

42% of the learners answered correctly.

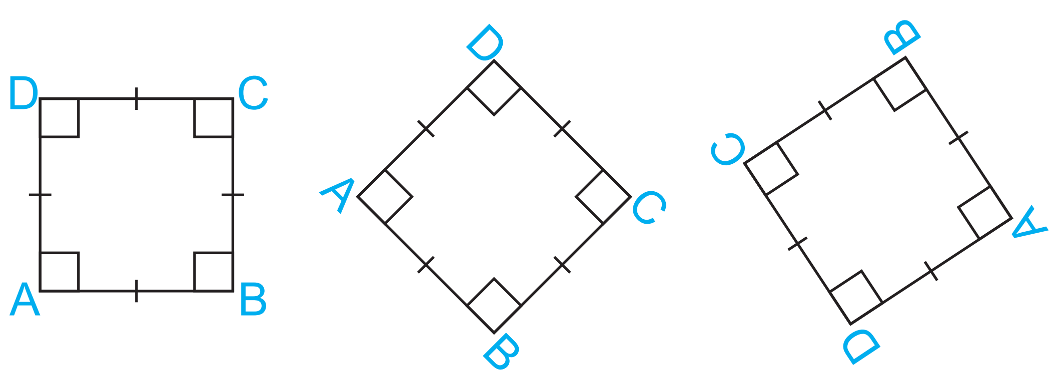
Factors contributing to the difficulty of the item

- Most learners only counted the small squares and did not realise that there are other embedded squares;
- This is a higher order question which needed a greater understanding of squares in order to identify the 5 squares which are made up of combinations of smaller squares.

Teaching strategies

Teaching the properties of squares

- Learners must be made aware of the properties of quadrilaterals with particular reference to squares.
    - All sides of a square are equal in length.
    - All angles in a square are equal to 90°.
- The following diagrams show a square in a few different positions.
- Show your learners many diagrams of squares in many different positions, so that they start to recognise a square wherever they see one because they know what properties are present in the shapes that they see and they know the properties of a square.



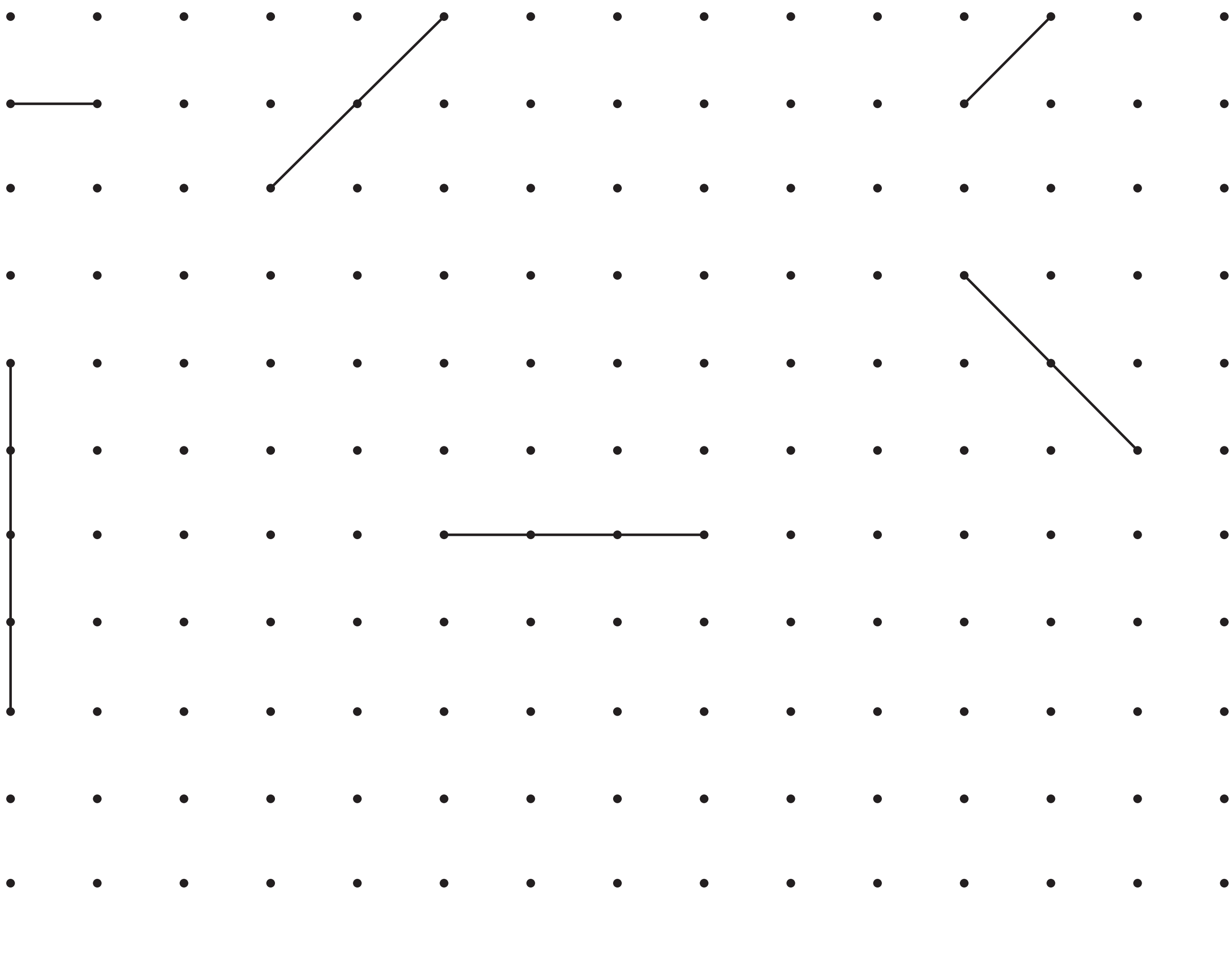
- In as much as we have to make learners conscious of the properties of squares, they must also be made aware of the fact that squares may be drawn in any orientation, not only one way.

Activity

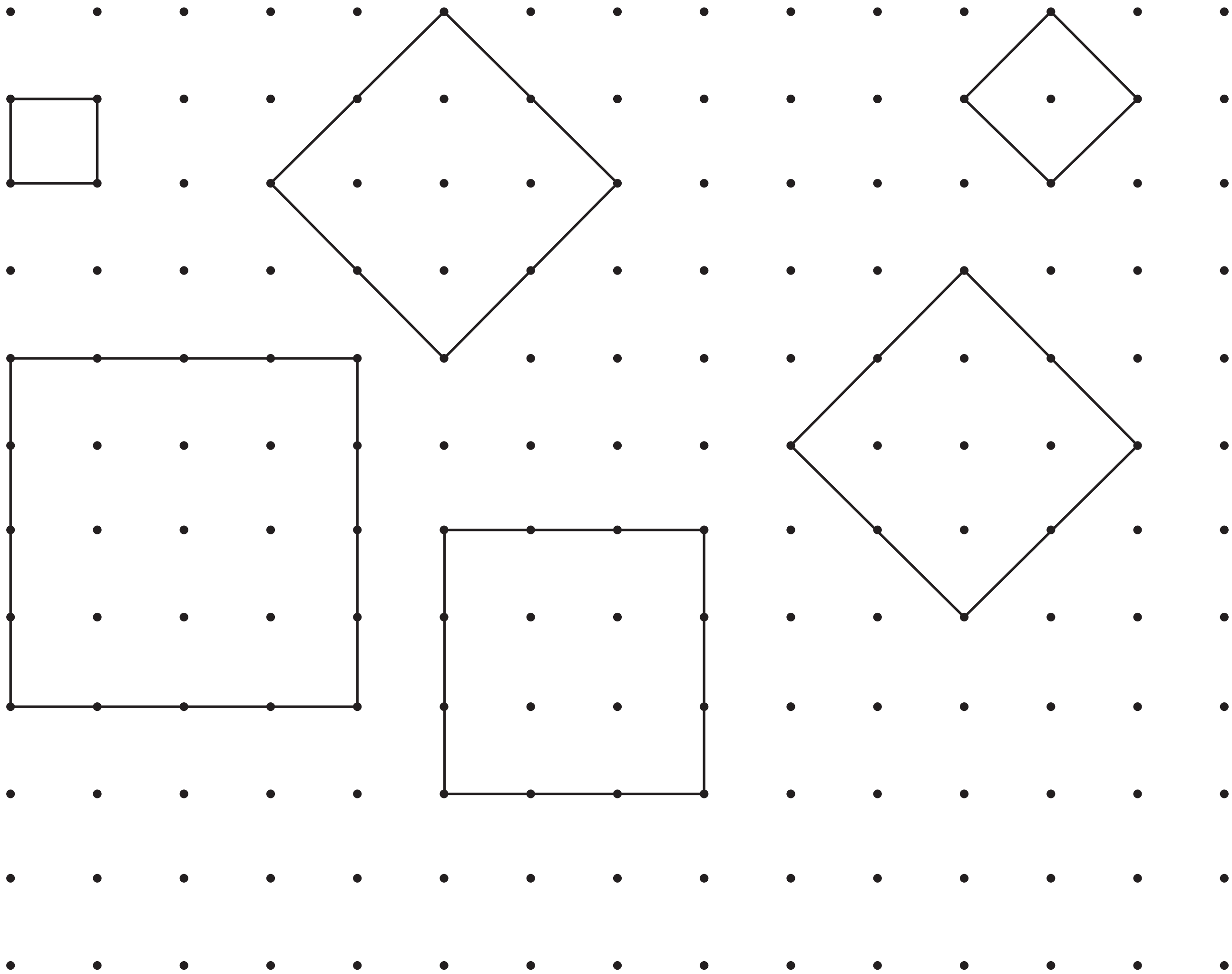
- Learners may be taken through some activities such as the one that follows to enhance their knowledge of and ability to work with squares. Learners should be encouraged to add more square sketches according to their own preference.

- Using dotted paper to draw squares.

- Copy and complete the following. Each shape that you draw must be a square.



Solution



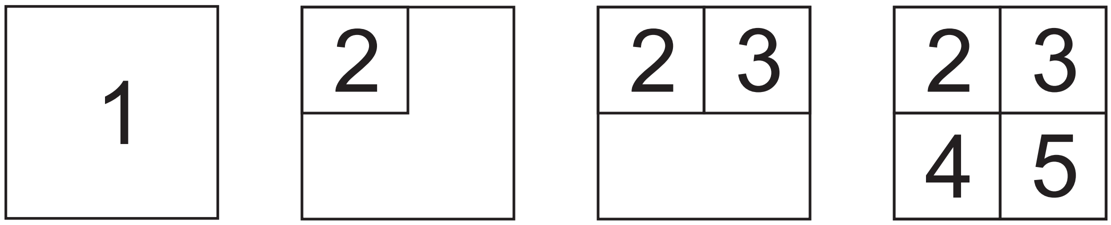
Using smaller squares/blocks to build larger squares

- Learners need to learn how to build bigger squares from given smaller squares. The smaller squares may be arranged in any possible way as long as the arrangement is not a square already.
- In addition, to identify single squares, learners may be asked to identify squares from composite diagrams.
- You can help them to see this kind of development using visual examples.

Examples

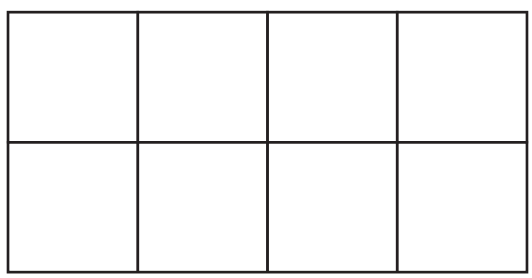
1). In the sequence of squares shown the number of squares in a diagram is increased one at a time.

- Draw the sequence of shapes on the board and then discuss the way in which the squares can be counted.



- In the first diagram, there is 1 square.
- One smaller square can be marked within the original square to give 2 squares.
- There is now 1 big square and 1 small square (which is inside the big one).
- Overall, we now have 2 squares.
- Other squares can be added to this shape to give us 3, 4 and 5 squares as shown.

2). The skill gained above can be applied to determine how many squares there are in the diagram below.



- There are 8 small squares.
- There are 3 bigger squares, each made up of 4 small squares:
  - 1 on the left, 2 blocks across and 2 blocks up;
  - 1 in the centre, 2 blocks across and 2 blocks up;
  - 1 on the right, 2 blocks across and 2 blocks up.
- Work though the example counting with your learners the 3 bigger squares.
- The total number of squares is 8 + 3 = 11.

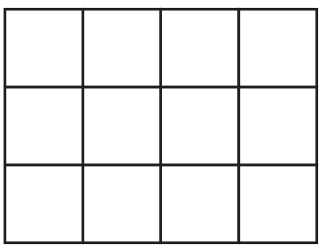
Activity: Identifying the squares

How many squares are in the following diagrams?

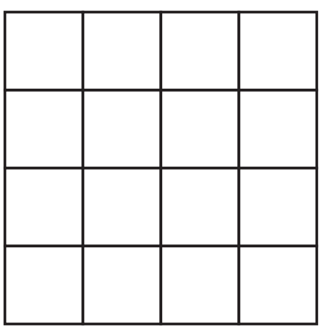
1).

2).

3).



4).



Solutions

1). 8 squares

2). 14 squares

3). 20 squares

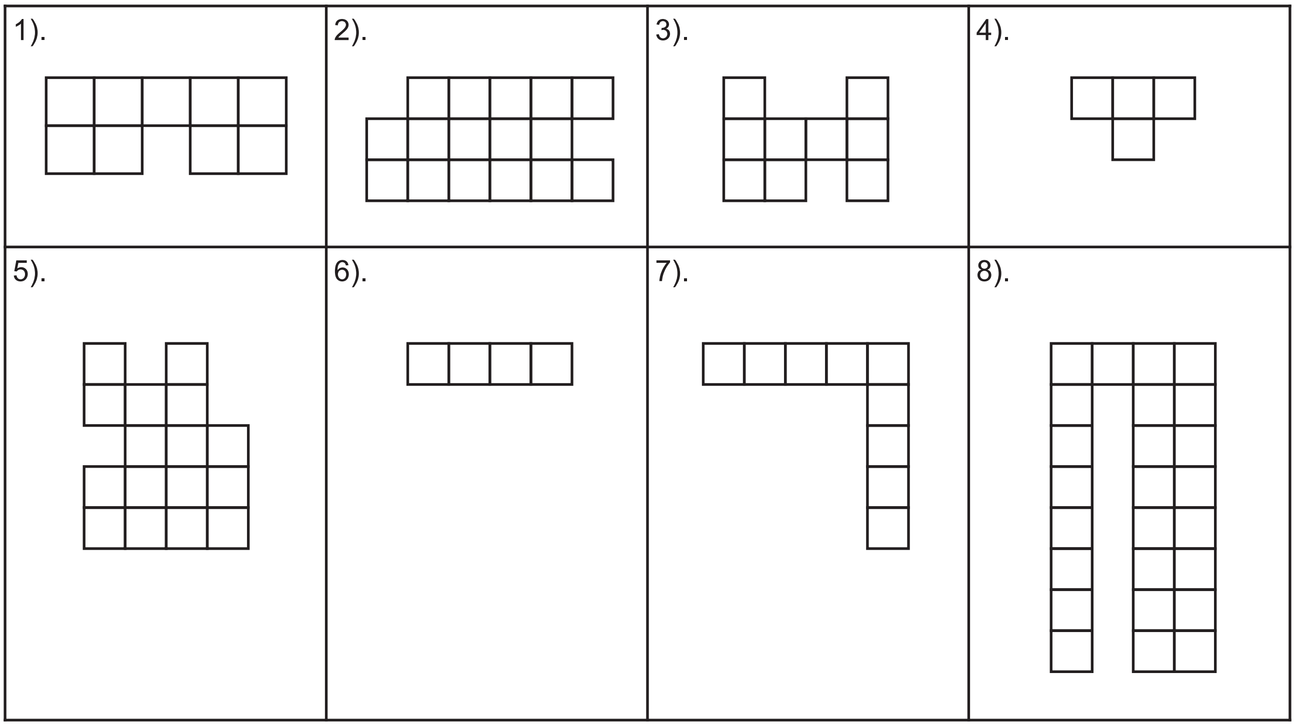
4). 31 squares

Building squares out of other squares

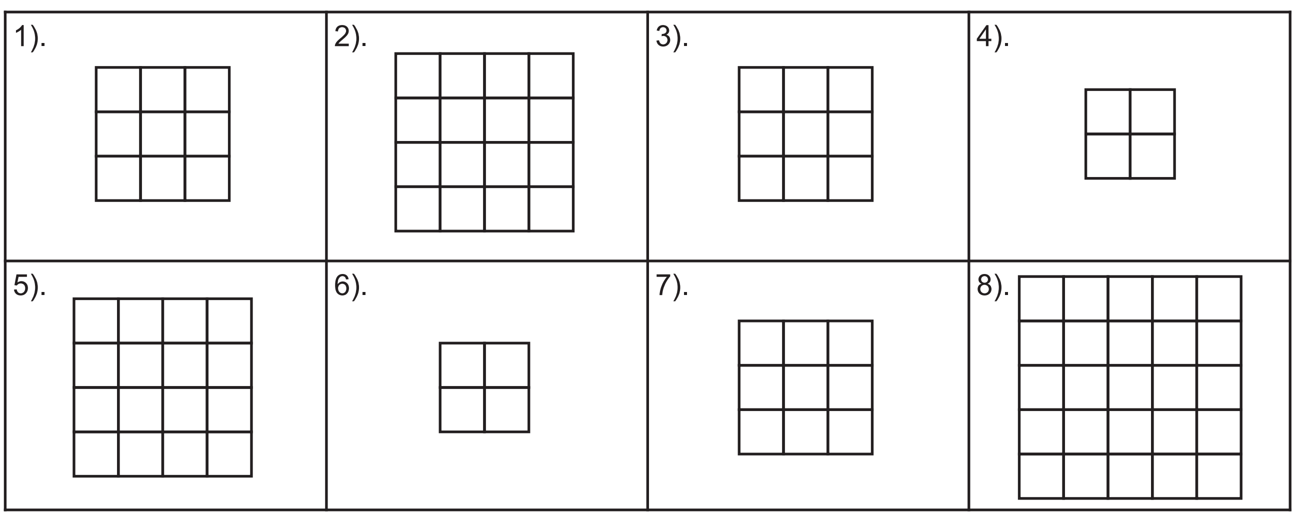
- Learners will also learn from building bigger squares from given smaller squares.
- This will help them to realise that a square has the same number of units of length across (in the columns) and up/down (in the rows).
- The number of units in the length and the breadth must be the same because the sides of a square are all equal in length.

Activity: Using small squares to build bigger squares

Complete the following. Use all the smaller squares in each group of squares given to build one big square.



Solutions



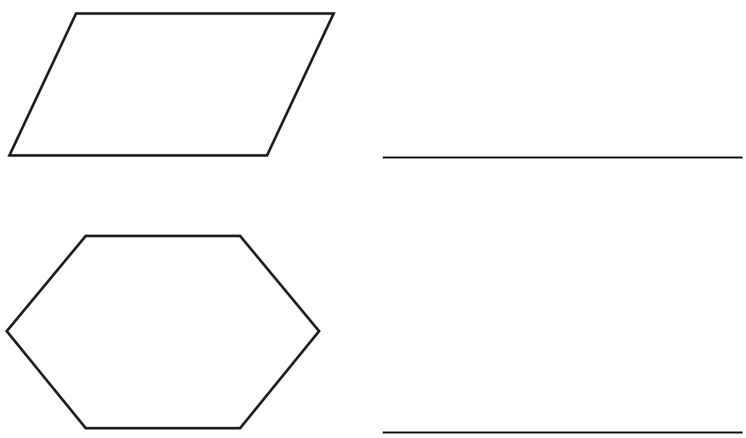
.

2-D Shapes – Quadrilaterals and other polygons

ANA 2013 Grade 6 Mathematics Item 29

29. Choose a word from the list to match the shape.

rectangle; parallelogram; pentagon; hexagon



[2]

What should a learner know to answer this question correctly?

Learners should be able to:

- Know the names of different 2-D shapes (rectangle, parallelogram, pentagon, hexagon);
- Differentiate between different 2-D shapes (rectangle, parallelogram, pentagon, hexagon);
- Match the given shapes to their names.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Space and Shape.

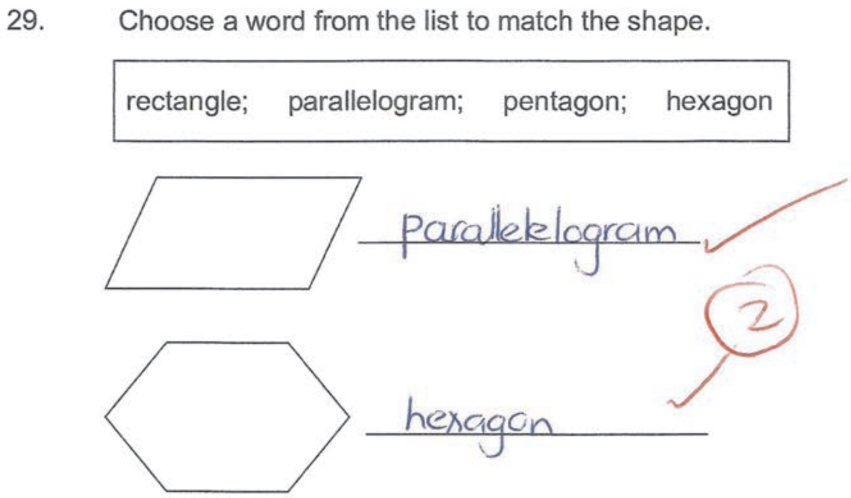
Topic: Properties of 2-D shapes.

Concepts and skills:

- Recognising and naming 2-D shapes.

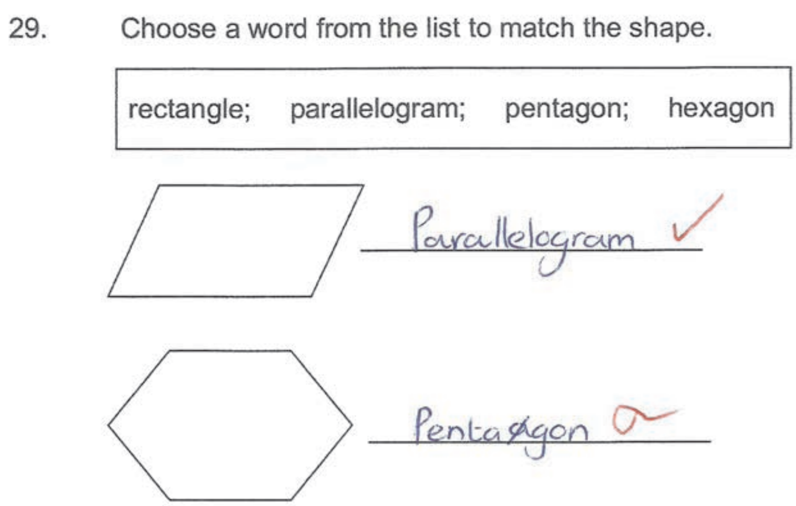
What would show evidence of full understanding?

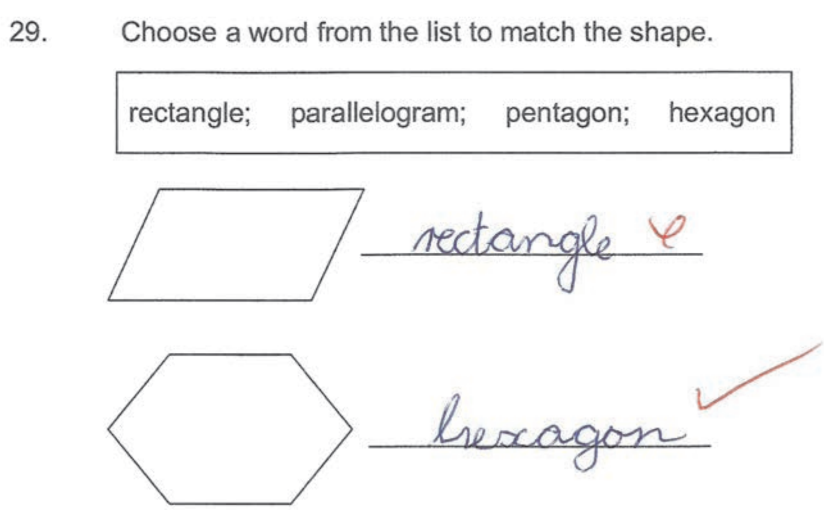
- If the learner named both the parallelogram and the hexagon correctly as shown in the example.



What would show evidence of partial understanding?

- If the learner named only one of the shapes correctly, as shown in the next example.



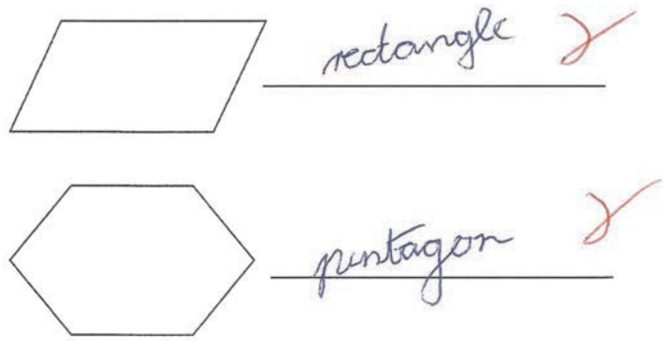


What would show evidence of no understanding?

- If the learner identified neither shape correctly as shown here.

29. Choose a word from the list to match the shape.

rectangle;	parallelogram;	pentagon;	hexagon



What do the item statistics tell us?

64% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not be able to differentiate between a rectangle and a parallelogram.
- Learners may confuse a pentagon (5-sided shape) and a hexagon (6-sided shape).

Teaching strategies

Definitions of Polygons

Rectangle: A closed plane figure with four straight sides and four right angles. Opposite pairs of sides of a rectangle are always equal. It may be different from a square in that its adjacent sides do not have to be equal. A square is a rectangle.

Parallelogram: A flat closed shape with opposite pairs of sides parallel and equal in length. Opposite pairs of angles are equal in size. Rectangles, squares and rhombuses are all parallelograms.

Pentagon: A closed plane figure with five straight sides and five angles.

Hexagon: A closed plane figure with six straight sides and six angles.

Properties of polygons and quadrilaterals

- Polygons are defined as closed 2-D shapes with straight sides.
- Given below are different types of polygons that Grade 6 learners need to know about.

- The quadrilaterals are one of the special groups of polygons which have 4 sides.
- In order to know the difference between a parallelogram, a rectangle and a square, learners need to know the properties of quadrilaterals.

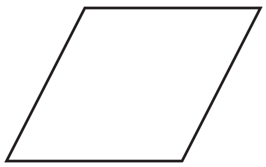
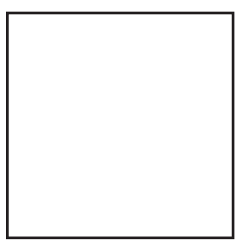
Polygons

Triangles	Quadrilaterals	Shapes with more than 4 sides
• Right-angled triangle • Scalene triangle • Isosceles triangle • Equilateral triangle	• Parallelogram • Rectangle • Square • Rhombus • Kite • Trapezium	• Pentagon • Hexagon • Heptagon • Octagon • Nonagon • Decagon

- In the table that follows detail is given about the characteristics of different quadrilaterals.
- You could write this table on the board and discuss the differences and similarities between the types of quadrilaterals.
- Some shapes have some of the same characteristics as other shapes. Look out for these in the table.

Notes:

Examples

Quadrilaterals	Features
1). Parallelogram 	• Opposite sides are equal in length • Opposite sides are parallel • Opposite angles are equal
2). Rhombus 	• All four sides are equal in length • Opposite sides are equal in length • Opposite sides are parallel • Opposite angles are equal • A rhombus is also a parallelogram!
3). Rectangle 	• Opposite sides are equal in length • Opposite sides are parallel • All angles are 90° • A rectangle is also a parallelogram!
4). Square 	• Opposite sides are equal in length • Opposite sides are parallel • All angles are 90° • All sides are equal • A square is a parallelogram! • A square is also a rectangle! • A square is also a rhombus!

Activity: Properties of quadrilaterals

1). Referring to the preceding table, what is the difference between a square and a rectangle?

2). Give three 2-D shapes that can be called parallelograms.

3). What is the difference between a square and a rhombus?

Solutions

1). In a square all sides are equal, whilst in a rectangle opposite sides are equal.

2). Rectangles
Squares
Rhombuses

3). In a square all angles are 90°, whereas a rhombus has 2 acute angles and 2 obtuse angles.

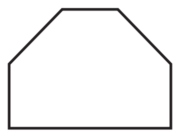
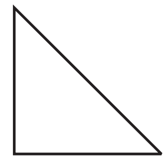
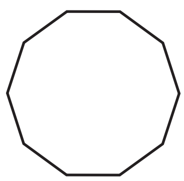
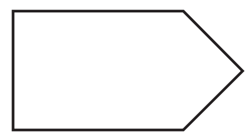
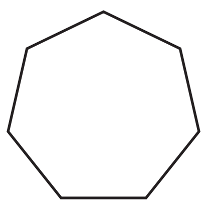
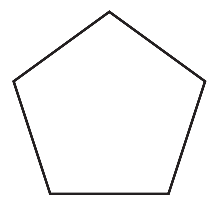
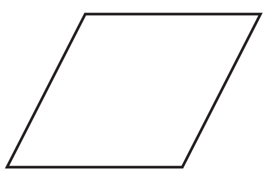
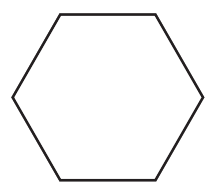
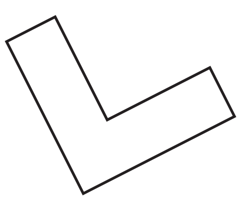
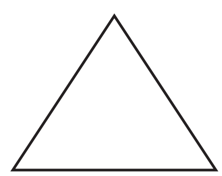
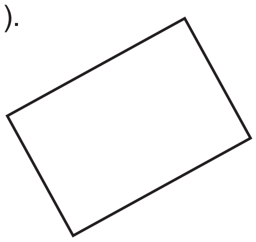
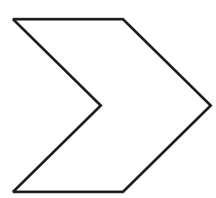
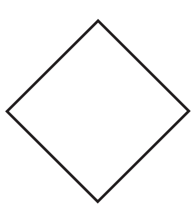
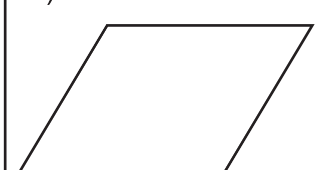
Method for naming of polygons

- Polygons are named according to the number of angles (and sides) that they have. The word polygon can be broken into two parts. The prefix 'poly' means many, whilst the suffix 'gon' means angle.
- The interesting thing is that the number of angles is always the same as the number of sides in a polygon.
- Learners need to know the prefixes that will help them to name certain important polygons. The prefixes relate to the number of angles and sides in the different polygons.

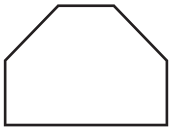
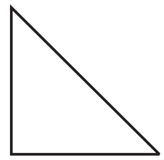
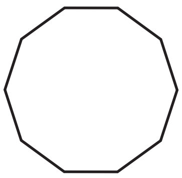
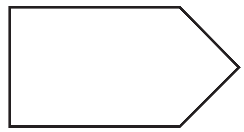
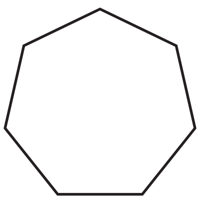
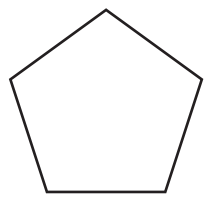
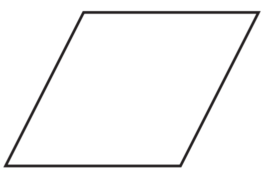
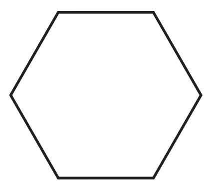
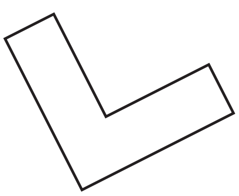
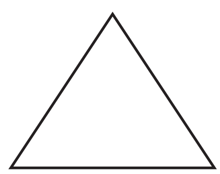
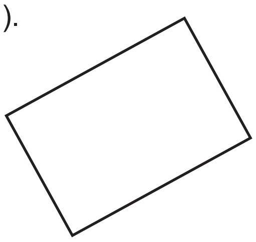
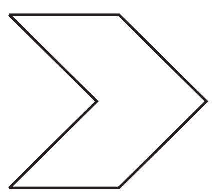
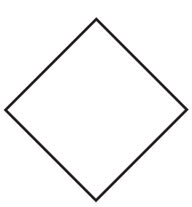
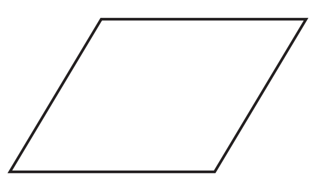
Prefix	Number of angles/sides of the polygon	Name of polygon
1. Tri-	3	Triangle
2. Quad-	4	Quadrilateral
3. Penta-	5	Pentagon
4. Hexa-	6	Hexagon
5. Hepta-	7	Heptagon
6. Octa-	8	Octagon
7. Nona-	9	Nonagon
8. Deca-	10	Decagon

Notes:

Activity: Identify the following polygons. Write the names in the spaces provided

Polygon	Name	Polygon	Name
1). 		2). 	
3). 		4). 	
5). 		6). 	
7). 		8). 	
9). 		10). 	
11). 		12). 	
13). 		14). 	

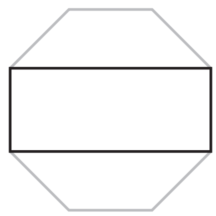
Solutions

Polygon	Name	Polygon	Name
1). 	Hexagon	2). 	Triangle
3). 	Decagon	4). 	Pentagon
5). 	Heptagon	6). 	Pentagon
7). 	Quadrilateral (or parallelogram or rhombus)	8). 	Hexagon
9). 	Hexagon	10). 	Triangle
11). 	Quadrilateral (or rectangle or parallelogram)	12). 	Hexagon
13). 	Quadrilateral (or square or parallelogram or rhombus)	14). 	Quadrilateral or parallelogram

Another example of how to test 2-D Shapes – quadrilaterals and other polygons

ANA 2014 Grade 6 Mathematics Item 18

18 Name the three different 2-D shapes in the diagram.



18.1 ____________________ [1]

18.2 ____________________ [1]

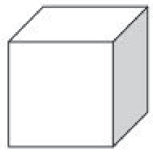
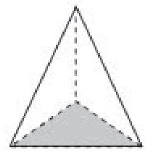
18.3 ____________________ [1]

Notes:

3-D Shapes

ANA 2013 Grade 6 Mathematics Item 19

19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	
Number of edges		6
Number of faces	6	

[3]

What should a learner know to answer this question correctly?

Learners should be able to:

- Differentiate between 3-D objects and 2-D shapes;
- Identify and work with the vertices, edges and faces of 3-D objects.

Where is this topic located in the curriculum? Grade 6 Term 2

Content area: Space and Shape.

Topic: Properties of 3-D objects.

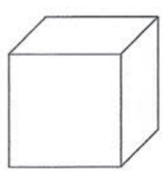
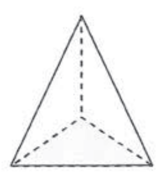
Concepts and skills:

- Describe 3-D objects according to the number and shape of faces, edges and vertices.

What would show evidence of full understanding?

- If the learner completed the table correctly for the cube and triangular pyramid.

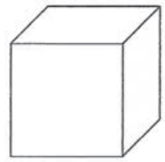
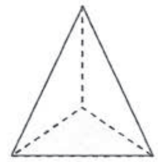
19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	4 ✓
Number of edges	12 ✓	6
Number of faces	6	4 ✓

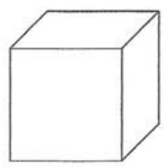
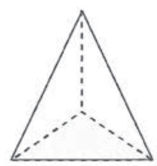
What would show evidence of partial understanding?

- If the learner gave one or two correct answers as shown in the following examples.

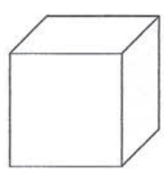
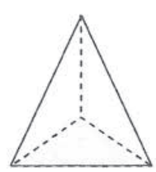
19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	4 ✓
Number of edges	12 ✓	6
Number of faces	6	3 ✗

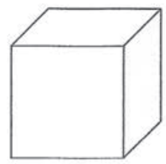
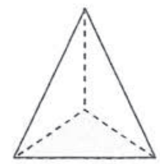
19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	4 ✓
Number of edges	8 ✗	6
Number of faces	6	3 ✗

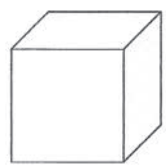
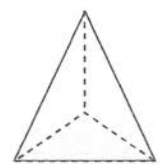
19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	5 ~~5~~ ~~4~~ ✗
Number of edges	12 ✓	6
Number of faces	6	10 ✗

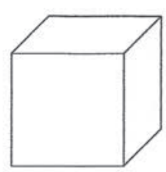
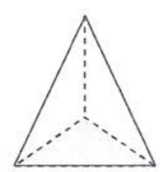
19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	5 ✗
Number of edges	12 ✓	6
Number of faces	6	4 ✓

19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	6 ✗
Number of edges	rectangle prism ✗	6
Number of faces	6	4 ✓

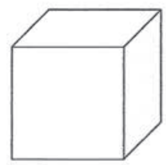
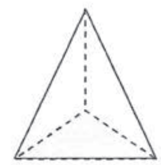
19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	4 ✓
Number of edges	10 ✗	6
Number of faces	6	4 ✓

What would show evidence of no understanding?

- If the leaner was unable to correctly identify and count the properties of the 3-D objects as shown in this answer.

19. Complete the table:

		
Name of 3-D object	Cube	Triangular pyramid
Number of vertices	8	10
Number of edges	10	6
Number of faces	6	3

What do the item statistics tell us?

40% of learners answered the question correctly.

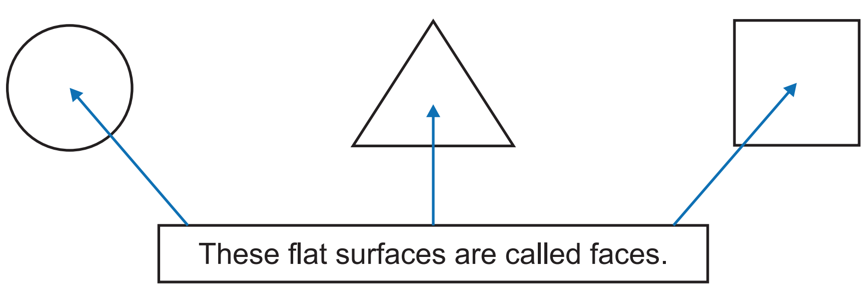
Factors contributing to the difficulty of the item:

- Not all the edges and faces of the cube are shown.
- Learners may be unfamiliar with 3-D objects.
- Learners may have difficulty in interpreting diagrams of 3-D objects correctly.
- Learners may be unfamiliar with the terminology used: edges, vertices and faces.

Teaching strategies

Definition of terms

- A face is a flat surface on a 3-D (3-dimensional) object.
  - 2-dimensional shapes are flat surfaces, but we do not call them faces unless we are speaking about them as the faces of 3-D objects.
  - The flat surfaces of 2-D shapes make up the faces of 3-D objects. A few common examples of 2-D shapes that make up the faces of 3-D objects are given here.



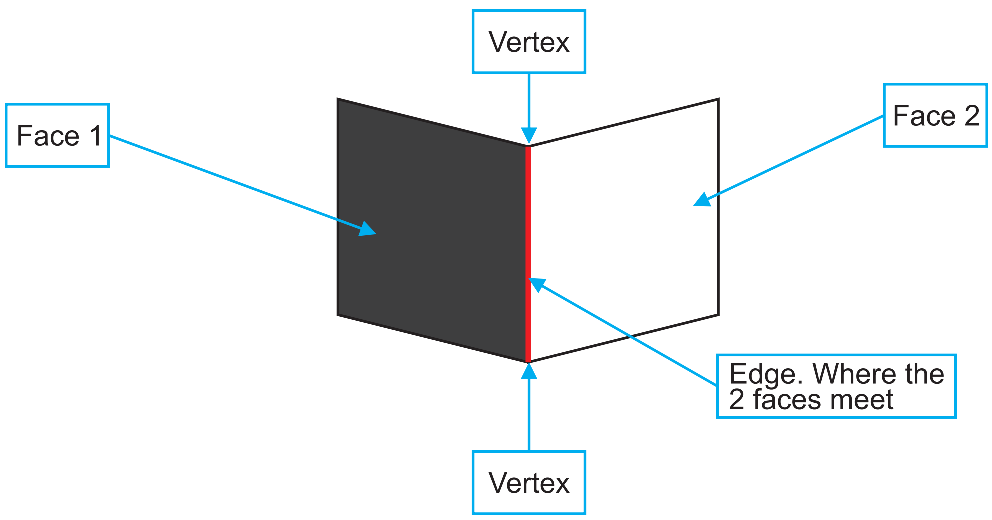
- Each time we talk about 3-D objects to the learners, reference has to be made to 2-D shapes, because 3-D objects are built up out of 2-D shapes.
- When you do a lesson on 3-D objects you should use models of the 3-D objects and 2-D shapes. You need to use these to match the 2-D shapes to the faces of the 3-D objects which they make up. Learners will then be able to see the differences between the 2-D shapes and 3-D objects being referred to and they will be able to identify the faces, edges and vertices of the 3-D objects.

- An **edge** is where two faces (which are the flat 2-D surfaces that make up the 3-D objects) meet.
- The corner or point where edges meet is called a **vertex**. Two or more corners are called **vertices**.

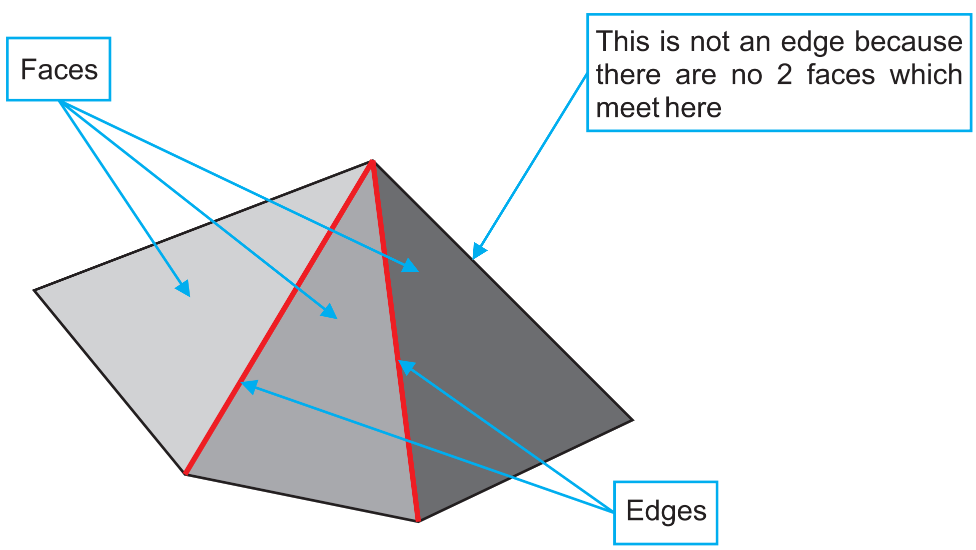
- The two sketches that follow illustrate the terminology of 3-D objects. Sketch the illustrations onto the board and use them to explain the terminology to your learners.

Examples using folded shapes to illustrate the terminology of 3-D objects

1). A piece of cardboard folded in the centre.



2). A net of a triangular pyramid (folded out and without the base).



Identification of faces, edges, vertices, 2-D shapes and 3-D objects

- You need to give your learners many activities that allow them to consolidate their knowledge of the terminology.
- Allow them to work with models of shapes when they work through these activities.
- Ultimately learners need to be able to use the preceding diagrams to identify and speak about both 2-D shapes and 3-D objects. They need to reach the level of abstraction necessary to work with these shapes confidently in mathematical activities and assessments.
- The three activities that follow can be used to consolidate this knowledge. Use your school textbooks and other resources to supplement these activities as much as you need to.

Activities: Identifying faces, edges, vertices, 2-D shapes and 3-D objects

1). Match the definition in column A with the correct words in column B

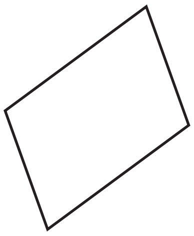
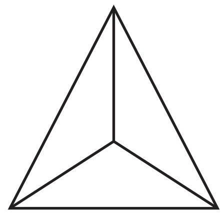
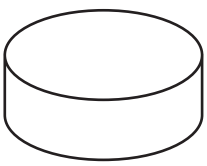
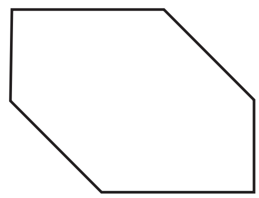
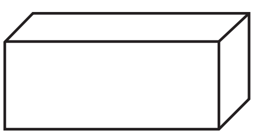
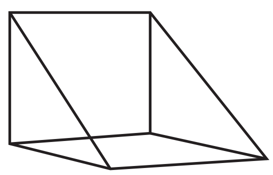
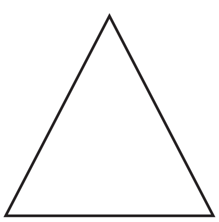
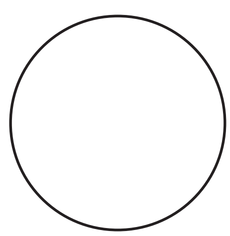
A. Definition	B. Word
A line where two surfaces of a 3-D object meet	Vertex
A point where 2 or more edges of a 3-D object meet	Face
A flat surface, a 2-D shape that builds up a 3-D object	Edge

Solutions

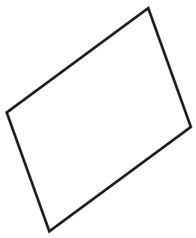
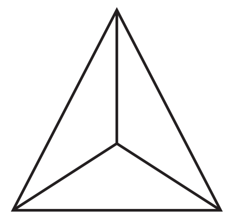
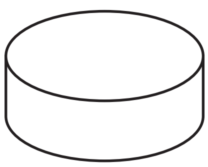
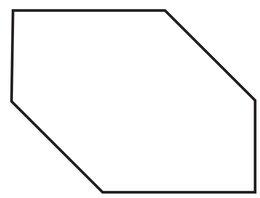
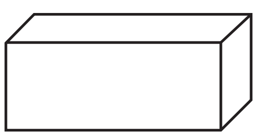
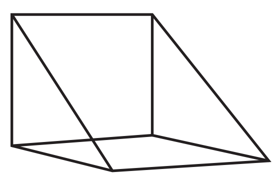
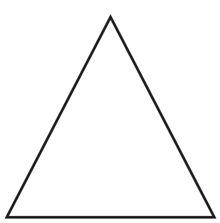
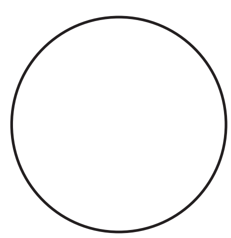
A. Definition	B. Word
A line where two surfaces of a 3-D object meet	Edge
A point where 2 or more edges of a 3-D object meet	Vertex
A flat surface, a 2-D shape that builds up a 3-D object	Face

Notes:

2). Name the shapes and state whether they are 2-D or 3-D

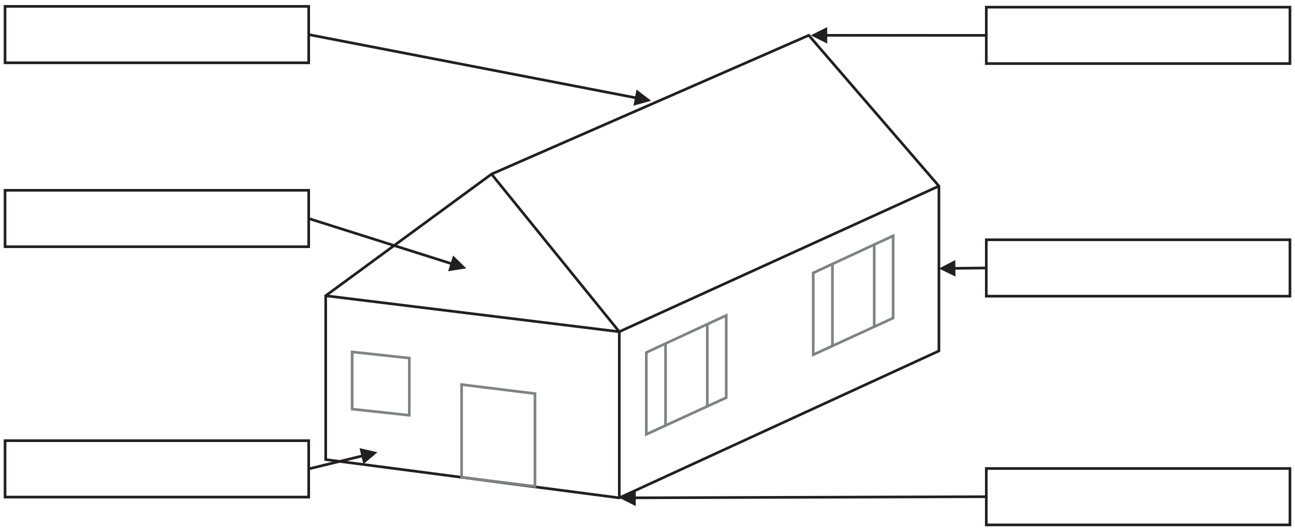
Shape	**2-D or 3-D Number of edges, faces and vertices (if 3-D)**	**Shape**	**Name2-D or 3-D Number of edges, faces and vertices (if 3-D)**
1). 		2). 	
3). 		4). 	
5). 		6). 	
7). 		8). 	

Solutions

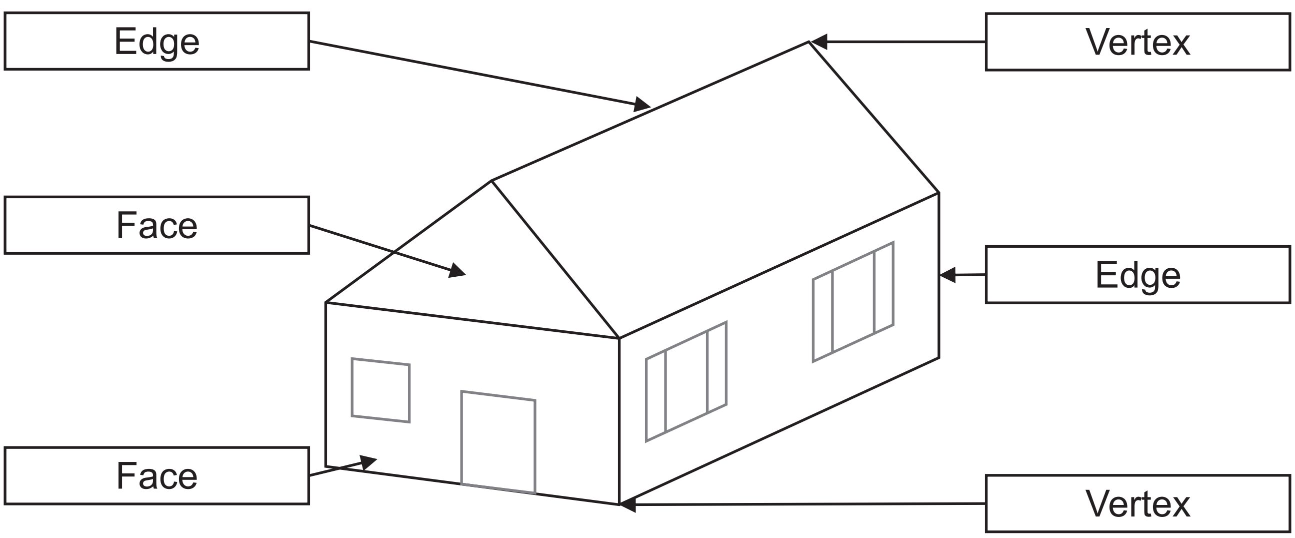
Shape	2-D or 3-D Number of edges, faces and vertices (if 3-D)	Shape	Name2-D or 3-D Number of edges, faces and vertices (if 3-D)
1). 	2-D quadrilateral parallelogram	2). 	3-D: triangular pyramid Faces: 4 Edges: 6 Vertices: 4
3). 	3-D: cylinder Faces: 2 Edges: n/a Vertices: n/a	4). 	2-D hexagon
5). 	3-D: Prism Faces: 6 Edges: 12 Vertices: 8	6). 	3-D: triangular prism Faces: 5 Edges: 9 Vertices: 6
7). 	2-D triangle	8). 	2-D circle

3). Label the diagram below using the words given

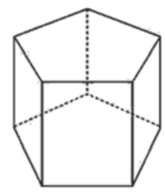
Edge;	Face;	Vertex



Solution



20. Complete the table.

Pentagonal prism	
Number of vertices	
Number of edges	
Number of faces	

[3]

Another example of how to test 3-D Objects

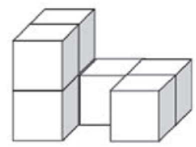
ANA 2014 Grade 6 Mathematics Item 20

Notes:

Views of 3-D objects

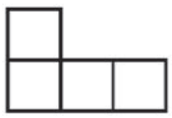
ANA 2013 Grade 6 Mathematics Item 1.10

1.10 Which 2-D shape below shows the left hand view of the 3-D object?

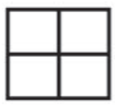


Front view

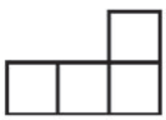
A



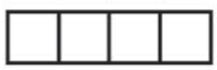
B



C



D



[1]

What should a learner know to answer this question correctly?

Learners should:

- Know from which side to view the object;
- Be able see the view of an object "in the mind's eye".

Where is this topic located in the curriculum? Grade 6 Term 3

Content area: Space and shape.

Topic: Viewing objects.

Concepts and skills:

- Position and views of geometric objects.

What would show evidence of full understanding?

- A learner who chose option B shows full understanding.

What would show evidence of partial understanding?

- A learner who chose the options A or C looked at the front or back views of the object showing some understanding of different views of objects.

What would show evidence of no understanding?

- A learner who chose option D showed little or no understanding of how to view objects from different positions.

What do the item statistics tell us?

24% of learners answered the question correctly.

Factors contributing to the difficulty of the item

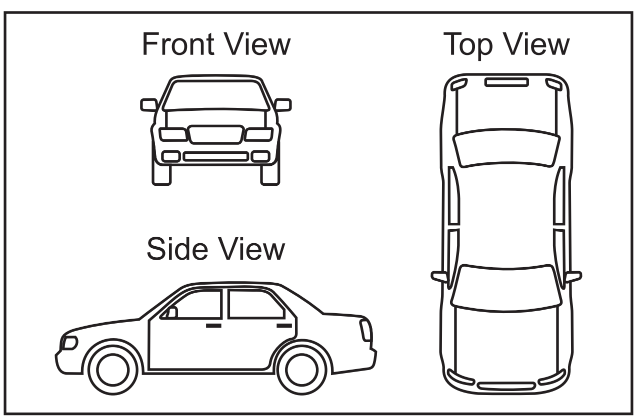
- Learners may have seen "front view" in the drawing and assumed that they had to draw the front view of the object;
- Learners may not be able to visualise objects from different perspectives.

Teaching strategies

Drawing 2-D views from 3-D drawings

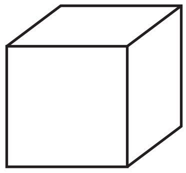
- You could be asked to draw the top, side or front view of a 3-D shape.
- The top is the view you would see if you were looking down onto the shape if you were positioned above the shape.
- This view may also be referred to as the birds-eye view, as a bird flying above the shape would see this view.

- Here are examples of the top, side and front view of a car.

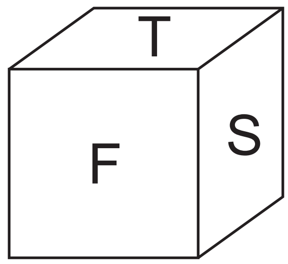


- When introducing the topic of views of 3-D objects it is best to start with simple shapes for which it is easy to visualise the different views.
- You must use real 3-D objects when you do this activity so that learners can refer to something tangible to help them to start to think about abstract views of shapes.

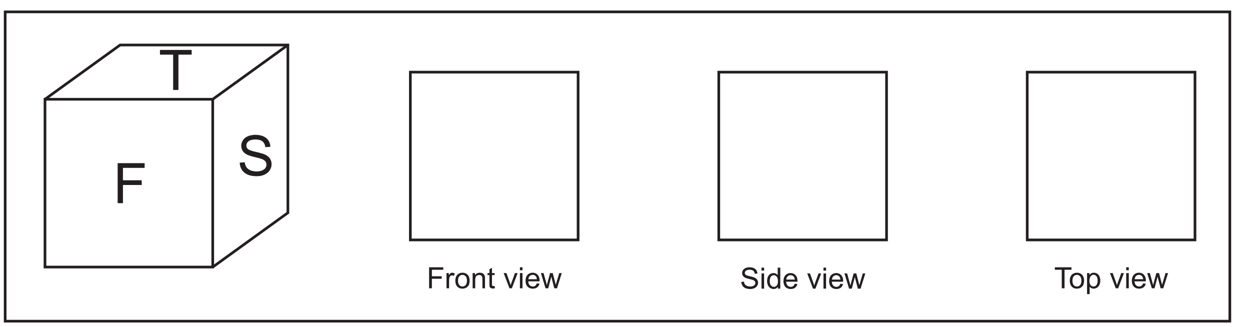
- You can use any 3-D cube to show learners the different faces of objects.



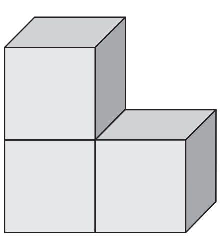
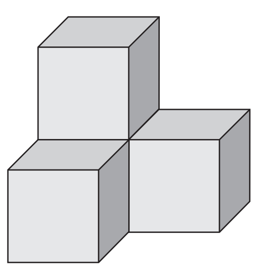
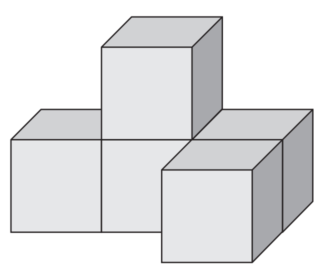

- Ask learners to show you the top, front and side views of the actual object.
- Draw a sketch of the object on the board and work with the learners to label the top, front and side views of the object as follows:



- Help learners to see that the views from these sides will be 2-D drawings and they will all be squares.
- Make sure that the learners can see how the views link to the object.



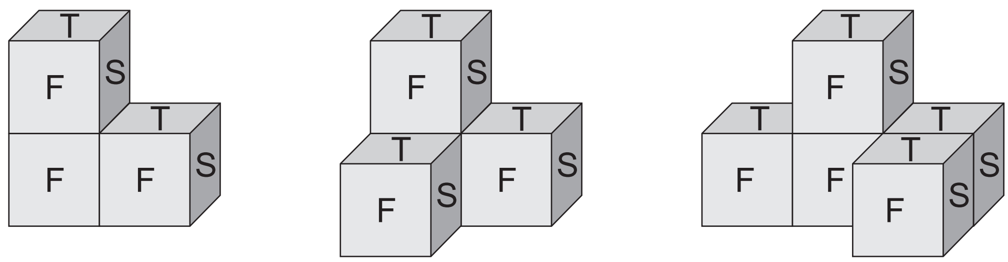
- When you think your learners are ready, move to views of more complex objects as shown in the following examples.

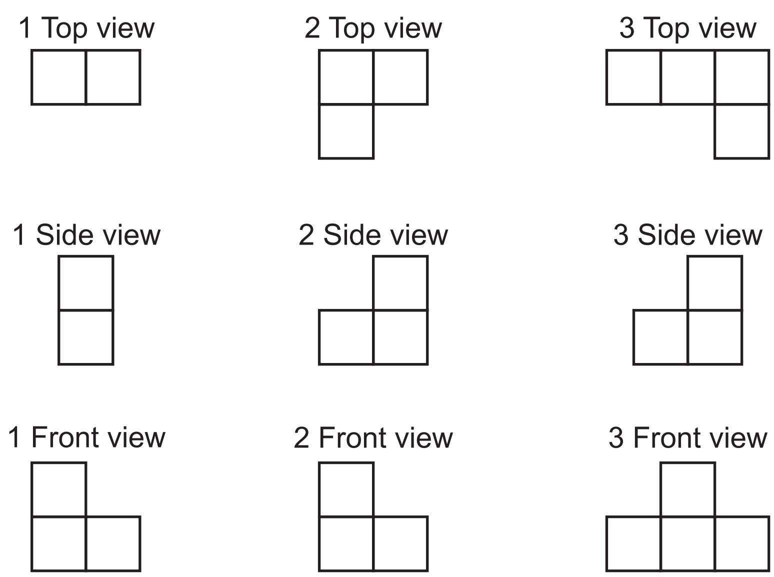
Example

- Allow learners to label the faces as top (T), front (F) and side (S).

1 2 3



- Learners can then draw the different views of the shapes by referring to the labels on the faces:

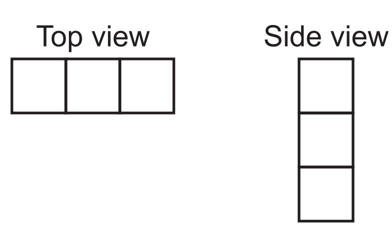


Drawing another view if 2 views are given

- Learners can also use views which are given to reconstruct 3-D objects.

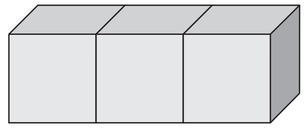
Example

If the following views are given, draw the front view of the structure.

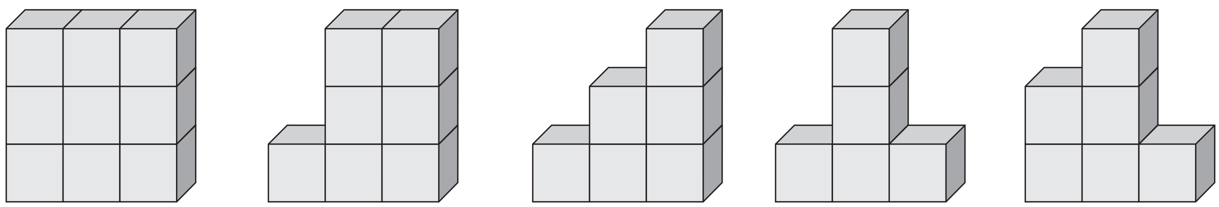


Solution

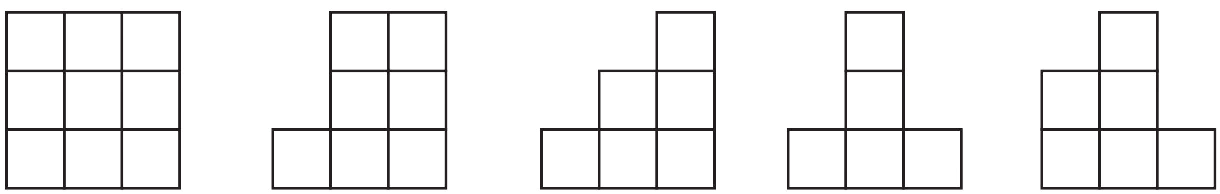
- There are various options for the structure.
- Start with the bottom row. Draw the three blocks as the “base” of your structure:



- Three blocks seen from above are needed for the given top view.
- The side view needs a column of three blocks. There are various ways in which you can build a side view of 3 blocks.
- Below are some possibilities.



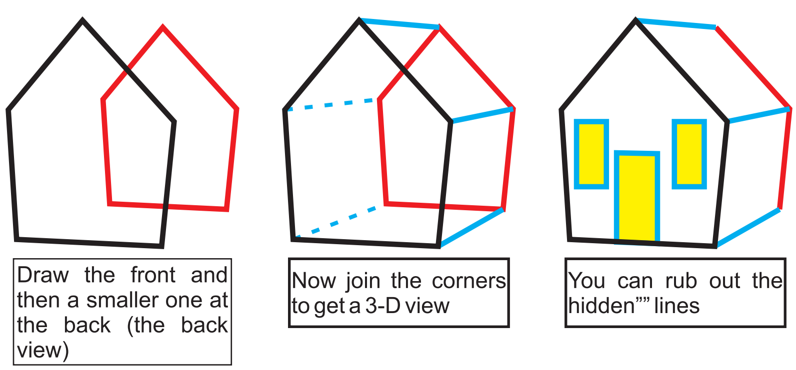
- Each of the above structures has a different front view.



Drawing three dimensional objects

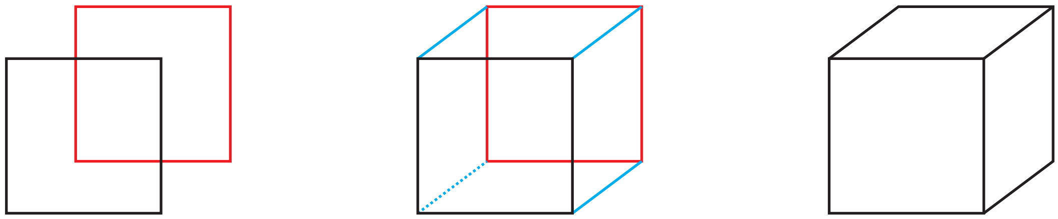
- Help learners to develop the skill of drawing simple 3-dimensional objects.
- Ask learners: If you draw a house, how do you do it?
- Learners might respond by drawing a typical house.

- Now draw a house on the black board, showing your learners how to draw the house from different perspectives:



- Learners who find it difficult to draw various views of 3-D objects should be guided to visualise 2-D drawings from a 3-D perspective.

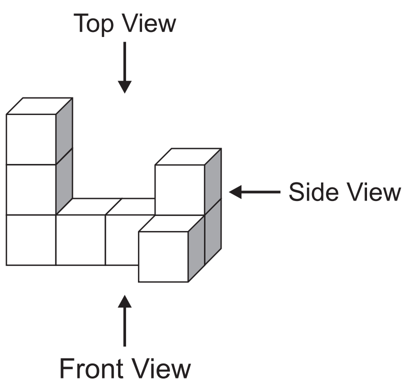
- In the same manner, we can draw a cube or rectangular prism.

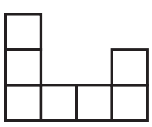


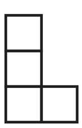
Another example of how to test views of 3-D objects

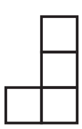
ANA 2014 Grade 6 Mathematics Item 1.8

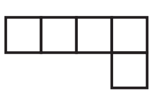
1.8 Which sketch represents the side view of the 3-D object?



A 

B 

C 

D 

[1]

Angles

ANA 2013 Grade 6 Mathematics Item 17

17. obtuse angle; acute angle; right angle

Use the above words to say what kind of angles are marked in the picture.

17.1 ____________________

17.2 ____________________

[2]

What should a learner know to answer this question correctly?

Learners should be able to:

- Explain the difference between an obtuse, acute and right angle;
- Recognise angles in the context of a diagram.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Space and Shape.

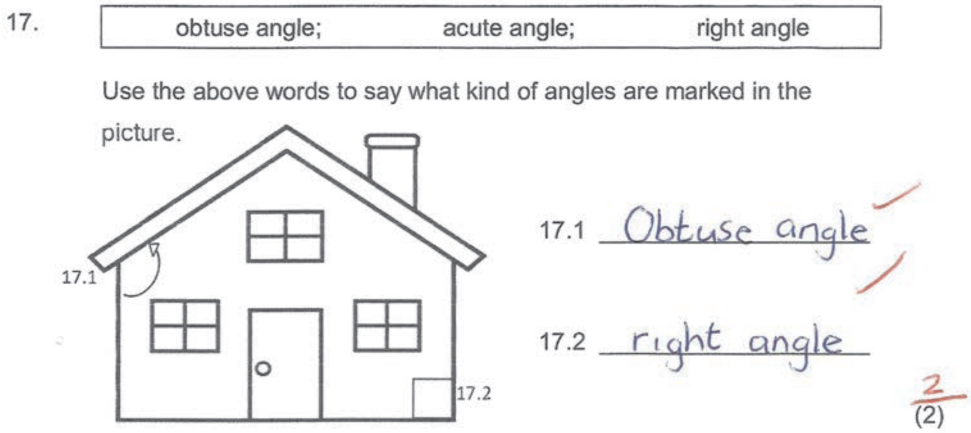
Topic: Properties of 2-D shapes.

Concepts and skills:

- Identify the angles given.

What would show evidence of full understanding?

- Correctly identifying the angles as shown in the following example.


17. obtuse angle; acute angle; right angle

Use the above words to say what kind of angles are marked in the picture.

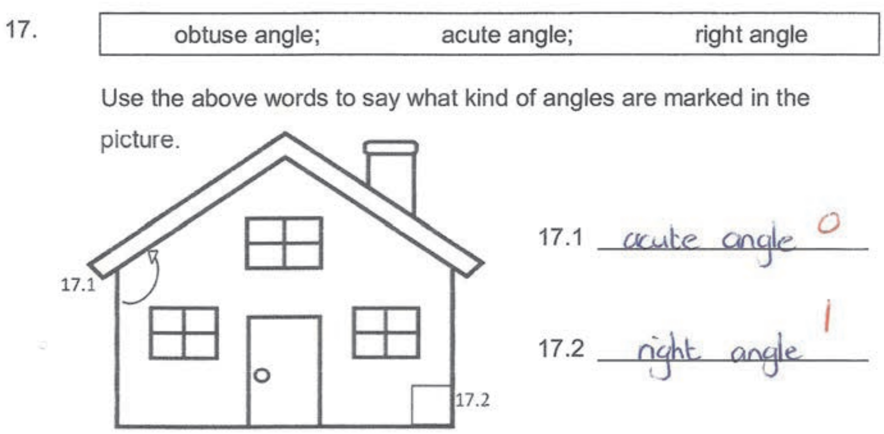
17.1 Obtuse angle

17.2 right angle

(2)

What would show evidence of partial understanding?

- Identifying only one of the angles correctly as in the next example.

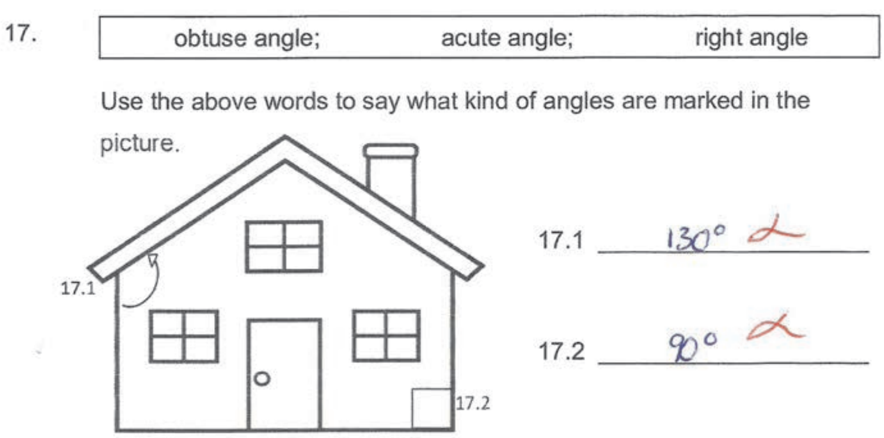

17. obtuse angle; acute angle; right angle

Use the above words to say what kind of angles are marked in the picture.

17.1 acute angle

17.2 right angle

- Writing approximate values of the angles instead of naming them, as shown in the next two examples.


17. obtuse angle; acute angle; right angle

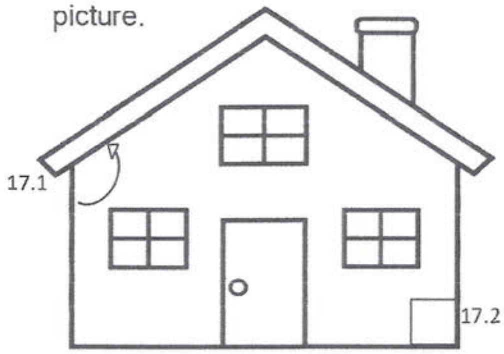
Use the above words to say what kind of angles are marked in the picture.

17.1 130°

17.2 90°

17. | obtuse angle; | acute angle; | right angle

Use the above words to say what kind of angles are marked in the picture.



17.1 95 Top

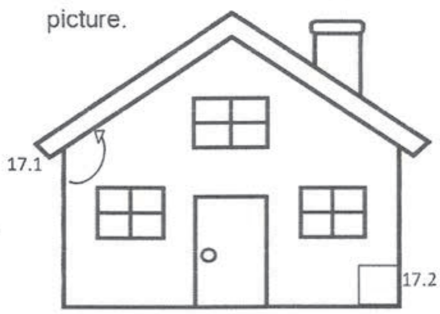
17.2 90 Side

What would show evidence of no understanding?

- Not managing to identify any of the angles, inter-changing the names or using other names as in these examples.

17. | obtuse angle; | acute angle; | right angle

Use the above words to say what kind of angles are marked in the picture.

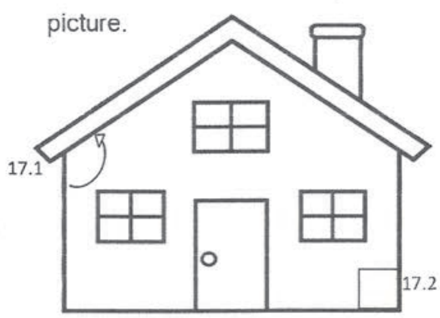


17.1 5 angles

17.2 7

17. | obtuse angle; | acute angle; | right angle

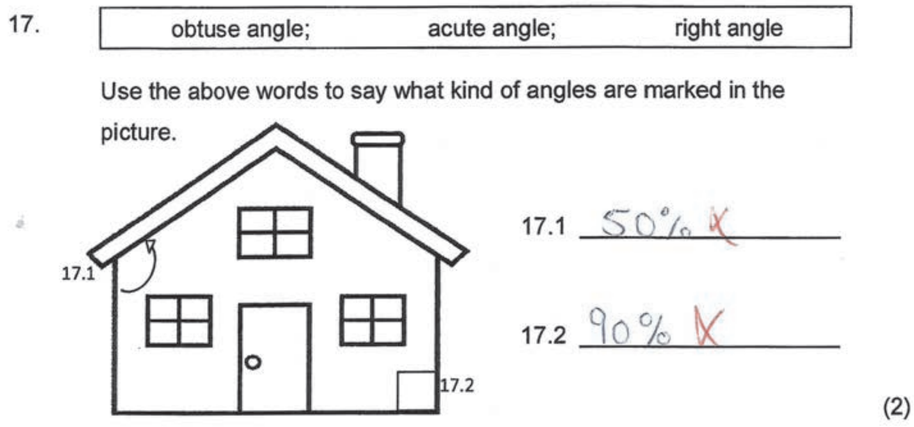
Use the above words to say what kind of angles are marked in the picture.



17.1 4cm

17.2 5cm

- Giving approximate angles as percentages instead of degrees as shown here.



17. | obtuse angle; | acute angle; | right angle |

Use the above words to say what kind of angles are marked in the picture.

(2)

What do the item statistics tell us?

Item 17.1

48% of learners answered the question correctly.

Item 17.2

58% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Angles were embedded inside a diagram. The angles were not drawn separately.

Teaching strategies

Definition of angles

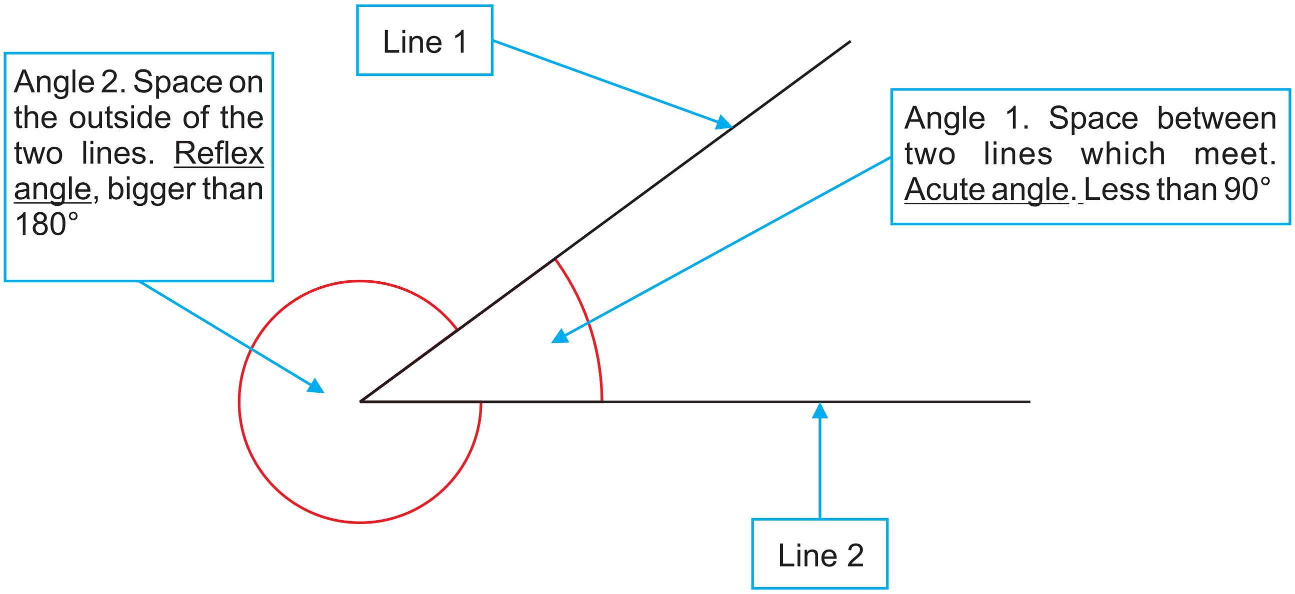
- Learners must know what angles are.
- The simplest definition for learners at this stage will be that 'an angle is the measure of the turn between two lines that meet at a point'. Angles are not straight lines but they are formed between lines.
- Illustrations should be provided so that the learners can link the theory or idea of angles to angles shown in a diagram. This will also aid pictorial or visual learners.

Examples

Draw all of the following diagrams on the board.

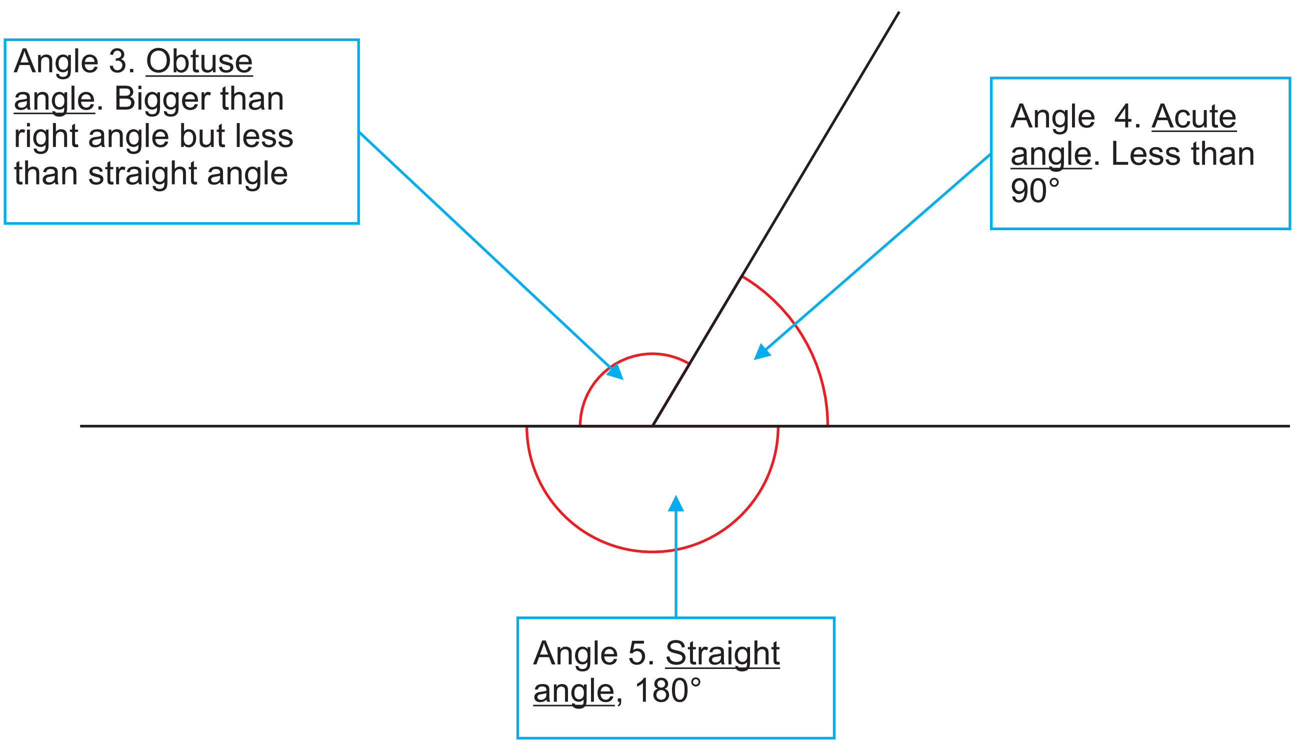
1). In the first sketch, illustrations of acute and reflex angles are shown.

- Angles are formed when lines meet. More than one angle may be formed and there are different kinds of angles that need to be identified by learners.
- Make sure that your learners can see these two types of angles and help them to pronounce the names of the angles correctly.



2). In the second sketch, illustrations of acute, obtuse and straight angles are shown.

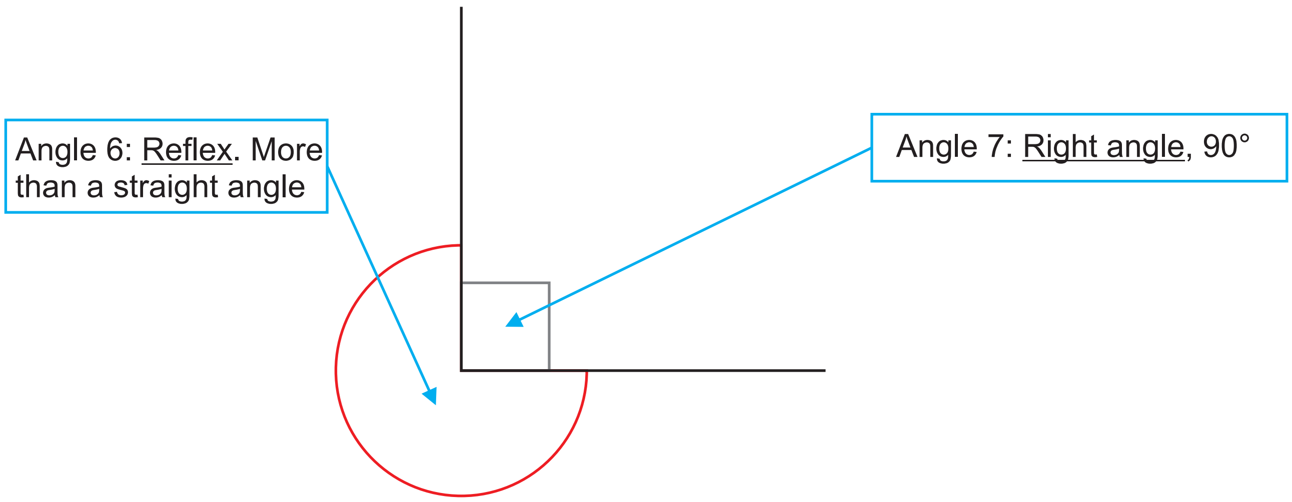
- Make sure that your learners can see these three types of angles and help them to pronounce the names of the angles correctly.



- Alternatively, the angles might be formed by the joining of the lines perpendicularly as shown in sketch 3.

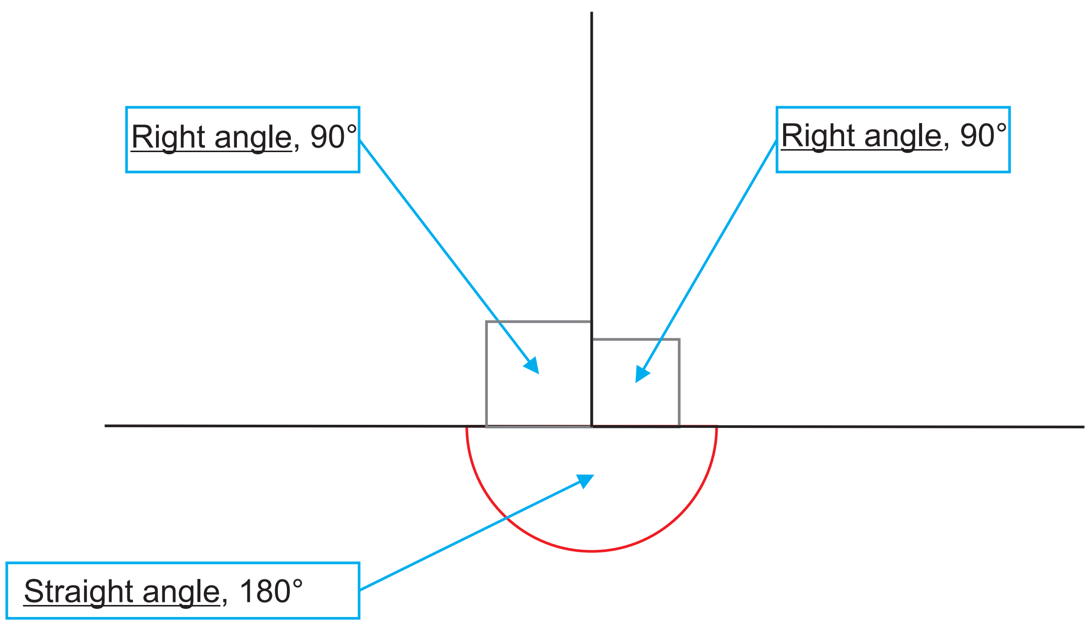
3). In the third sketch, illustrations of a right angle and a reflex angle are shown.

- Make sure that your learners can see these two types of angles and help them to pronounce the names of the angles correctly.

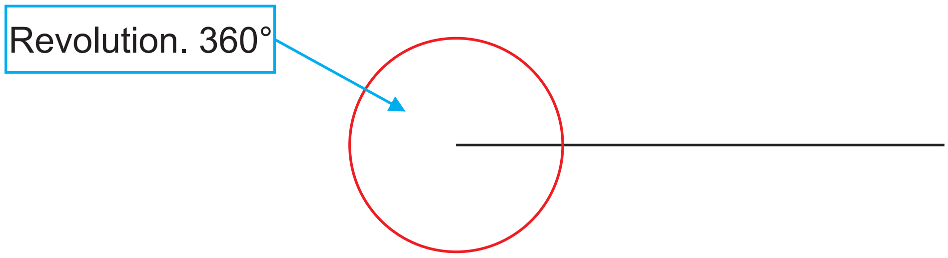


4). The fourth sketch illustrates two right angles and a straight angle formed in this way.

- Make sure that your learners can see these two types of angles and help them to pronounce the names of the angles correctly.



5). A revolution is formed when a full rotation is made around a point. We illustrate this by drawing a circle around the point on the line where the rotation is made, as shown in the next sketch.



- Use a variety of examples similar to those given above to illustrate angles.
- You could use real life examples such as houses as in the ANA item.
- You could relate the 90° angle to the corner of a page. All angles with this shape will be right angles and all straight angles will be in a straight line. These are the two reference angles.
- Identify an ACUTE angle as being less than a right angle.
- Identify an OBTUSE angle as being bigger than a right angle but smaller than a straight angle.
- A REFLEX angle is identified as being bigger than a straight angle but smaller than a revolution.
- A REVOLUTION is like a full circle and is bigger than a reflex angle.

Refer to the diagrams given above until your learners are able to recognise and name all the different types of angles.

Activity: Ordering of angles

Write the following angles from biggest to smallest.

Straight Angle; Right Angle; Obtuse Angle; Acute Angle; Reflex Angle; Revolution.

Solution

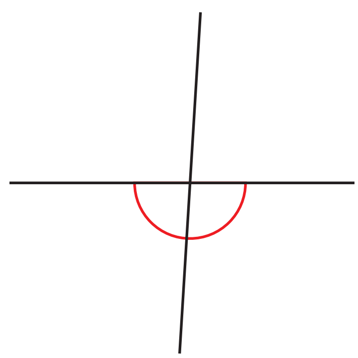
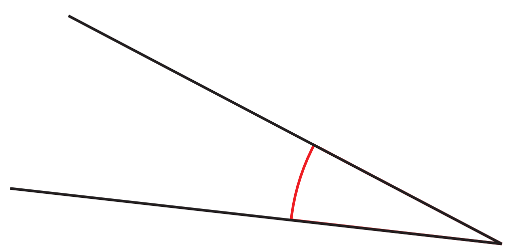
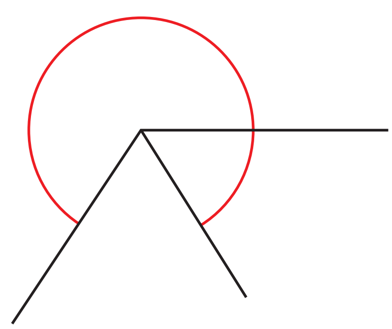
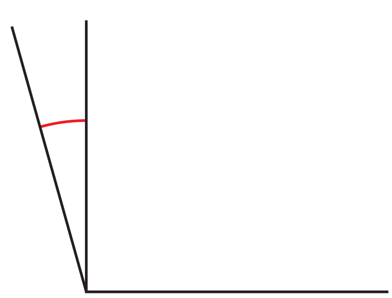
Biggest ◄────► Smallest

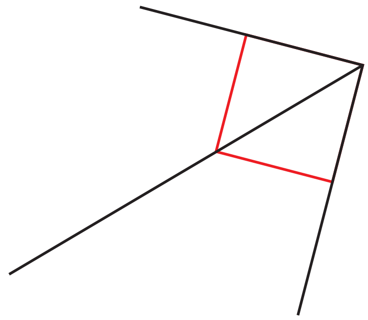
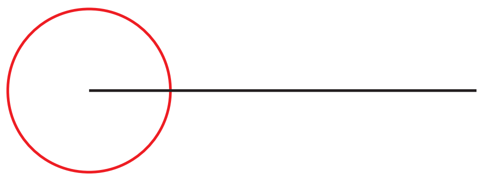
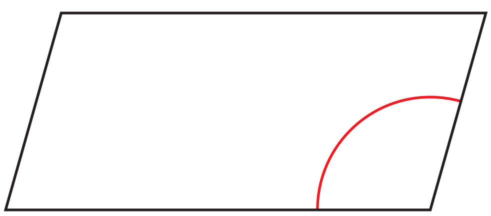
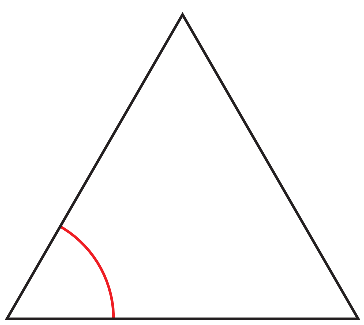
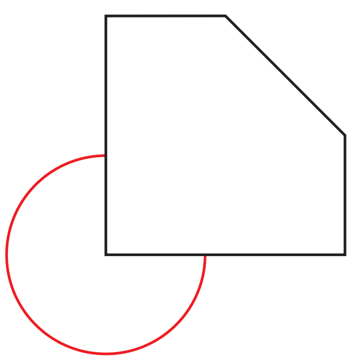
Revolution	Reflex	Straight	Obtuse	Right	Acute

Activity: Identification of angles

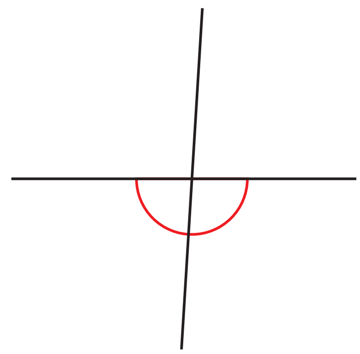
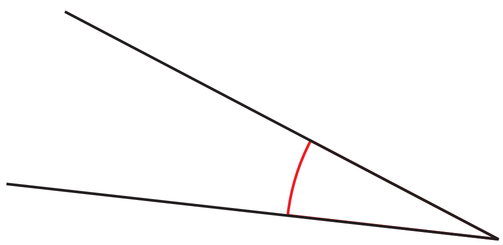
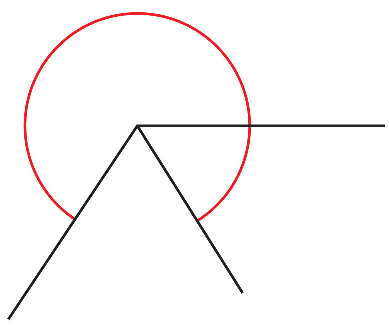
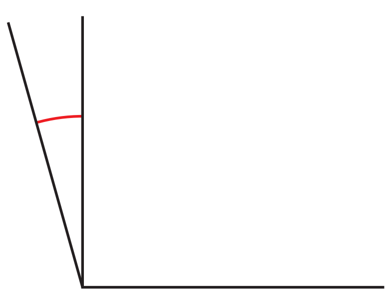
Learners must be able to identify all the angles shown in the following table using the explanations above.

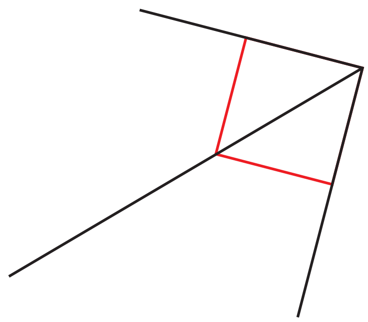
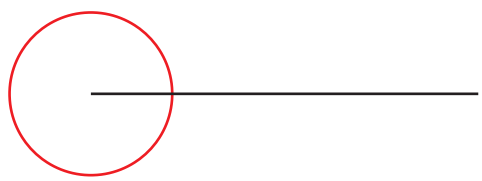
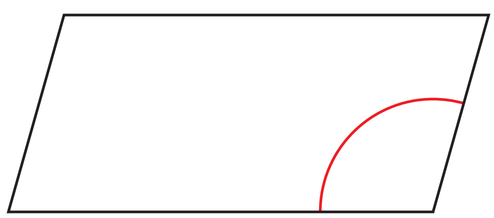
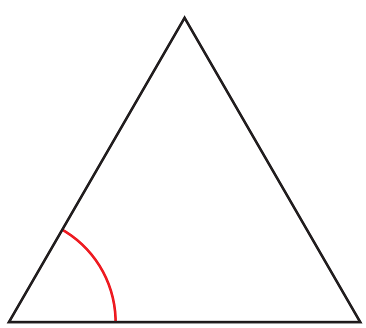
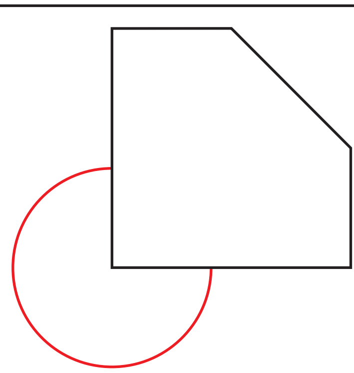
Name the angles indicated by the arc in each of the diagrams below.

Angle	Name
1). 	____________
2). 	____________
3). 	____________
4). 	____________

5). 	
6). 	
7). 	
8). 	
9). 	

Solutions

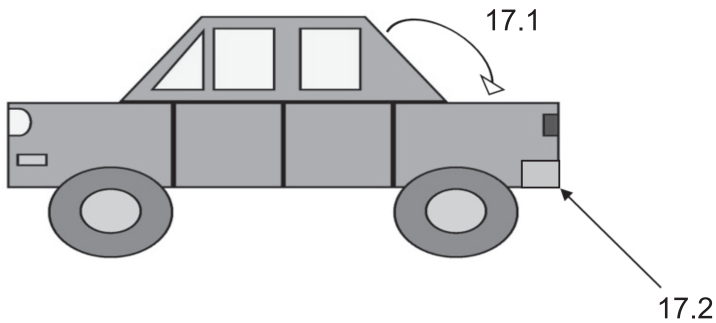
Angle	Name
1). 	Straight angle
2). 	Acute angle
3). 	Reflex angle
4). 	Acute angle

5). 	Right angle
6). 	Revolution
7). 	Obtuse angle
8). 	Acute angle
9). 	Reflex angle

Another example of how to test angles

ANA 2014 Grade 6 Mathematics Item 17

17. Name the kind of angles that are indicated with the arrows below.



17.1 ____________________ [1]

17.2 ____________________ [1]

Notes:

Transformation geometry: Symmetry

ANA 2013 Grade 6 Mathematics Item 20

20 Draw the line(s) of symmetry in the picture.

[1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Draw a line of symmetry to divide a drawing into two identical parts;
- Realise that if a drawing is folded along the line of symmetry, then the two parts will overlap exactly.

Where is this topic located in the curriculum? Grade 6 Term 2

Content area: Space and shape.

Topic: Symmetry.

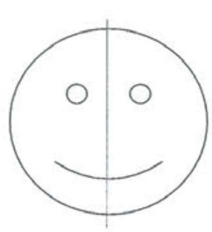
Concepts and skills:

- Recognise, draw and describe lines of symmetry in 2-D shapes.

What would show evidence of full understanding?

- If the learner drew the line of symmetry vertically as in the example given.

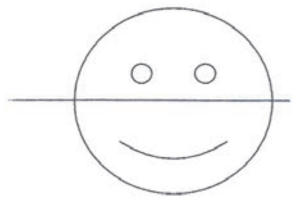
20. Draw the line(s) of symmetry in the picture.



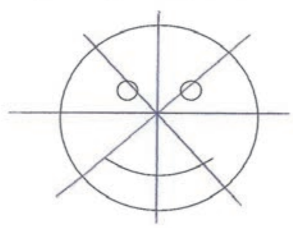
What would show evidence of partial understanding?

- Some learners ignored the picture in the circle and drew lines of symmetry through the centre of the circle. They understood that there are other lines of symmetry that can be drawn through the circle.
- Some included one correct line of symmetry together with other lines that were not correct.

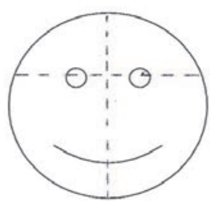
20. Draw the line(s) of symmetry in the picture.



20. Draw the line(s) of symmetry in the picture.



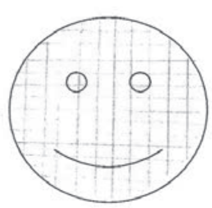
20. Draw the line(s) of symmetry in the picture.



What would show evidence of no understanding?

- If the learner drew lines that did not go through the centre of the circle as seen in the following example. This indicates that the learner does not understand the notion of symmetry.

20. Draw the line(s) of symmetry in the picture.



What do the item statistics tell us?

52% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The face on the circle had to be considered when drawing the line of symmetry.

Teaching strategies

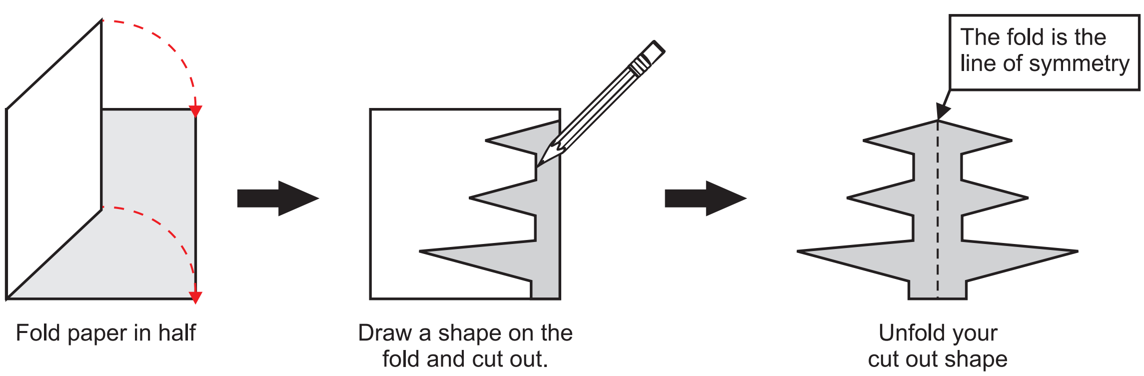
Understanding symmetry

- When we say symmetry we mean **line symmetry**.
- We say line symmetry because of the line of symmetry – the line (or axis) about which the symmetry occurs.
- Line symmetry is also called reflection symmetry (because it has a lot to do with reflections) and bilateral symmetry (because of the 'two-sided' nature of symmetrical figures).
- When two points are symmetrical to each other we say that the one is the reflection of the other.
- A figure needs to have at least ONE axis of symmetry to be symmetrical, although it may have MORE THAN ONE axis of symmetry.
- It is good to use practical activities such as the two examples that follow to consolidate learners' understanding of symmetry.

Examples

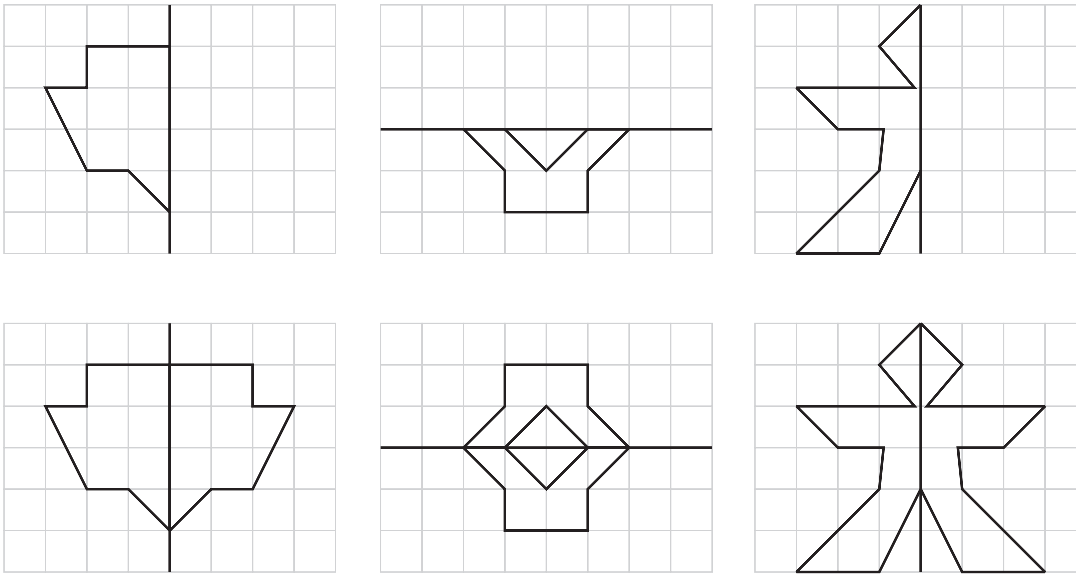
1). Take a piece of paper and fold it in half.

- Ask learners to draw a shape next to the fold.
- Then cut out the shape and unfold.
- Allow learners to paste their pictures on the wall of the classroom.



2). Use blocked paper with shapes on one side of the line of symmetry.

- Learners must draw the shape on the other side of the line of symmetry.

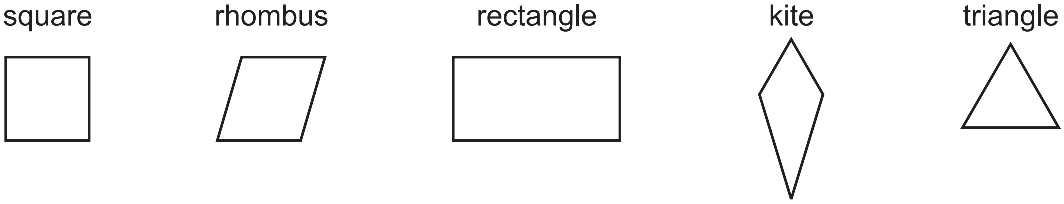


Drawing lines of symmetry in polygons

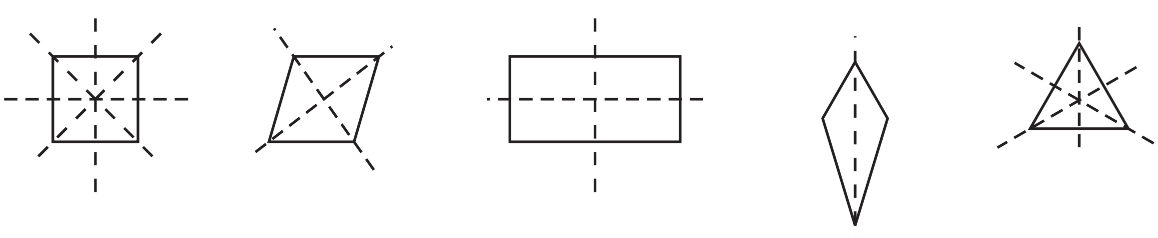
- Learners need to practice drawing lines of symmetry in a wide range of different geometrical shapes.
- Use the following activity to give them this opportunity. Draw the shapes on the board and allow the learners some time to draw in the lines of symmetry.

Activity: Draw lines of symmetry in the following polygons:

square rhombus rectangle kite triangle

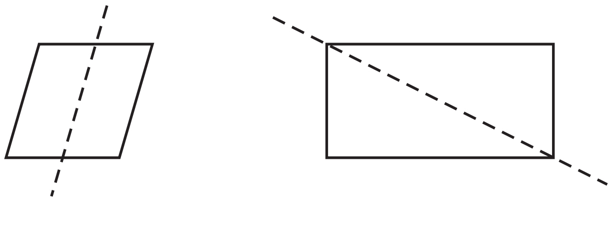


Solutions



- These solutions are correct.
- What if learners gave other responses? How can we test that our lines of symmetry are correct?

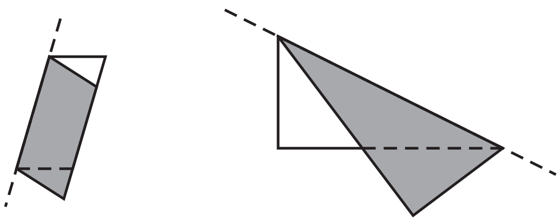
- Learners might have given the following answers:



- Cut out each of the shapes from scrap paper.
- Fold each shape along the line that you think might be a line of symmetry.
- The two pieces of the diagram must overlap exactly for the fold line to be a line of symmetry.
- Which of these lines are lines of symmetry?

Answer:

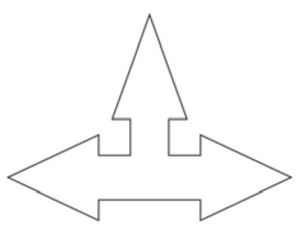
- None are lines of symmetry as the two parts do not overlap exactly when folded.



Another example of how to test symmetry

ANA 2014 Grade 6 Mathematics Item 1.6

1.7 How many lines of symmetry can be drawn on the shape below?



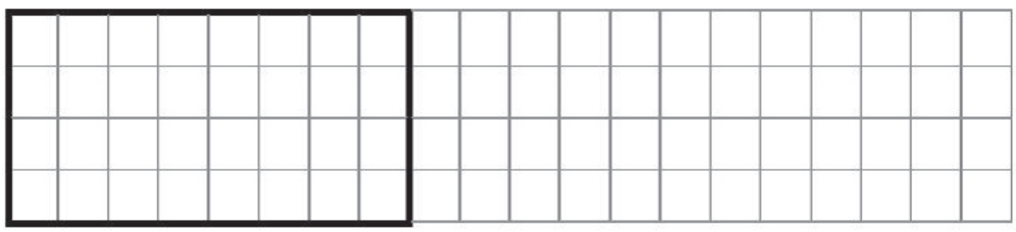
A 1
B 3
C 5
D 2

[1]

Transformation geometry: enlargement and reduction

ANA 2013 Grade 6 Mathematics Item 21

21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.



[2]

What should a learner know to answer this question correctly?

Learners should be able to:

- Reduce the size of a given shape, that is, make a smaller diagram in the same proportion as the original diagram;
- Understand that the word "size" refers to the area of the rectangle;
- Find a quarter of a rectangle by dividing the original rectangle into four equal parts.

Where is this topic located in the curriculum? Grade 6 Terms 3 and 4

Content area: Space and Shape (Term 3) and Measurement (Term 4).

Topic: Transformations.

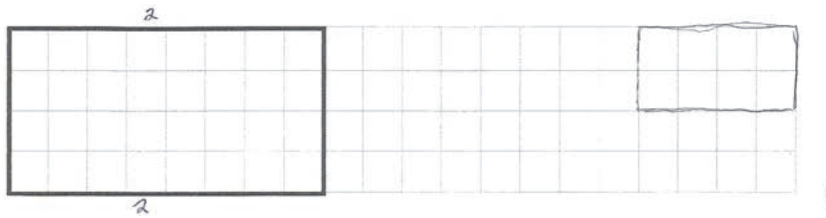
Concepts and skills:

- Transformations (Term 3) and perimeter, area and volume (Term 4);
- Enlargement and reductions (Term 3) and measurement of area (Term 4).

What would show evidence of full understanding?

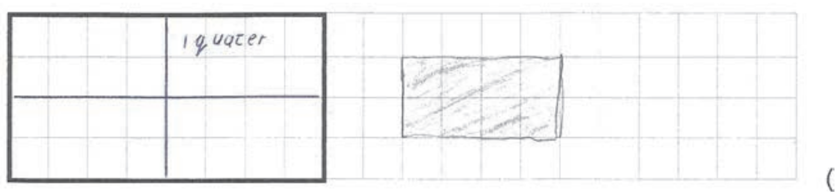
- Full understanding is demonstrated if the new rectangle drawn by the learner is $\frac{1}{4}$ the size of the original one and the lengths of the sides are in the same ratio as in the following example

21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.



- In the next example the learner showed clear reasoning and full understanding of the concept of one quarter of the original size.

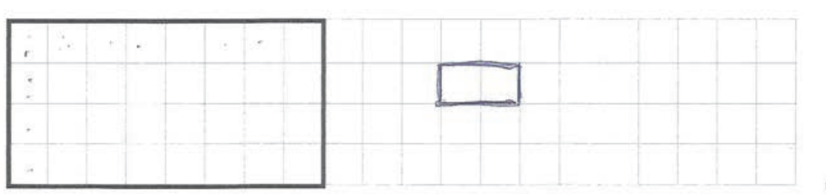
21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.



What would show evidence of partial understanding?

- In this example the learner showed his or her reasoning. The dimension, length and breadth were divided by 4 (to find one-quarter of the dimensions). To this learner the "<u>size</u>" which was referred to in the question related to the lengths of the sides and not specifically to the "<u>area</u>". The learner understood the question as "reduce the diagram by a factor of 4".

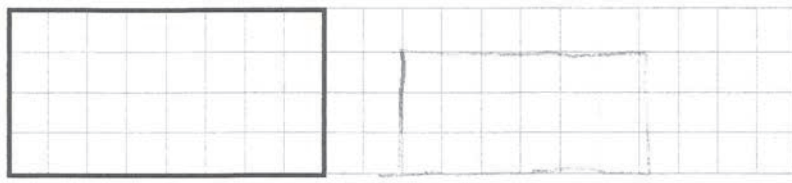
21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.



What would show evidence of no understanding?

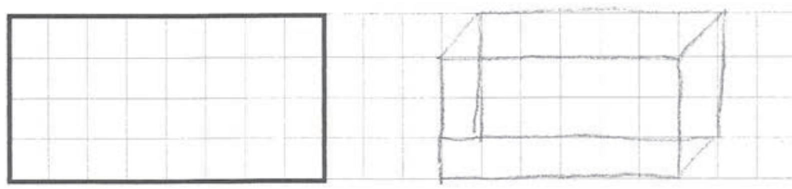
- In the following example the learner did not show any understanding of what was required to reduce the diagram to make it a quarter of its original size. The learner just made the shape one block shorter and one block narrower.

21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.



- In the next example the learner tried to draw a three dimensional prism, having not understood the question at all.

21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.



What do the item statistics tell us?

26% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- The wording of the question could have confused learners. They have not yet encountered the "reduction in size" type of question at this stage.
- The ANA question referred to a reduction of the size of the shape but the CAPS requirement for enlargement and reduction refers drawing a larger or a smaller shape by increasing or decreasing the lengths of the sides by the same ratio.

Teaching strategies

Scale drawings

- Always use blocked paper when enlargements or reductions are drawn. The blocks will make it easier for learners to work with a scale and decide what size the new shape should be.

Enlargement

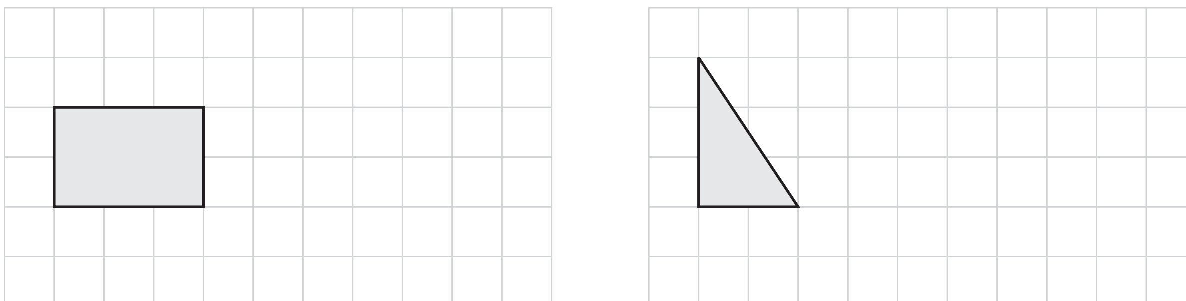
- Enlarged shapes get bigger.
- You can enlarge a shape by increasing the lengths of the sides of the shape in the same ratio.
  - For example, when enlarging a shape by making the sides double their original length, the new shape must have all sides double the length of the old shape.

- You can enlarge a shape by increasing the area of the shape.
  - For example, when enlarging a shape by making it 2 times bigger (double its size), the area of the new shape must be twice as big as the area of the original shape.

- Enlargements in the area of a shape are related to enlargements done according to the lengths of the sides. The following activity shows the relationship between enlarging a shape using size (area) and lengths of sides together to decide on how to draw the required enlarged shape.

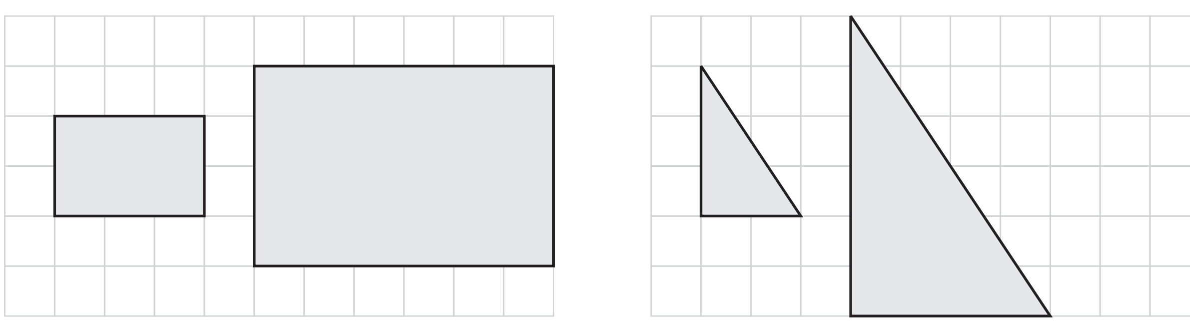
Activity: Enlarging shapes

Enlarge the figures on the grid by making them 4 times bigger.



- Look carefully at the dimensions of the rectangle and the triangle. The dimensions must be increased so that the size of the shape in the new diagram is 4 times bigger than in the diagram given.
- The size of the shape (area) on the left is 6 blocks. 4 times bigger means we need an area of 6 x 4 = 24 blocks.
- The dimensions of the new shape need to be 6 blocks by 4 blocks.
- This means that all of the dimensions must be double what they were before (old dimensions: 3 blocks by 2 blocks; new dimensions: 6 blocks by 4 blocks).
- Learners should use the blocks to work out the new dimensions and draw the enlarged shape next to the original shape as shown.

Solution



Reflect:

- Look at the drawings of the enlarged shapes.
- In each case, the new shape is 4 times bigger than the old shape.

- It is the area of the shape that we look at when we talk about the size of the shape.
- Notice that the size gets 4 times bigger when the lengths of the sides are doubled.

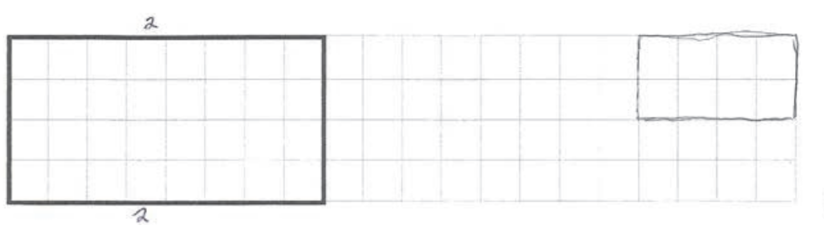
Reduction

- Reduced shapes get smaller.
  - For example, when reducing a shape to a quarter of its original size, the shape must be divided into four equal parts. Each of these parts represents a reduction of the original shape to a quarter of its size.
- You can reduce a shape by decreasing the lengths of the sides of the shape in the same ratio.
  - For example, when decreasing a shape by making the sides half their original length, all sides of the new shape must be half the length of the sides in the old shape.

- In the ANA question learners had to make the shape a quarter of the size of the original shape.
- To find the size of the new shape learners had to work with the areas of the shapes.
- The area of the original shape was 32 squares.
- The area of the reduced shape had to be one quarter of 32 squares = 8 squares.

- The correct solution to the ANA question is shown again here.

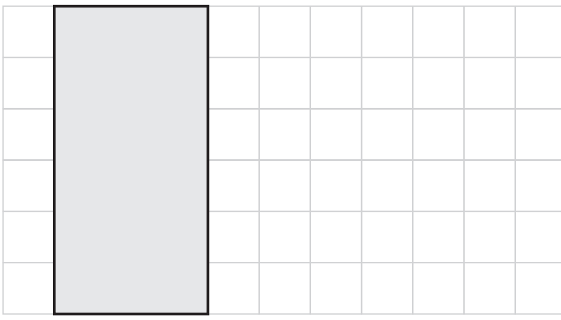
21. Draw a reduction of the rectangle on the grid so that the size of the new rectangle is a quarter of the size of the original one.

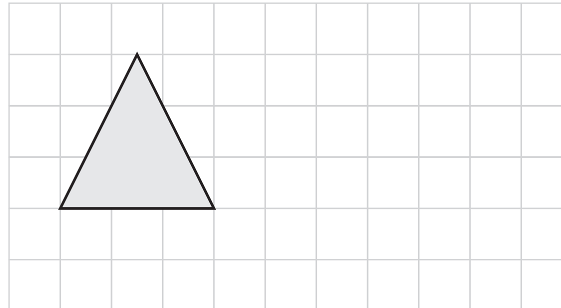


- The reduced shape is a quarter of the size of the original shape.
- The lengths of the sides of the reduced shape are half the lengths of the sides of the original shape.
- This shows us that halving the lengths of the sides reduces the area of the shape to a quarter of the original size.

Activity: Reducing shapes

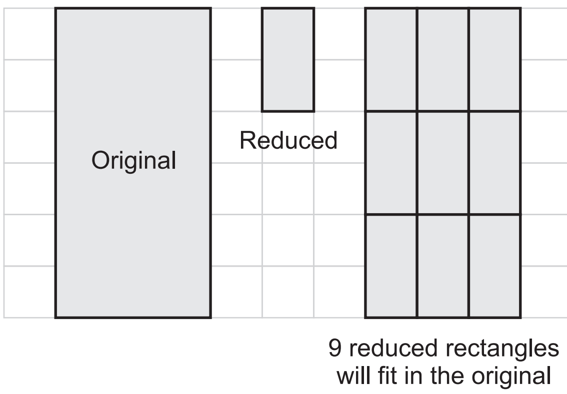
- Reduce the figures on the grid to one ninth of their original size.
- Ask learners "How many of the reduced diagrams will fit into the original diagrams?"
- The new shapes must be one ninth of the size of the original shapes. This means that 9 reduced shapes must fit into the original shapes.
- Each reduced shape must be $\frac{1}{9}$ of the original shape.

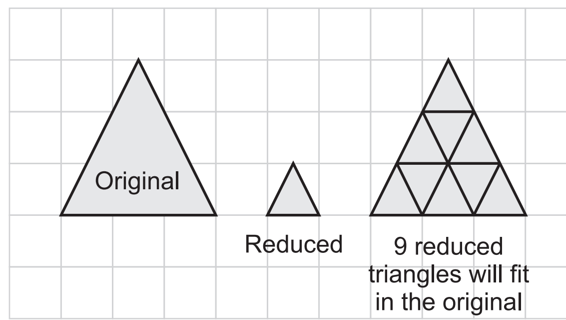




Solution

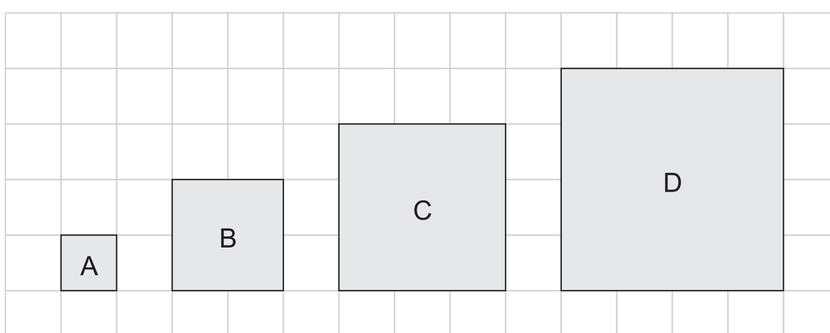
- In each case, 9 of the reduced shapes must fit into the original shape.
- The dimensions of the reduced shapes must be $\frac{1}{3}$ of the dimensions of the original shape to produce reduced shapes that are $\frac{1}{9}$ of the size of the original shape.





Activity: Enlargements and reductions

What enlargement or reduction results in the change in size of the shapes in the following diagram?



1). Square A to square B?
2). Square A to square C?
3). Square A to square D?
4). Square B to square D?
5). Square D to square B?
6). Square D to square A?

Solution

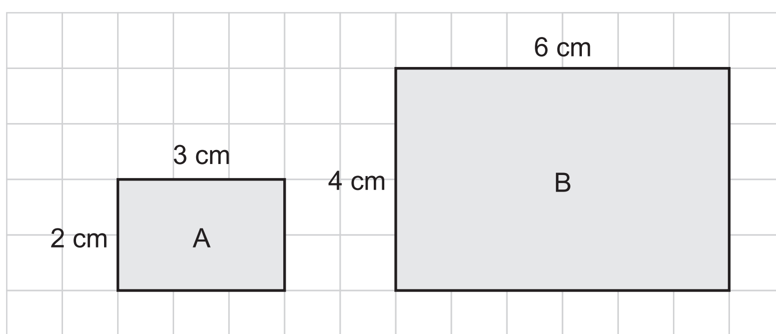
- Square A has been made 4 times bigger to get square B. The lengths of the sides are 2 times longer (double) the lengths of the sides of the original shape.
- Square A has been increased to 9 times its original size to get square C. The lengths of the sides are 3 times longer (triple) the lengths of the sides of the original shape.
- Square A has been made 16 times bigger to get square D. The lengths of the sides are 4 times longer than the lengths of the sides of the original shape.
- Square B has been made 4 times bigger to get square D. The lengths of the sides are 2 times longer (double) the lengths of the sides of the original shape.
- Square D has been made 4 times smaller to get B. The lengths of the sides are 2 times shorter than (half of) the lengths of the sides of the original shape.
- Square D has been made 16 times smaller to get square A. The lengths of the sides are 4 times shorter than (a quarter of) the lengths of the sides of the original shape.

Working with ratios and enlargements/reductions

- When we enlarge or reduce a diagram the lengths of the sides of the original diagram and the new diagram will always be in the same proportion. That means the ratios are the same.

Examples

1). In the diagram that follows what is the ratio of the sides of the shapes?



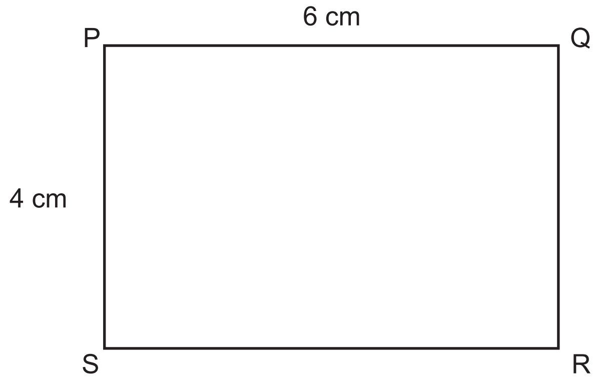
- Length of rectangle A : Length of rectangle B = 3 : 6 = 1 : 2
- Breadth of rectangle A : Breadth of rectangle B = 2 : 4 = 1 : 2
- Therefore the ratio of the sides of the shapes is 1 : 2.

2). Investigate the ratio of the areas.

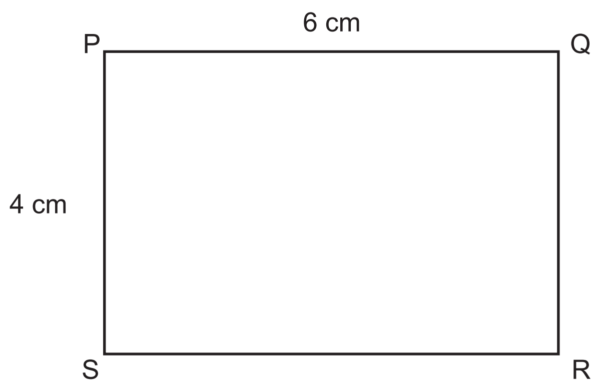
- Count the number of squares in rectangle A: area of A = 6 blocks
- Count the number of squares in rectangle B: area of B = 24 blocks
- Area of A : Area of B = 6 : 24 = 1 : 4

Activity: Ratios, enlargement and reduction

1). You want to enlarge the rectangle so that the side PS is 6 cm long and the sides of the old and new shapes are proportional. How long will the side PQ be after the enlargement?



2). You want to reduce the rectangle that follows so that the side PS is 2 cm long and the sides of the old and new shapes are proportional. How long will the side PQ be after the reduction?



Solutions

1). Old PS : New PS = 4 : 6 = 2 : 3

The ratio for Old PQ : New PQ must be the same

Old PQ : New PQ = 2 : 3 = 6 : ?

× 3   × 3

The new length of PQ will be 9 cm.

2). Old PS : New PS = 4 : 2 = 2 : 1

The ratio for Old PQ : New PQ must be the same

Old PQ : New PQ = 2 : 1 = 6 : ?

× 3 × 3

The new length of PQ will be 3 cm.

Notes:

Length

ANA 2013 Grade 6 Mathematics Items 1.3 and 23

1.3 What is the length of the pencil shown below?

The length of the pencil is …

A 56 cm

B 5,6 mm

C 0,56 m

D 5,6 cm [1]

23. Convert to the units as indicated:

23.1 3 kℓ = ______________ ℓ

23.2 43,5 cm = ______________ m [1]

What should a learner know to answer these questions correctly?

Learners should be able to:

Item 1.3

- Read from a diagram;
- Identify units of measurements;
- Measure using a ruler.

Item 23

- Convert kilolitres to litres;
- Convert centimetres to metres.

Where is this topic located in the curriculum? Grade 6 Term 3

Item 1.3

Content area: Measurement.

Topic: Length.

Concepts and skills:

- Use of measuring instruments: Rulers;
- Identify units of measurements.

Item 23

Content area: Measurement.

Topic: Capacity/Volume.

Concepts and skills:

- Conversion between units.

What would show evidence of full understanding?

Item 1.3

- If the learner selected option D (5,6 cm).

1.3 What is the length of the pencil shown below?

The length of the pencil is ...

A 56 cm

B 5,6 mm

C 0,56 m

(D) 5,6 cm ✓

Item 23

- If the learner converted correctly as shown in the answer that follows.

23. Convert to the units as indicated:

23.1 3 kℓ = 3000 ℓ ✓

23.2 43,5 cm = 0,435 m ✓

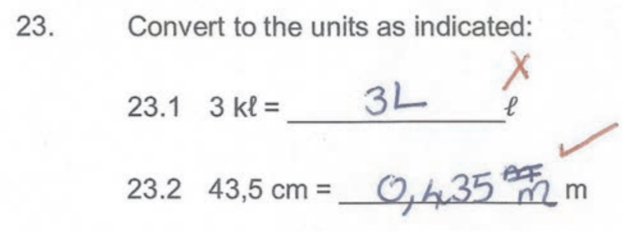
What would show evidence of partial understanding?

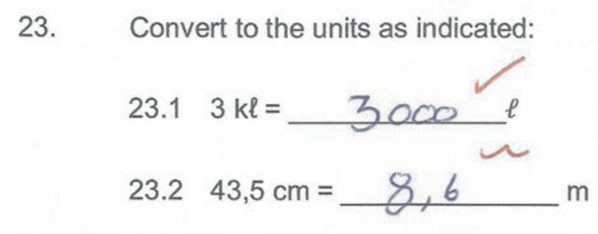
Item 1.3

- If the learner selected option B (5,6 mm): the learner might have read the answer correctly as 5,6 but did not realise that in option B the units are given as mm instead of cm. The learner did not take the units of measurement into consideration.

Item 23

- If the learner only managed to convert one of the units as shown in the following solutions.


23. Convert to the units as indicated:
23.1 3 kℓ = 3L ℓ
23.2 43,5 cm = 0,435 m m


23. Convert to the units as indicated:
23.1 3 kℓ = 3000 ℓ
23.2 43,5 cm = 8,6 m

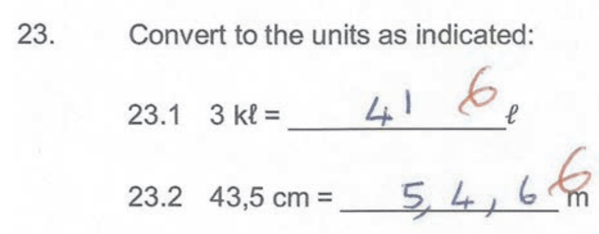
What would show evidence of no understanding?

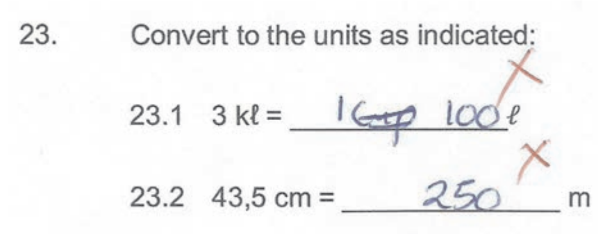
Item 1.3

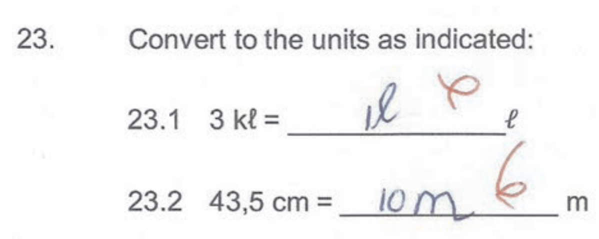
- If the learner selected A (56 cm) or C (0,56 m): this exhibits a lack of understanding of measurements or of the relative sizes of the units involved. (In this case the pencil would be more than half a metre long!)

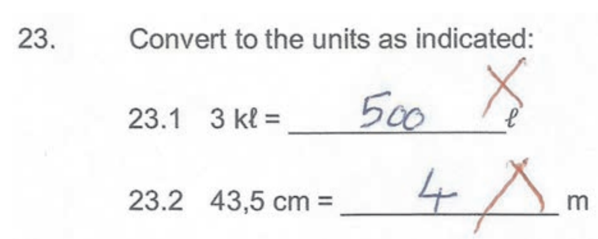
Item 23

If answers written have no apparent link to what the learner was asked to do as in the next examples.


23. Convert to the units as indicated:
23.1 3 kℓ = 41 ℓ
23.2 43,5 cm = 5,4,6 m


23. Convert to the units as indicated:
23.1 3 kℓ = 100 ℓ
23.2 43,5 cm = 250 m


23. Convert to the units as indicated:
23.1 3 kℓ = 1ℓ ℓ
23.2 43,5 cm = 10m m


23. Convert to the units as indicated:
23.1 3 kℓ = 500 ℓ
23.2 43,5 cm = 4 m

What do the item statistics tell us?

Item 1.3:

53 % of learners answered the question correctly.

Item 23.1:

27% o of learners answered the question correctly.

Item 23.2:

10% of learners answered the question correctly.

Factors contributing to the difficulty of the items:

Item 1.3

- Learners may not know how to read from the ruler scale;
- Learners do not know the magnitude of the units.
- Learners do not understand the units of measurement.

Item 23.1

Learners may not know what k*l* and *l* stand for.

Learners may not know how to convert k*l* to *l*.

Item 23.2

Learners may not understand the relationship between m and cm.

Teaching strategies

The meaning of the word kilo

The word kilo is a scientific term that means 1 000. This means if the prefix kilo comes before a unit, we have to multiply by 1 000.

For example, kilolitres or k*l* means 1 000 *l*.

The word kilo is called a prefix, because it is put at the front of a unit to indicate 1 000 times that unit.

Converting between units of measurement

Example

Copy and complete the following table

Value to be converted	Unit to be converted to:	Answer
1 km	m	1 000 m
0,022 km	m	
300 m	km	
1 k*l*	*l*	
0,65 k*l*	*l*	
900 *l*	k*l*	
0,001 k*l*	*l*	

Solution

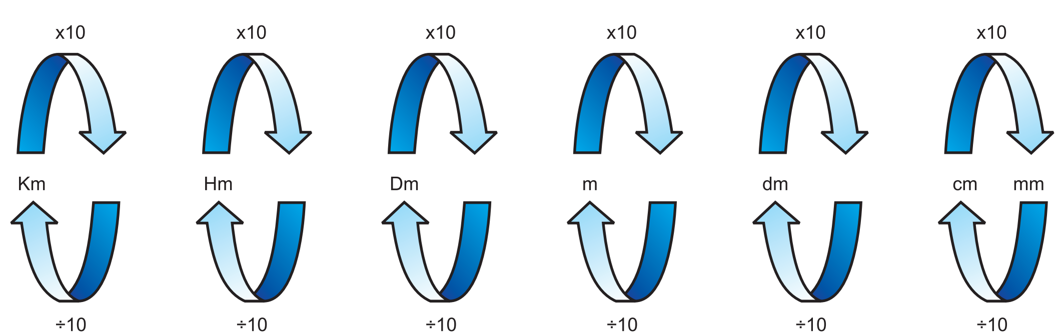
Value to be converted	Unit to be converted to:	Answer
1 km	m	1 000 m
0,022 km	m	22 m
300 m	km	0,3 km
1 k*l*	*l*	1 000 *l*
0,65 k*l*	*l*	650 *l*
900 *l*	k*l*	0,9 k*l*
0,001 k*l*	*l*	1 *l*

The relationship between progressive units of measurement

- Learners must be able to relate and convert between different units of measurement of length: mm, cm, and m.
- The most common units used are kilometres, metres, centimetres and millimetres.
- The other units should be known as they create the links (in multiples of ten) between consecutive units.
- Discuss all of the units, but be sure that learners become familiar with those that are most commonly used.

- The following flow diagram can be used to convert between units of measurement. Show your learners the flow chart drawing to help them to visualise the different prefixes in the correct order.
- Explain the relationship between the units as they get bigger and smaller in the diagram.
- When you read from left to right, the units of measurement get smaller.
- For example centimetres are bigger than millimetres. To convert from millimetres (smaller unit) to centimetres (bigger unit) you must divide by 10.
- When you read from right to left, the units of measurement get bigger.
- For example metres are smaller than kilometres. To convert from kilometres (bigger unit) to metres (smaller unit) you must divide by 1 000.

			reference unit			
kilo-	hecto-	deca-	metre	deci-	centi-	milli-



- Once you have explained the names of the units and how they are related to each other, learners can be given an exercise in which to apply the above flow chart.
- Below is an example of such an exercise, you could use it as it is or

Activity: Converting between units of measurement

Measurement	Converted to:	Answer
5 cm	cm	
350 cm	m	
0,005 6 km	m	
398 900 mm	km	
300 m	km	
100 cm	m	
12 cm	mm	

Solution

Measurement	Converted to:	Answer
5 cm	cm	500 cm
350 cm	m	3,50 m
0,005 6 km	m	5,6 m
398 900 mm	km	0,3989 km
300 m	km	0,3 km
100 cm	m	1 m
12 cm	mm	120 mm

- You can give your learners other activities involving converting between units of length.
- The activities should involve the use of different units of measurement so that learners can understand the size of each unit that they are dealing with.
- The following activity may be used or adapted to suit your needs:

Activity: More conversions between units

To convert cm to mm we multiply by 10 (x10) Example: Convert 69 cm to mm = 69 x 10 = 690 mm	**To convert m to cm we multiply by 100 (x100)** Example: Convert 5 m to cm = 5 x 100 = 500 cm	**To convert km to cm we multiply by 100 000 (x100 000)** Example: Convert 43 km to cm = 43 x 100 000 = 4 300 000 cm
Convert 12 cm, 19 cm and 560 cm to mm	Convert 0,3 m, 0,012 m and 48 m to cm	Convert 19 km, 0,43 km and 0,000 071 km to cm
To convert from mm to cm we divide by 10 (÷10) Example: Convert 80 mm to cm = 80 ÷ 10 = 8 cm	**To convert from mm to m we divide by 1 000** (**÷ 1000**) Example: Convert 3 689 mm to m = 3 689 ÷ 1 000 = 3,689 m	**To convert from mm to km we divide by 1 000 000** (**÷ 1 000 000**) Example: Convert 5 000 000 mm to km = 5 000 000 ÷ 1 000 000 = 5 km
Convert 65 mm, 10 000 mm, 0,578 9 mm to cm	Convert 690 mm, 8 300 mm, 200 000 mm to m	Convert 9 800 000 mm 200 000 mm, 16 000 mm to km
To convert m to mm we multiply by 1 000 (x 1 000) Example: Convert 0,006 5 m to mm = 0,006 5 x 1 000 = 6,5 mm	**To convert m to km we divide by 1 000 (÷ 1 000)** Example: Convert 2 300 m to km = 2 300 ÷ 1 000 =2,3 km	**To convert cm to m we divide by 100 (÷100)** Example: Convert 30 cm to m = 30 ÷ 100 = 0,3 m
Convert 3 m, 0,94 m, 0,0754 m to mm	Convert 15 m, 100 m, 4 000 m to km	Convert 15 cm, 1 cm, 300 cm to m
To convert cm to km we divide by 100 000 (÷100 000) Example: Convert 100 000 cm to km = 100 000 ÷ 100 000 = 1 km	**To convert km to mm we multiply by 1 000 000 (x 1000 000)** Example: Convert 1 km to mm = 1 x 1 000 000 = 1 000 000 mm	
Convert 300 000 cm, 30 000cm, 3 000 cm to km	Convert 5 km, 0,005 km, 0,000 005 km to m	

Solutions

Example given	Measurement to be converted to mm	Answer
Convert 69 cm to mm = 69 **x 10** = 690 mm	12 cm	120 mm
	19 cm	190 mm
	560 cm	5 600 mm
Example given	**Measurement to be converted to cm**	**Answer**
Convert 5 m to cm = 5 **x 100** = 500 cm	0,3 m	30 cm
	0,012 m	1,2 cm
	48 m	4 800 cm
Example given	**Measurement to be converted to cm**	**Answer**
Convert 43 km to cm = 43 **x 100 000** = 4 300 000 cm	19 km	1 900 000 cm
	0,43 km	43 000 cm
	0,000 071 km	7,1 cm
Example given	**Measurement to be converted to cm**	**Answer**
Convert 80 mm to cm = 80 **÷ 10** = 8 cm	65 mm	6,5 cm
	10 000 mm	1 000 cm
	0,578 9 mm	0,057 89 cm
Example given	**Measurement to be converted to m**	**Answer**
Convert 3 689 mm to m = 3689 **÷ 1 000** = 3,689 m	690 mm	0,690 m
	8 300 mm	8,3 m
	200 000 mm	200 m
Example given	**Measurement to be converted to km**	**Answer**
Convert 5 000 000 mm to km	9 800 000 mm	9,8 km

= 5 000 000 ÷ **1 000 000** = 5 km	200 000 mm	0,2 km
	16 000 mm	0,016 km
Example given	**Measurement to be converted to mm**	**Answer**
Convert 0,006 5 m to mm = 0,006 5 m **x 1 000** = 6,5 mm	3 m	3 000 mm
	0,94 m	940 mm
	0,0754 m	75,4 mm
Example given	**Measurement to be converted to km**	**Answer**
Convert 2 300 m to km = 2300 ÷ **1000** =2,3 km	15 m	0,015 km
	100 m	0,1 km
	4 000 m	4 km
Example given	**Measurement to be converted to m**	**Answer**
Convert 30 cm to m = 30 ÷ **100** = 0,3 m	15 cm	0,15 m
	1 cm	0,01 m
	300 cm	3 m
Example given	**Measurement to be converted to km**	**Answer**
Convert 100 000 cm to km = 100 000 ÷ **100 000** = 1 km	300 000 cm	3 km
	30 000 cm	0,3 km
	3 000 cm	0,03 km
Example given	**Measurement to be converted to mm**	**Answer**
Convert 1 km to mm = 1 **x 1 000 000** = 1 000 000 mm	5 km	5 000 000 mm
	0,005 km	5 000 mm
	0,000 005 km	5mm

Activities involving measuring

- To familiarise your learners with the size of the units of measurement you should allow them to do some practical activities.
- This will help them to visualise the units of measurement (such as 1 cm) and understand that a pencil (as in the ANA example) could not be 56 cm in length.
- The following activity may be used or adapted for use to suit your needs.
- Discuss the choice of units with your class.
- Make sure that they understand the importance of choosing the appropriate unit to measure something.

Activity: Measuring

- Measure the following lengths and record your measurements in the table.

Dimension to be measured	Record the length you have measured in			Best unit of measurement to use
	mm	cm	m	
Thickness of a pencil				
Length of a learner's desk				
Breadth of an A4 exercise book				
Length of a classroom				
Breadth of a classroom				
Length of a teacher's desk				

Solutions

Note: The measurements that follow are relative and not representative of all possible measurements.

Dimension to be measured	Record the length you have measured in			Best unit of measurement to use
	mm	cm	m	
Thickness of a pencil	8	0,8	0,008	mm
Length of a learner's desk	1 200	120	1,2	cm
Breadth of an A4 exercise book	207	20,7	0,207	cm
Length of a classroom	15 000	1 500	15	m
Breadth of a classroom	9 000	900	9	m
Length of a teacher's desk	920	92	0,92	cm

Another example of how to test measurement

ANA 2014 Grade 6 Mathematics Item 25.2

25 Use the kilogram scale below to answer the questions.

25.2 Convert the above mass to grams. ______________ [1]

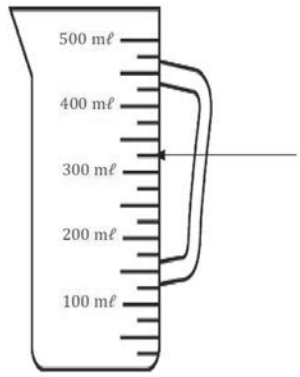
Notes:

Capacity and Temperature

ANA 2013 Grade 6 Mathematics Items 1.8 and 25

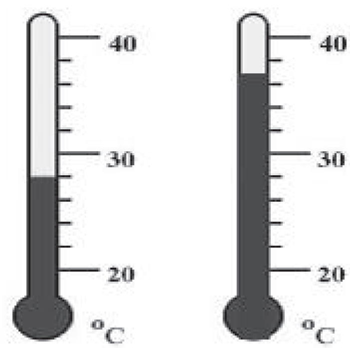
1.8 What capacity does the arrow on the jug indicate?

A 310 ml
B 325 ml
C 320 ml
D 3,1 ml



[1]

25 What is the difference in the temperatures indicated on the two thermometers?



[1]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Read from a diagram and estimate the calibrations;
- Read numbers from differently calibrated scales;
- Subtract two-digit numbers.

Where is this topic located in the curriculum? Grade 6 Term 3

Content area: Measurement.

Topic: Temperature.

Concepts and skills:

- Read temperature measurement;
- Do calculations and problem solving relating to temperature.

What would show evidence of full understanding?

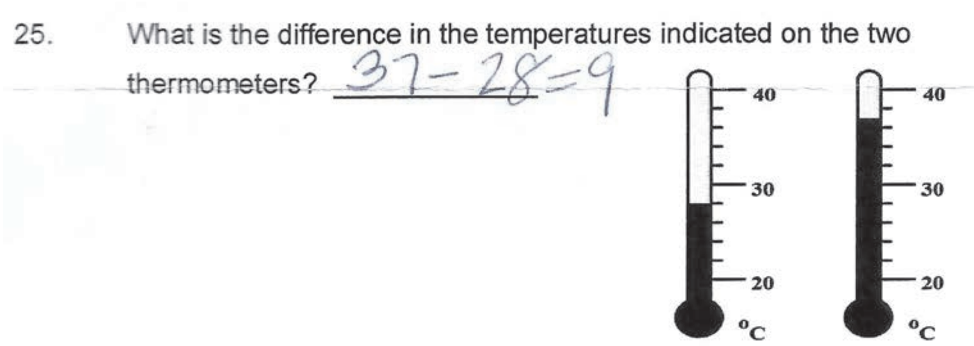
- If the learner correctly interpreted the demarcations on the scale of the measuring cup and took the correct reading;
- If the learner read off the two temperatures and found the difference between them correctly. The learner may have shown the working on the script or simply written down the correct answer.

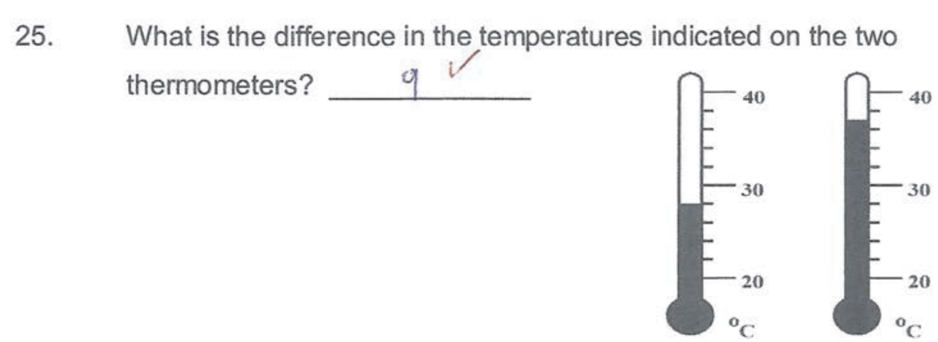
Item 1.8

An answer of 325 m*l* (B) indicates full understanding.

Item 25

An answer of 9 indicates full understanding.





What would show evidence of partial understanding?

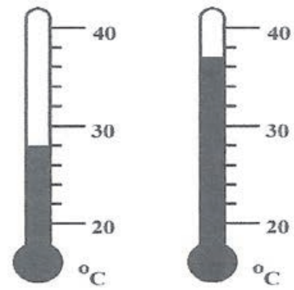
Item 1.8

- If the learner chose 310 m*l* (A) or 320 m*l* (C): this shows the learner could not interpret the scale correctly.

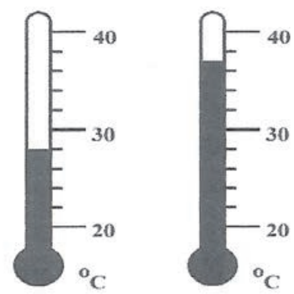
Item 25

- If the learner recorded one or two temperatures correctly, but did not fully answer the question: this indicates that the learner could read off the temperatures but did not know how to complete the question, as shown in the following examples.

25. What is the difference in the temperatures indicated on the two thermometers? 37

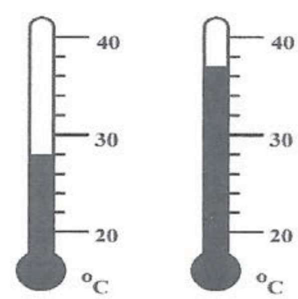


25. What is the difference in the temperatures indicated on the two thermometers? 28 and 37

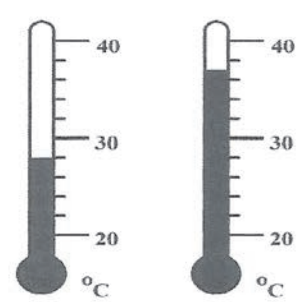


- If the learner subtracted two values, one or none of which was correct: this shows that the learner knew that to find the difference the values had to be subtracted, but did not know how to read those values from the diagrams given. The next three examples demonstrate this.

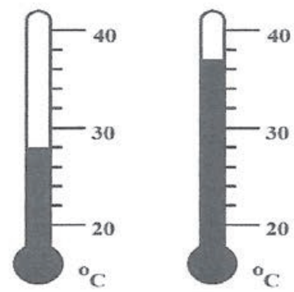
25. What is the difference in the temperatures indicated on the two thermometers? 43 - 35 = 8



25. What is the difference in the temperatures indicated on the two thermometers? 33 - 29 = 14



25. What is the difference in the temperatures indicated on the two thermometers? 35-28 = 7



What would show evidence of no understanding?

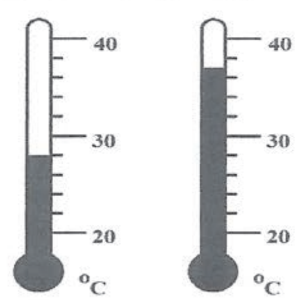
Item 1.4

- An answer of 3,1 ml shows no understanding of the concept of measurement. If the learner chose D as the answer, the learner has little understanding of the size of a millilitre.

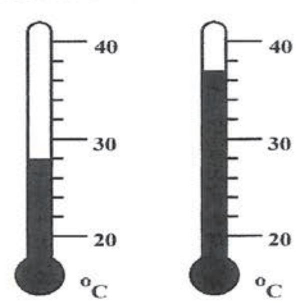
Item 25

- If the learner rounded off the values but did not subtract them, as shown in the 3 examples that follow. The examples show that the learners' lack confidence in their ability to read off the values correctly and made no attempt to find the difference.

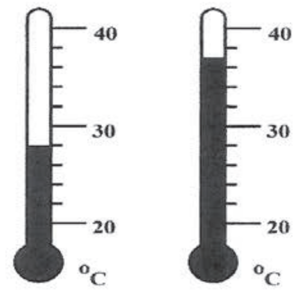
25. What is the difference in the temperatures indicated on the two thermometers? 20c° and 30°



25. What is the difference in the temperatures indicated on the two thermometers? is 25 and 35

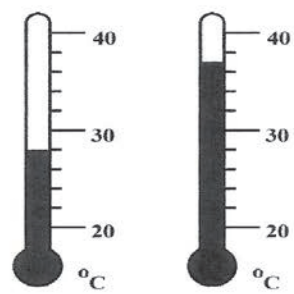


25. What is the difference in the temperatures indicated on the two thermometers? 30 degrees and 25°



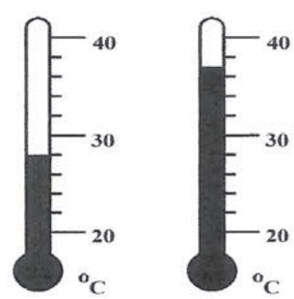
- If the learner gave numbers with decimals, for example, 30,3℃ and 20,4℃: this also points to the learner's inability to estimate between demarcations in temperature. In the following example the learner seems to think that the demarcations indicate decimals.

25. What is the difference in the temperatures indicated on the two thermometers? 20,4°C 3,3½°C

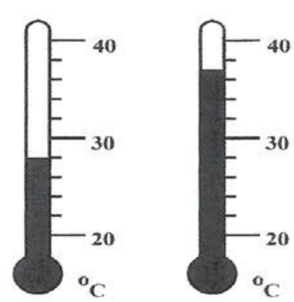


- If, as in these examples, the learner wrote down values or information unrelated to the two thermometers or did not answer the question fully.

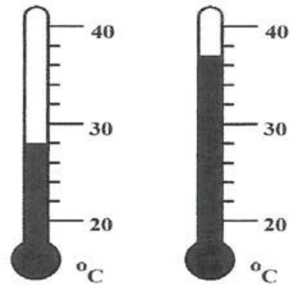
25. What is the difference in the temperatures indicated on the two thermometers? 39°C



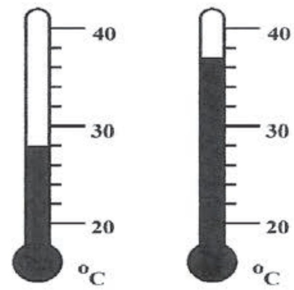
25. What is the difference in the temperatures indicated on the two thermometers? 33



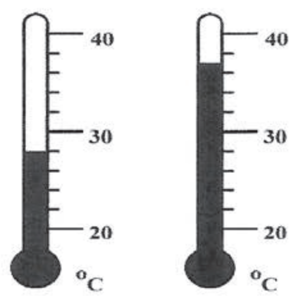
25. What is the difference in the temperatures indicated on the two thermometers? in numbers



25. What is the difference in the temperatures indicated on the two thermometers? They are not the same



25. What is the difference in the temperatures indicated on the two thermometers? it is no



What do the item statistics tell us?

Item 1.8:

34% of the learners answered the question correctly.

Item 25:

9% of the learners answered the question correctly.

Factors contributing to the difficulty of the items

- Learners may have difficulty in reading scales with different calibrations.
- Learners may not understand that they must read and then subtract the two temperatures shown on the thermometers.
- Learners may not be able to read the temperatures shown.
- Learners may not understand the meaning of the word 'difference'.

- Not much time is allocated to the topic temperature (1 hour) in the CAPS document and hence teaching and learning may not have been thorough.

Teaching strategies

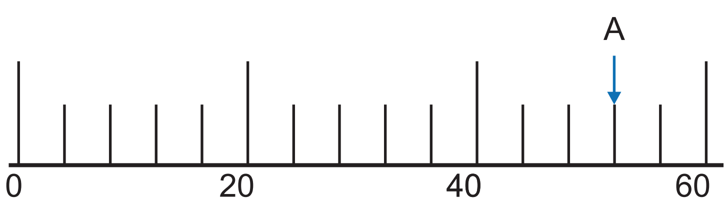
Use of number lines scaled in different calibrations

- We have to give learners guidelines about interpretation of scales.
- To calibrate a measuring instrument means to make little marks on the instrument according to a scale so that the instrument can be used to measure things to a certain degree of accuracy.

- Give learners number lines calibrated differently.
- As a class calculate the value of each demarcation on the scale. To do this you need to follow the steps below:
- Look at the major demarcations.
- Count the number of spaces between these demarcations.
- Calculate: $\frac{\textit{difference between major demarcations}}{\textit{number of spaces between major demarcations}}$
- This gives you the value of each little demarcation.

Example

- Find the value of each little demarcation on the following number lines then give the value of the number represented by a letter on the number line.
- Difference between major demarcations = 20 – 0 = 20
- Number of spaces between major demarcations = 5



$\frac{\textit{difference}}{\textit{number of spaces}} = \frac{20}{5} = 4$
Each little demarcation is 4 units

Value of A: 40 + 4 x 3 = 52

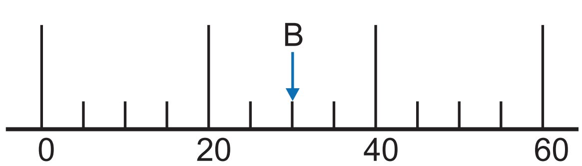
Activities

- Now ask your learners to do the following in the same manner to consolidate what they have learnt.

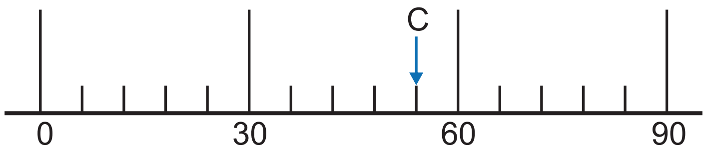
Activity: Reading from a scale

1). Read off the values of letters B to D on the number lines given below

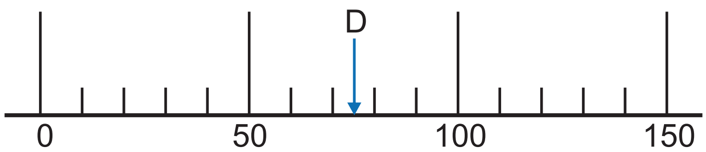
a).



b).

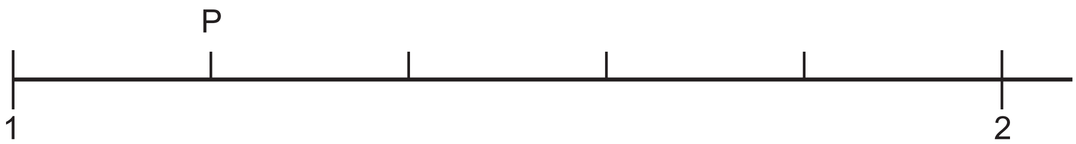


c).

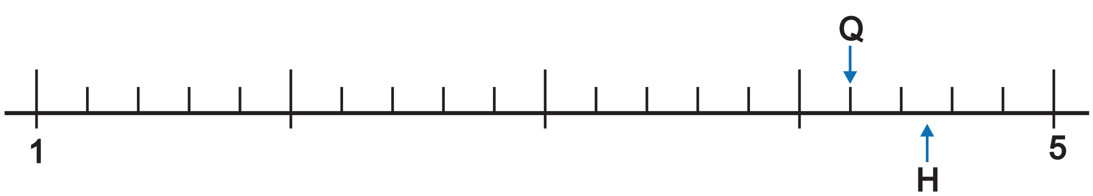


2). Identify the numbers marked by the letters P, Q and H on the number lines below

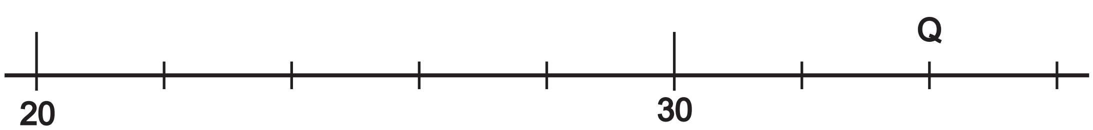
a).



b).



c).



Solutions

1. a).

$$\frac{difference}{number\ of\ spaces} = \frac{20}{4} = 5$$

Each little demarcation is 5 units

Value of B: 30

b).

$$\frac{difference}{number\ of\ spaces} = \frac{30}{5} = 6$$

Each little demarcation is 6 units

Value of C: 54

c).

$$\frac{difference}{number\ of\ spaces} = \frac{60}{5} = 10$$

Each little demarcation is 10 units

Value of D: 75

2).

	Letter	Number represented
a).	P	1,2
b).	Q	4,2
	H	4,5
c).	Q	34

Taking readings from measuring instruments

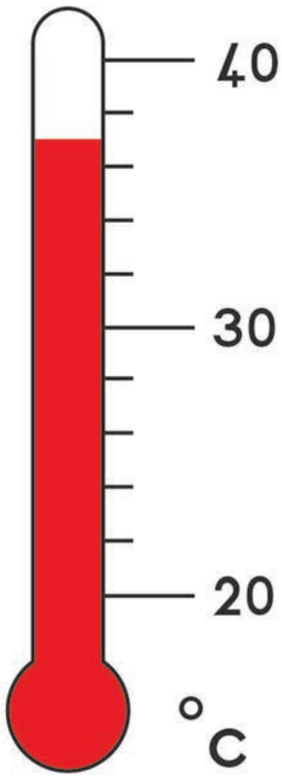
- Provide analogue and or digital thermometers in diagrammatic form.
- Learners can work in small groups to do activities on reading and recording the temperatures they see.
- Discuss learners' work and go over the ways in which they read from the scales so that you are sure they know how to read correctly from a scale.

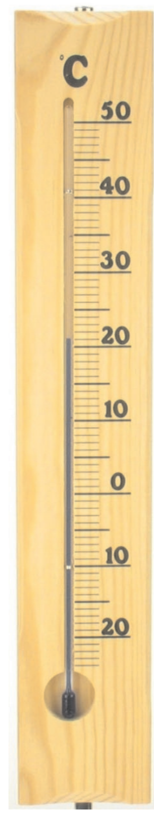
Activity: Find the difference in each of the following pairs of temperature readings:

Set 1

Set 2

Set 3





Set 4

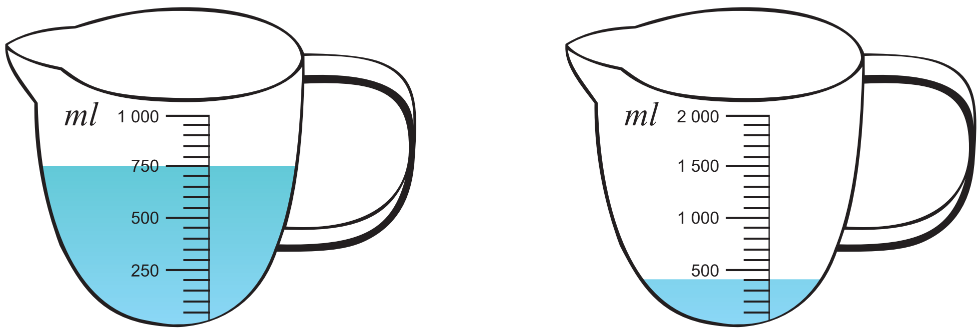
Solutions

Set	**Difference**
1	37 – 33 = 4
2	37 – 24,5 = 12,5
3	37 – 21 = 16
4	25 – 23,7 = 1,3

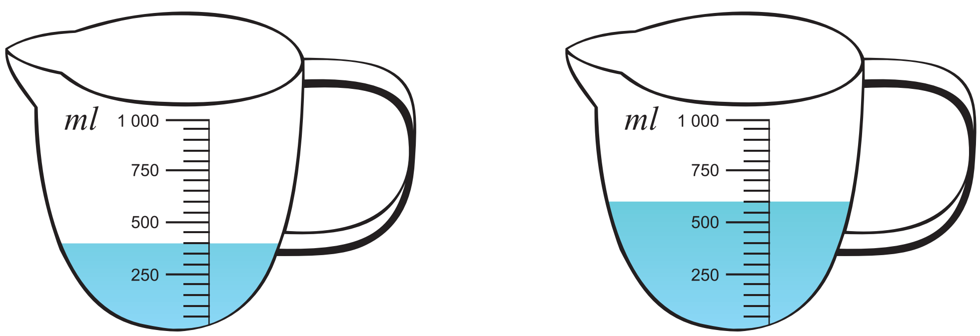
Working with capacity

Examples

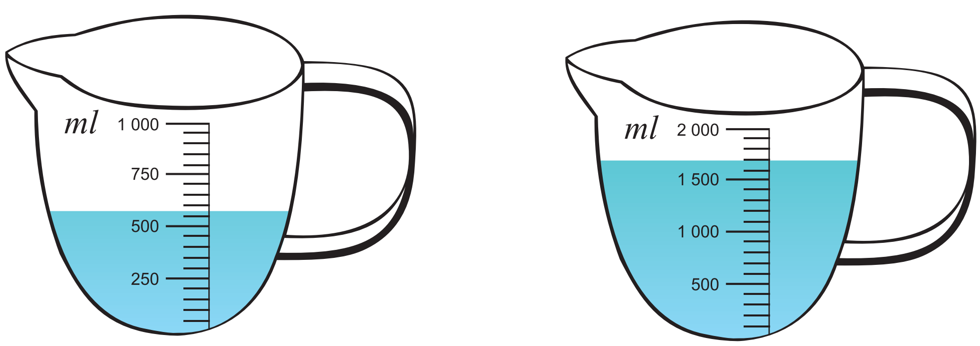
1). If you add 200 ml of water to each jug what would be the new reading?



2). How much water is required to fill each jug up to 800 ml?



3). How much water should you pour out from each jug to drop the water level down to 300 ml?



Solutions

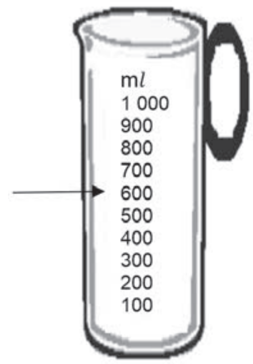
(Learners answers might vary since they might read the scales differently, but their answers should be fairly close to the given answers.)

1). 950 ml ; 600 ml
2). 400 ml; 200 ml
3). 300 ml; 1 400 ml

Another example of how to test measurement – capacity and temperature

ANA 2014 Grade 6 Mathematics Item 23

23 Convert the number of millilitres indicated on the jug to litres. ___________



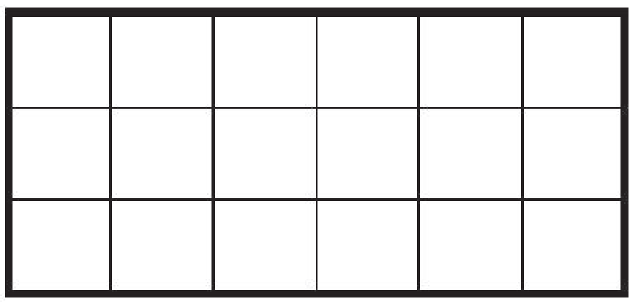
[1]

Notes:

Perimeter and area

ANA 2013 Grade 6 Mathematics Item 26

26. The grid below shows the sketch of a rectangular garden which must be fenced. The squares on the grid are each 1,5 m long. What is the length of the fence around the garden?



[2]

What should a learner know to answer this question correctly?

Learners should be able to:

- Understand that "length around" has the same meaning as "perimeter";
- Add or multiply decimal numbers;
- Calculate the perimeter of a rectangle.

Where is this question located in the curriculum? Grade 6 Term 4

Content area: Measurement.

Topic: Length, perimeter.

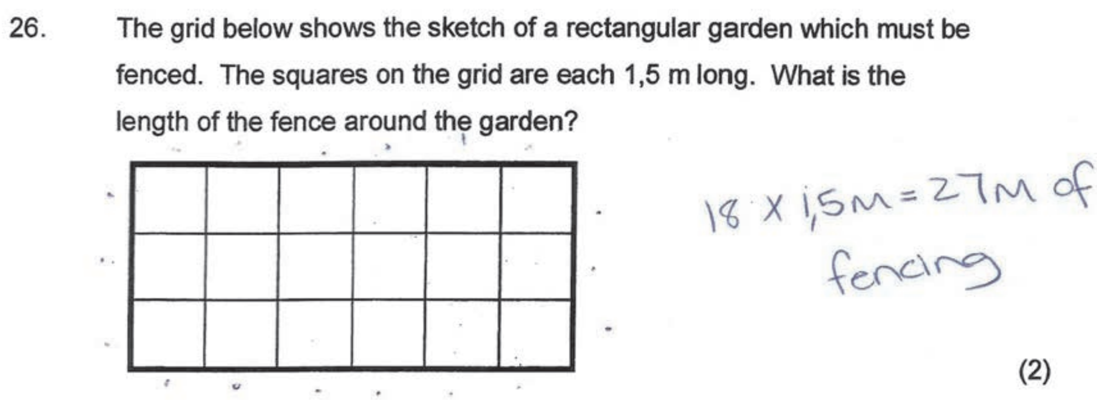
Concepts and skills:

- Solve problems in context related to length.

What would show evidence of full understanding?

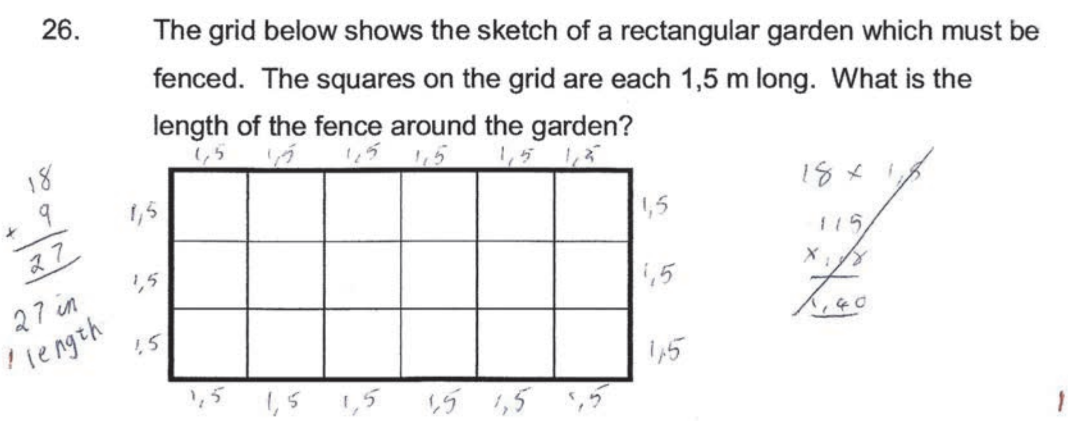
- If the learner calculated the perimeter of the rectangle correctly and gave the answer 27 m, this shows full understanding. However, the same answer could have been obtained by counting the number of blocks in the rectangle and multiplying by 1,5. Therefore, the method of calculation the learner used will give insight into the learner's understanding of the item.

- The following example illustrates that the learner added the pieces of the fence required to get 18 and then multiplied by 1,5.



26. The grid below shows the sketch of a rectangular garden which must be fenced. The squares on the grid are each 1,5 m long. What is the length of the fence around the garden?

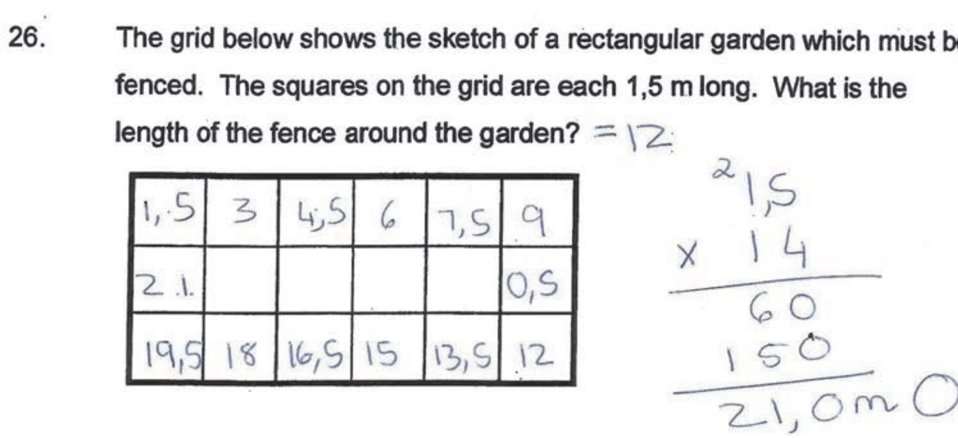
- In the next example the learner understood the question and was able to work out the solution using his/her own algorithm. The learner wrote out the length of each square, then went further to add the 2 lengths of 9 m each to get 18 m. Adding the 2 breadths gives 9 m. At the end the learner added 18 + 9 to get 27. This shows full understanding.



26. The grid below shows the sketch of a rectangular garden which must be fenced. The squares on the grid are each 1,5 m long. What is the length of the fence around the garden?

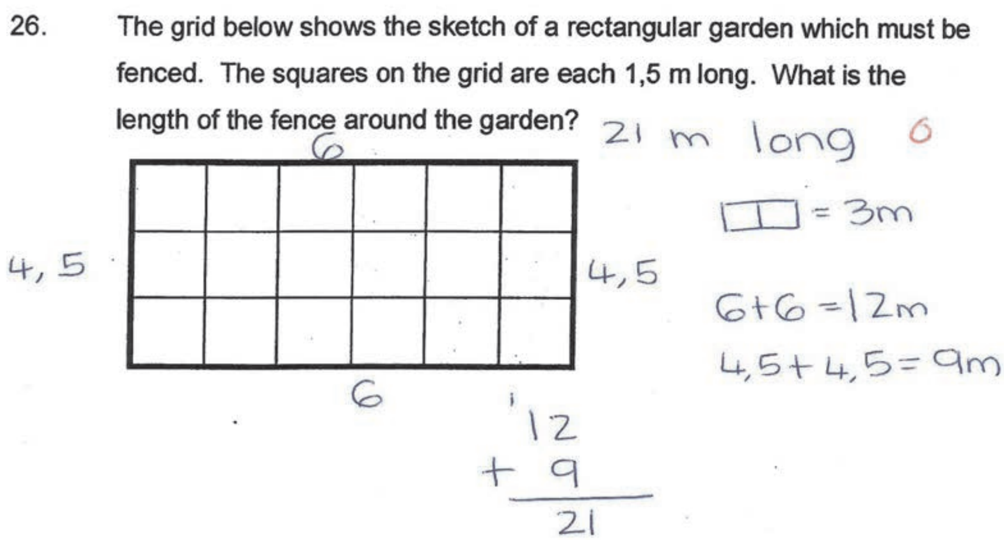
What would show evidence of partial understanding?

- If the learner calculated correctly, but did not count all the pieces of the fence around the edge of the rectangle as in the following example. The learner counted 14 pieces of fence and calculated 14 × 1,5 = 21. This learner did not realise that the squares at the corners must be counted twice because they contribute two lengths to the perimeter. Miscounting the pieces was found to be a common error.



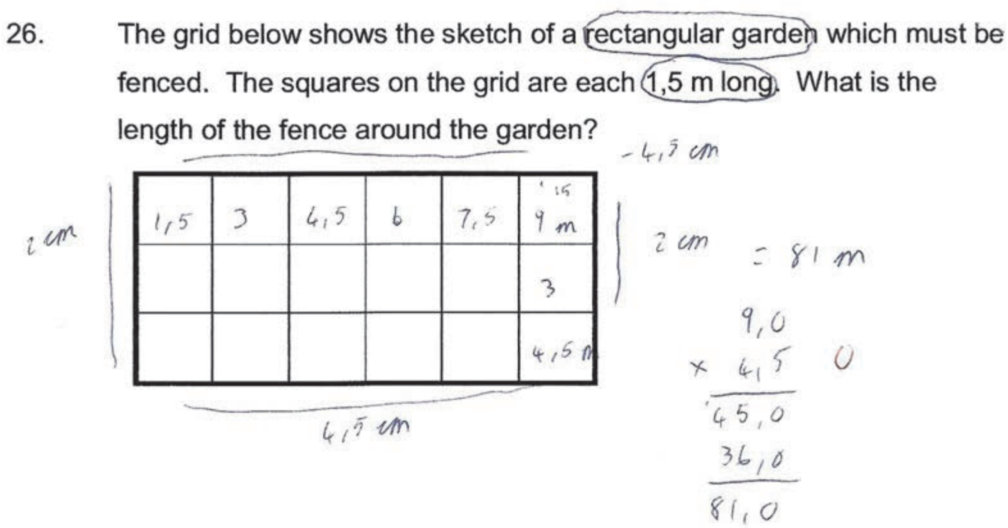
26. The grid below shows the sketch of a rectangular garden which must be fenced. The squares on the grid are each 1,5 m long. What is the length of the fence around the garden? = 12

- In the next example the learner did not take into account that each square in the grid was 1,5 m long. The learner's calculation was correct for the measurements the learner used.



What would show evidence of no understanding?

- The following example shows no understanding of the concepts tested. Note that the learner used cm and m indiscriminately in the calculation. Since learners only encounter multiplication of decimals in Grade 7 it is important to note how they attempt to multiply decimals without formal teaching.
- This learner tried to use the vertical algorithm without taking the place values of the digits into account. Further, the calculation does not represent "length x breadth" and there is no connection between the lengths marked around the shape and the numbers inside the shape.



What do the item statistics tell us?

7% of learners answered the question correctly.

Factors contributing to the difficulty of the item

- Learners may not understand the meaning of "length of fence around";
- Some learners just read "length of the fence" and then calculated 6 x 1,5;
- Learners have not yet done multiplication of decimals at this stage, but many of them attempted to multiply 1,5 by the number of blocks in the perimeter;
- There was no space provided in the question for learners to write their answers.

Teaching strategies

Understanding the measurement of length, area and volume

- Teachers should be aware of the different concepts involving these three measurements – length, area and volume. They are related yet they are different.

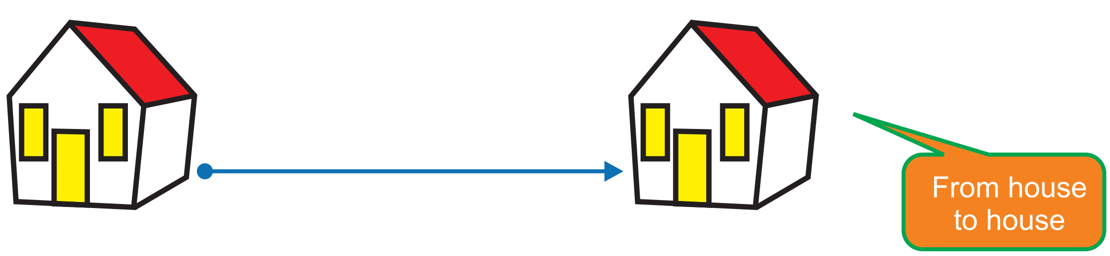
- All these measurements refer to an **amount** of what is being measured.
    - **Length** is the amount of distance.
    - **Area** is the amount of surface covered.
    - **Volume** is the amount of space occupied.

Ask learners, "Which of the following can you relate to the three types of measurements above?"

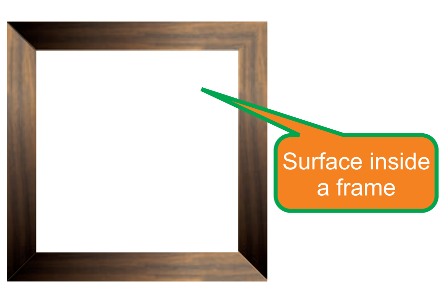
- How far did you walk? (length)
- What is the size of the tile? (area)
- How much water did you drink? (volume)

Examples

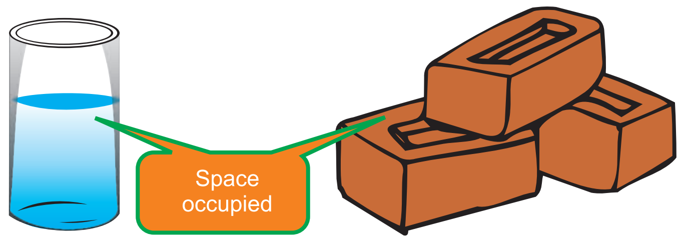
Length: "how far"



Area



Volume



- Ask learners to find objects in the classroom relating to these questions.

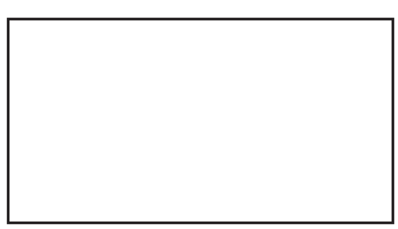
Perimeter

- Perimeter is the measurement of the length of the border/outline of a shape. It is the length of the sides of the shape.
- Perimeter is a one-dimensional measurement of length. (The length "around" the shape).
- Perimeter is measured in units of length – mm, cm, m, km and so on.
- Allow your learners time to work on activities in which they calculate the perimeter of different shapes.
- While they do this remind them that what they are doing is finding the length of the border of a shape.
- The concept of perimeter needs to be consolidated before you can move onto activities about area of a shape.
- Activities that involve measuring the lengths of the sides of shapes will reinforce the concept of perimeter.

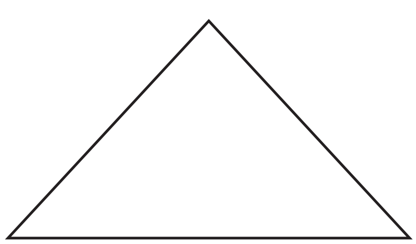
Activity: Perimeter

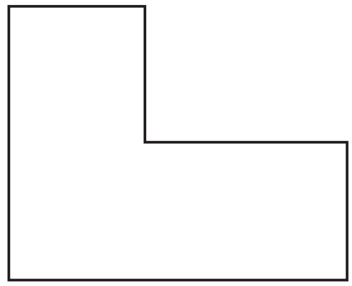
1). In the diagrams below, measure the length of each side and calculate the perimeter

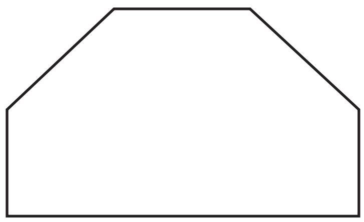
a).

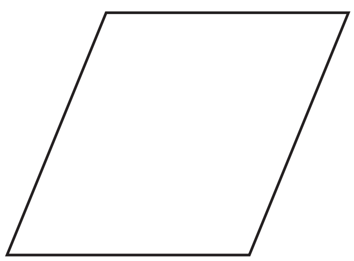


b).



c). 

d). 

e). 

2). Calculate the perimeter of

a). A square of side 12,3 mm
b). A rectangle with length = 5,4 cm and breadth = 4,9 cm
c). A soccer field whose length is 99,5 m and a breadth of 78 m
d). A runway whose length is 2,3 km and a breadth of 0,2 km.

Solutions

- Learners' answers might vary for the first activity above – check their actual measurements and discuss the way the measurements were taken. This is a practical activity.
- Answers will also depend on the unit of measurement the learner used (mm or cm).

1. a). P = 3,8 cm + 3,8 cm + 2 cm + 2 cm = 11,6 cm
   b). P = 2,9 cm + 2,9 cm + 4 cm = 9,8 cm
   c). P = 14 mm + 14 mm + 21 mm + 14 mm + 34 mm + 27 mm = 124 mm
   d). P = 15 mm + 15 mm +15 mm + 11 mm + 36 mm + 11 mm = 103 mm
   e). P = 2,8 cm + 2,3 cm + 2,8 cm + 2,3 cm = 10,2 cm

2. a). P =12,3 mm + 12,3 mm + 12,3 mm + 12,3 mm = 49,2 mm

b). P = 5,4 cm + 5,4 cm + 4,9 cm + 4,9 cm = 20,6 cm

c). P = 99,5 m + 99, 5 m + 78 m + 78 m = 355 m

d). P = 2,3 km + 0,2 km + 2,3 km + 0,2 km = 5 km

The relationship between perimeter and area

- Learners often confuse perimeter and area.
- Remember: Perimeter is the measurement of the border of a shape. It is the length of the sides of the shape.
- Area is the measurement of the amount of surface covered by the shape.
- If learners are given enough practical experience finding that perimeter, which is a length, is a one-dimensional attribute and that area is a two-dimensional attribute, the differences should be easier for them to grasp and remember.
- Learners should also be made aware that each is expressed using different standard units:
  - Perimeter is a measurement of length and is measured in m, cm (and so on).
  - Area is a measurement of surface covered and is measured in m^2 and cm^2 (and so on).

- Some learners calculated the area of the shape in the ANA question rather than the perimeter.
- The activity that follows is one you could use to allow learners to grapple with the relationship between perimeter and area.
- Write the activity questions on the board and allow the learners' time to try them all out.
- You should then go over the activity interactively with the class.
- The solutions to the activity are given with diagrammatic explanations and comments – you should use these solutions as a guide for a discussion with your learners.
- This discussion will consolidate learners' understanding of the relationship as well as the differences between the perimeter and the area of a shape.

Activity: The relationship between perimeter and area

What happens when you double the dimensions?

Decide which of these statements are true and which are false. Show your reasoning by making use of drawings.

A. If you double the dimensions of a rectangle you double its perimeter.

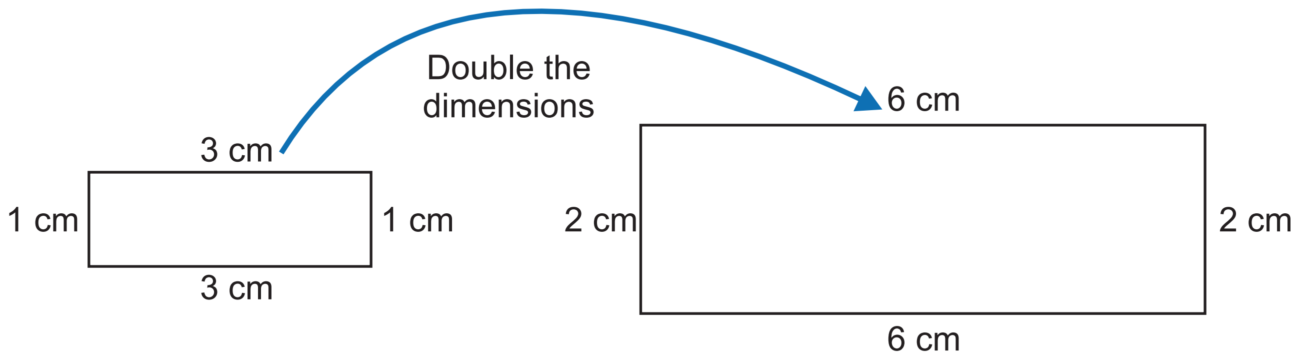
B. If you double the dimensions of a rectangle you double its area.

C. If you double the lengths of the sides of a rectangle, but leave the width the same, you double the area.

D. If you double the dimensions of a cube you double its volume.

Solution

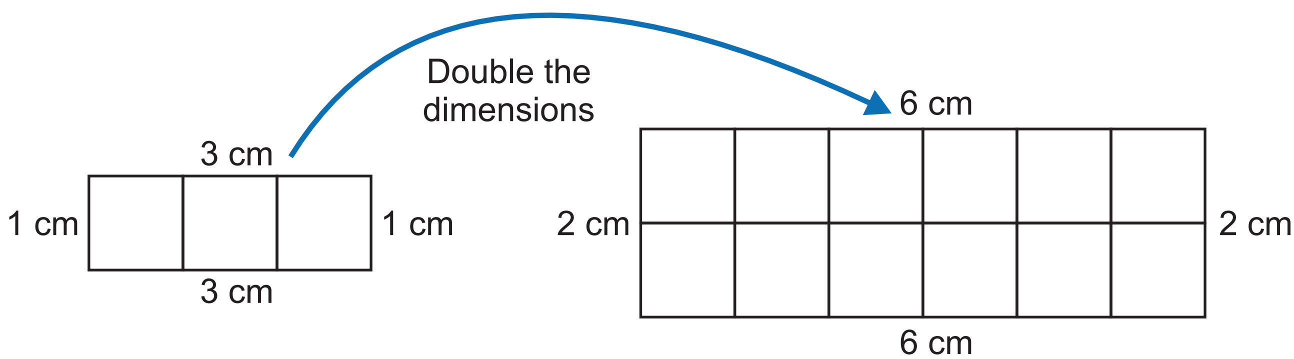
Statement A is true: When you double the dimensions you double the perimeter.



Perimeter (length around the rectangle) = 8 cm

Perimeter (length around the rectangle) = 16 cm

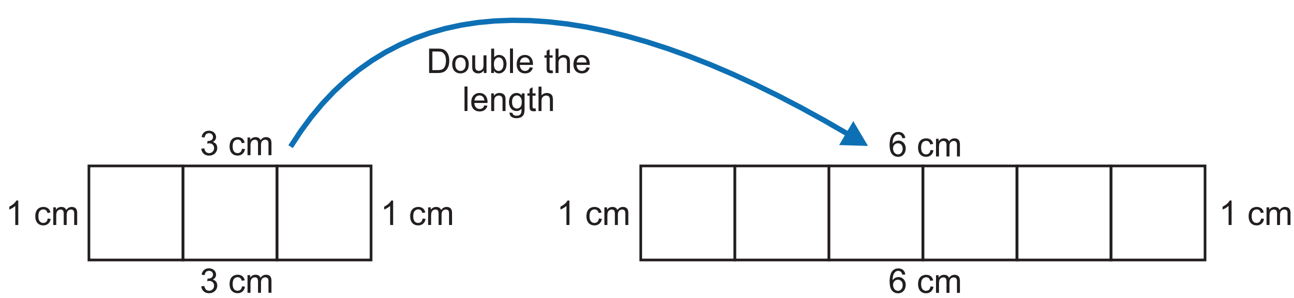
Statement B is false: When you double the dimensions the area becomes four times the original.



Area of rectangle = 3 cm^2

Area of rectangle = 12 cm^2

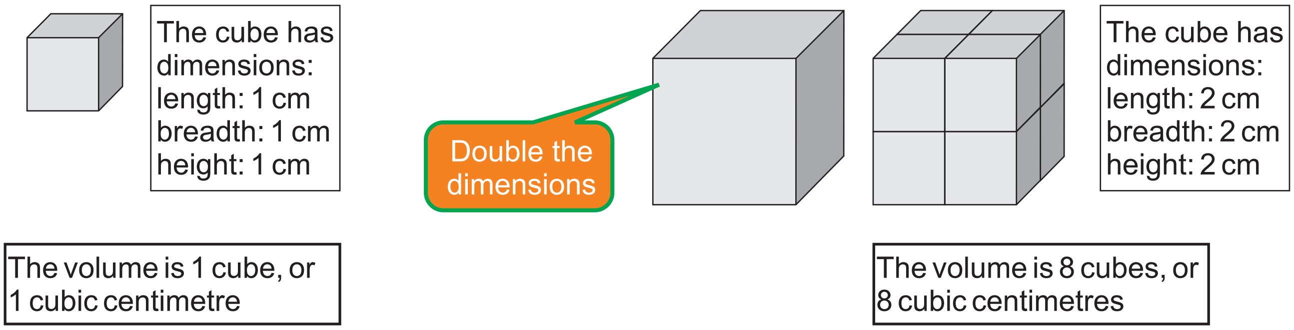
Statement C is true: When you double only the length of the rectangle the area is also doubled.



Area of rectangle = 3 cm^2

Area of rectangle = 6 cm^2

Statement D is false: If you double the dimensions of a cube then the volume is 8 times bigger than that of the original cube.



The dimensions of the cube:

Original volume: $1 \times 1 \times 1 = 1$ cubic centimetre

New volume: $2 \times 2 \times 2 = 8$ cubic centimetres

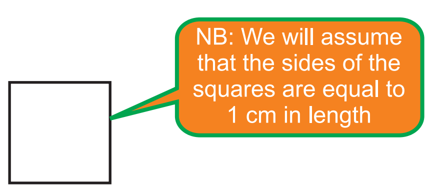
Constructing rectangles when area or perimeter is given

- Practical work is best when the concepts relate to topics that are linked to real-life contexts. This is the case for perimeter and area.
- The next activity gives learners another opportunity to grapple with the relationship between perimeter and area.
- As with the previous activity, you should write the activity questions on the board or prepare a worksheet and then allow the learners time to try the activities out.
- Explain to the class that in the activity they are going to build rectangles using squares that they will cut out. You could cut out the squares yourself in preparation for the lesson. Make sure you have enough to give each group of learners about 20 squares.
- The solutions to the activity are given with diagrammatic explanations and comments – you should use these solutions to guide you in your discussion with your learners. This discussion will consolidate learners' understanding of the relationship as well as the differences between the perimeter and area of a shape.

Activity

Learners should cut out several small squares to use in this activity **(see printable)**.

Group leaners into groups of 4 to 5 learners.



Each group must get about 20 squares.

Build the following rectangles:

1). A rectangle with perimeter (length around) of 10 cm
Write down how many squares you need: __________
The area of your rectangle is: __________

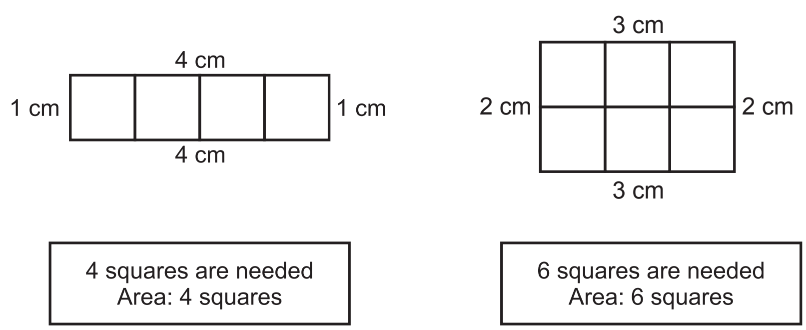
2). A rectangle with an area of 18 squares
Write down the length and breadth of the rectangle:
Length: ________ Breadth: _________
Perimeter: ___________

3). A rectangle with perimeter (length around) of 14 cm
Write down how many squares you need: __________
The area of your rectangle is: __________

Solutions

- Learners may arrive at a variety of solutions.
- You should use the answers below and go through the learners' solutions in a class discussion.
- Remind learners that the perimeter is found by counting the number of units around the edge of the shape.

1). If the perimeter is 10 cm then the dimensions could be:

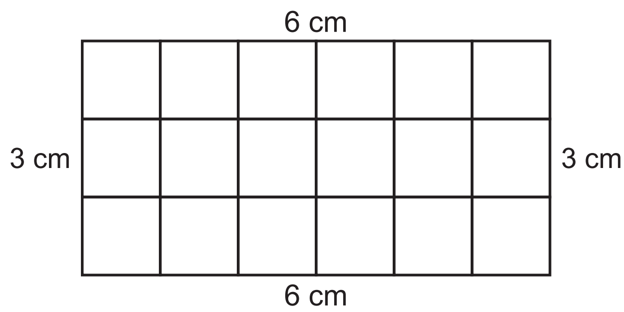


2). If the area is 18 square centimetres then the dimensions could be:
1 by 18: Perimeter = 38 cm
2 by 9: Perimeter = 36 cm
3 by 6: Perimeter = 18 cm

- The 3 by 6 rectangle is illustrated.
- Draw all of the alternatives on the board if your class would like to see them and find the area and

perimeter in all the drawings.

- The class will then start to become more confident about how to count and calculate the perimeter and area of a rectangle.



Length: 6 cm
Breadth: 3 cm
Perimeter: 18 cm

3). A rectangle with perimeter (length around) of 14 cm

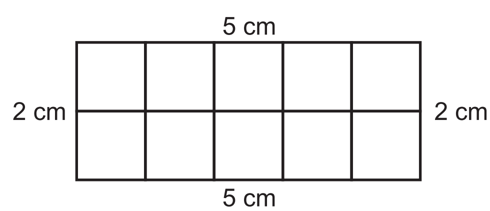
The following rectangles could be constructed:

a). 5 cm by 2 cm: Area = 10 square centimetres

b). 3 cm by 4 cm: Area = 12 square centimetres

c). 1 cm by 6 cm: Area = 6 square centimetres

- The 5 by 2 rectangle is shown here.
- Again, you should draw all of the alternatives on the board if your class would like to see them and use the drawings to find the areas and perimeters.



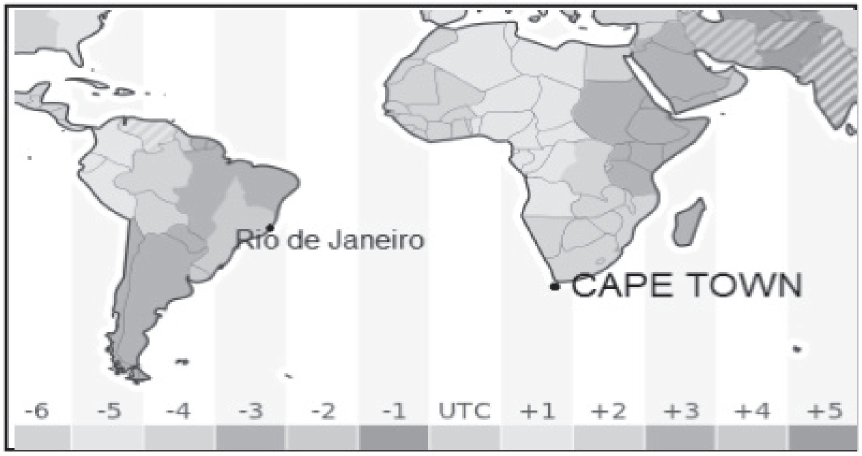
Notes:

Printable: Squares grid

Time zones

ANA 2013 Grade 6 Mathematics Item 22

Examine the map below and answer the questions that follow.



22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? __________

[1]

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? __________

[1]

What should a learner know to answer this question correctly?

Learners should be able to:

- Read information about time zones from a map or number line;
- Select the appropriate time zone from a given diagram;
- Subtract or add the relevant times

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Measurement.

Topic: Time.

Concepts and skills:

- Read time zone maps and do calculations using time zones.

What would show evidence of full understanding?

- If the learner gave the correct time difference between the two cities;
- If the learner found the correct time in Cape Town by adding 4 hours to 11:00 hrs to get 15:00 hrs or 3 pm.

22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? It is 4 hours (1)

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? It is 3p.m (1)

What would show evidence of partial understanding?

- In the following examples the learners managed to read the time intervals of the two cities, but subtracted the time intervals incorrectly.

22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? 3-1 = 2

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? 13:00

22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? -3 and +1 0

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? 1:00 p.m. 0

- In the next example the learner could not answer 22.1 correctly but managed to answer 22.2 correctly.

22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? -6 0

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? 15:00 1

- Some learners managed to work out the time difference as 4 hours, but did not add it to 11:00 hours to get 15:00 hours as in these examples.

22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? 4 hours 1

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? +4 0

What would show evidence of no understanding?

- If a learner gave answers completely unrelated to the question as shown in the two examples that follow.

22.1 What is the time difference in hours between Cape Town and Rio de Janeiro? Province 0

22.2 If it is 11:00 a.m. in Rio de Janeiro, what is the time in Cape Town? 6 pm 0

- If a learner identified the time zones incorrectly.

What do the item statistics tell us?

Item 22.1

8% of learners answered the question correctly.

Item 22.2

8% of learners answered the question correctly.

Factors contributing to the difficulty of the item

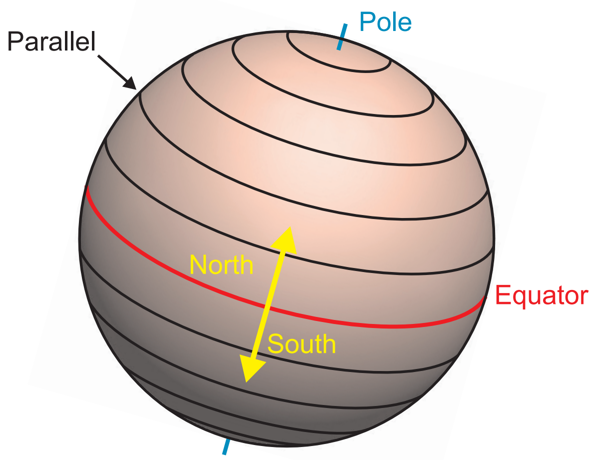
- Learners struggled with identifying correct time zones;
- Learners may not know how to work with a negative value (-3) since this is not prescribed for Grade 6 in the CAPS.
- There is not very much time allocated to time zones in the CAPS – only 4 hours. Hence teaching on this topic might be insufficient.

Teaching strategies

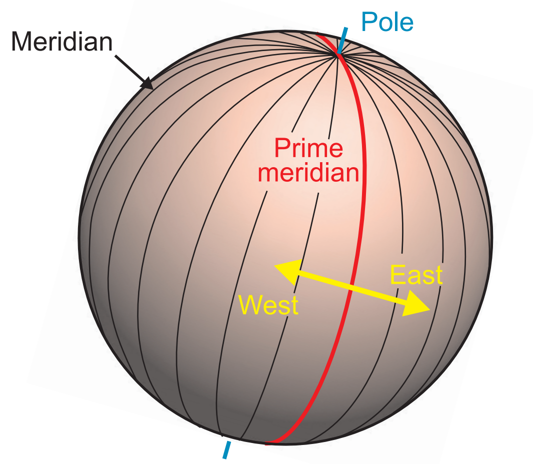
Learning about time zones

- A time zone is a region that has uniform standard time for legal, commercial and social purposes.
- Time zones arise because all the places on earth are not clustered at one point and therefore see the sun at different times.
- Some places are more to the east than others and some are more to the west than others.
- Time zones are defined better by using lines of latitude and lines of Longitude
- Learners need to know about the lines of latitude and longitude around the globe, with lines of latitude affecting weather patterns and lines of longitude affecting time zones.

- The illustration shows the basic globe with lines of latitude and longitude.
- Learners need to know the difference between lines of latitude and longitude and that it is the lines of longitude that are used in the calculation of time zones.

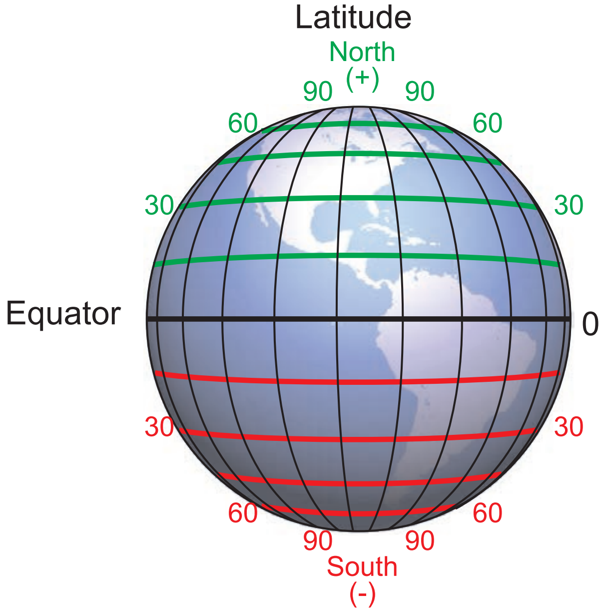


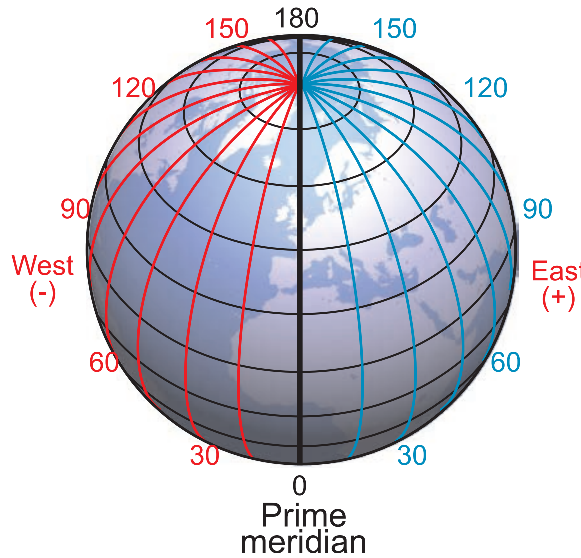
Latitude



Longitude

- The following diagram is another representation of the two sets of lines – latitude and longitude.
- Allow learners time to look at the illustrations and notice how the lines of latitude run parallel to the equator – marking out degrees to the north and south of the equator – while the lines of longitude run from the north pole to the south pole of the earth and mark out degrees to the east and west of a line called the Prime Meridian.



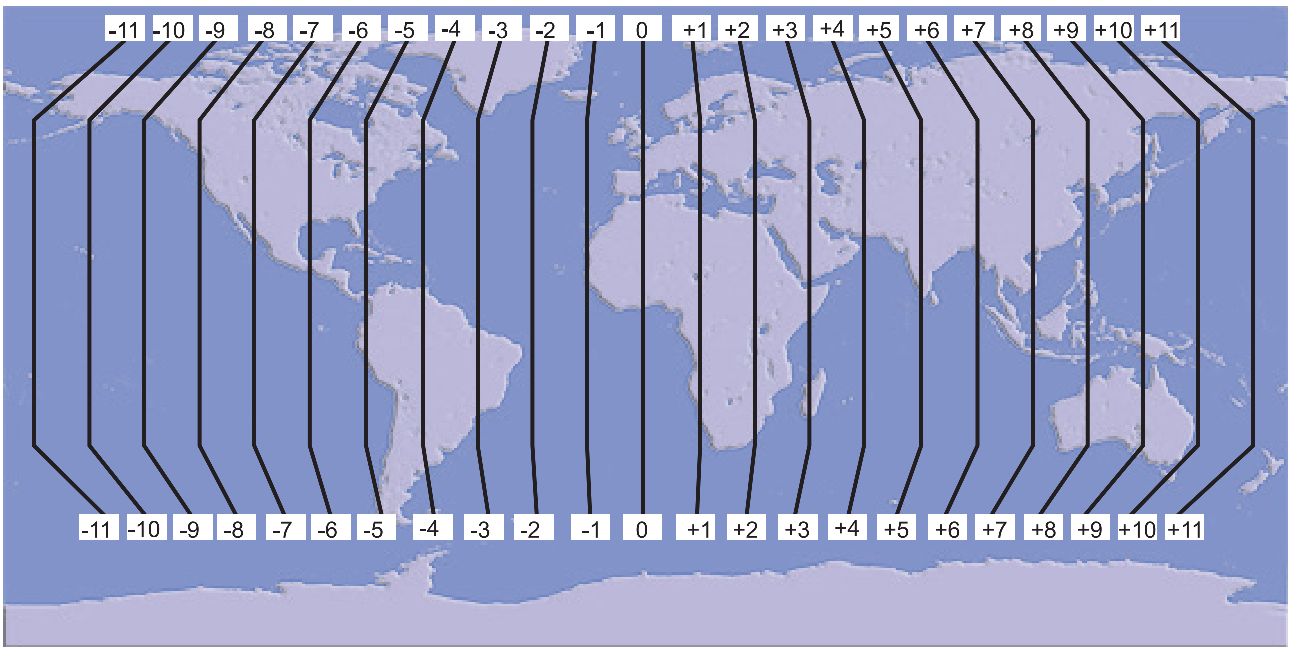


- The map that follows shows the reference point of lines of longitude, the Prime Meridian (or the zero line).
- The sun is first seen on the right (east) of the zero line or Prime Meridian. Therefore, the countries to the east are ahead in time (hours) because they see the sun first!
- The positive numbers to the east represent the degrees they are ahead, as translated into time zones.

- Places to the west of the zero line (to the left) are behind in time (hours) because they see the sun later than those along the zero line.

- The numbers to the west represent the degrees those places are behind, as translated into time zones.

- Each time zone is defined by specific lines of longitude.
- The time on the zero degree line is always regarded as the reference time.
- To the right (east) time is positive (ahead of the zero line) and to the left (west), time is negative (behind the zero line).



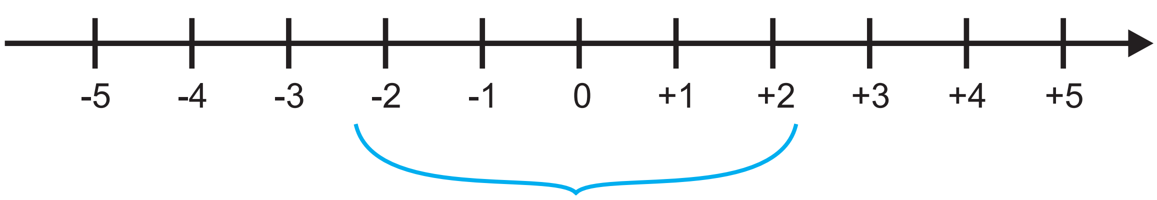
Calculating time intervals

In order to calculate the time intervals consider the following example.

- Consider the numbers on the number line below as time zones.
- Using the number line you can simply count the number of spaces between any two points in order to calculate the time difference between them.

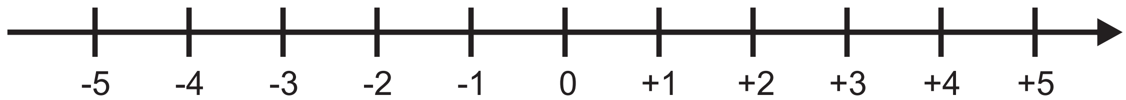
Examples

- The difference in time zones of +2 and -2 is 4 hours. To check this, count the spaces from +2 to -2 on the number line: there are 4 spaces.



- The difference in time zones of +2 and +5 is 3 hours. To check this, count the spaces from +2 to +5 on the number line: there are 3 spaces. (Or algebraically $5 - 2 = 3$).

- The difference in time zones of -4 and -1 is 3 hours. To check this, count the spaces from -4 to -1 on the number line: there are 3 spaces..



+2 – (-2) = 2 + 2 = 4. This is found by summing time on the east (+) and time on the west (-).

+5 – 2 = 3. This is found by subtracting.

-1 – (-4) = 3. This is found by subtracting.

- A different way to think about time differences when times zones are on either side of the zero line is to think about distances.
- For example, if one car is 5 km to the left of a given point and one car is 5 km to the right of a given point, what is the distance between the two cars? It is 10 km.
- We get this total distance between the cars by adding the distances apart, not subtracting.

- To sum up, there are two types of differences between time zones that we need to calculate.
- If the times zones have the **same sign** (i.e. the times are on the same side of the zero line) one just subtracts the two values, disregarding the signs.
- To find the time difference between time zones that have a **different sign** (i.e. the time zones are on the either side of the zero line) one just adds the values, disregarding the signs.

For example:

1). What is the time difference between zone +3 and zone +2?

2). What is the time difference between zone -3 and zone -2?

- In these examples the two time zones given have the same signs: they are on the same side of the zero line. So to find the time difference we just subtract one value from the other:

Time Difference = 3 – 2 = 1 hour.

3). What is the time difference between zone +3 and zone -2?

- If the two time zones have different signs (i.e. the two times are on different sides of the zero line) then we simply add up the two times, disregarding the signs.

- The two times in this example are on different sides of the zero line (they have different signs) so we just add the two times.

Time Difference = 3 + 2 = 5 hours.

Activities: Calculating the differences between time zones

1). Calculate the difference in time between the following time zones

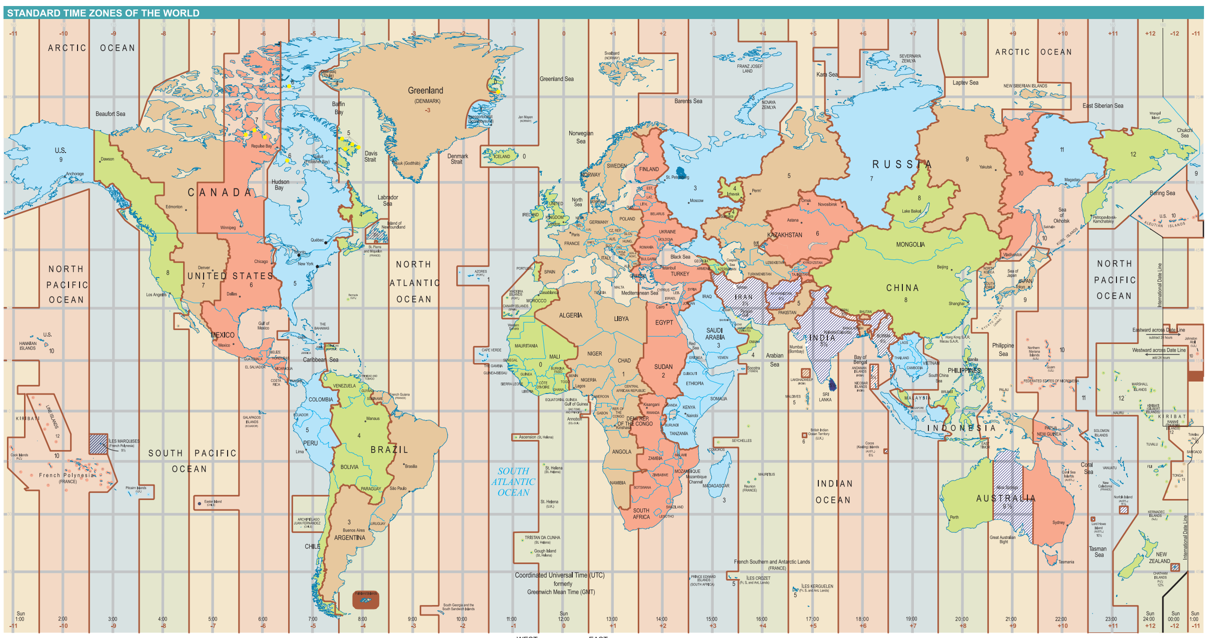
	Time Zones	Difference in the time
1).	zone +2 and zone -2	Signs are different, therefore we add +2 +2 = 4 Hours. (Or simply count the spaces between +2 and -2 on the above number line.)
2).	zone -5 and zone -3	
3).	zone +3 and zone +2	
4).	zone -7 and zone -2	
5).	zone -11 and zone +12	

Solutions

	Time Zones	Difference in the time
1).	zone +2 and zone -2	Signs are different, therefore we add +2 +2 = 4 Hours. (Or simply count the spaces between +2 and -2 on the above number line.)
2).	zone -5 and zone -3	Signs are the same, we subtract the times 5 – 3 = 2 hours
3).	zone +3 and zone +2	Signs are the same, we subtract the times 3 – 2 = 1 hour
4).	zone -7 and zone -2	Signs are the same, we subtract the times -7 – 2 = 5 hours
5).	zone -11 and zone +12	Signs are different, therefore we add 12 + 11 = 23 hours

2). Identifying correct time zones

- Time zones are read on the horizontal line (x-axis), which is just like the horizontal number line used in the preceding example
- Use the map to answer the questions that follow.
- A printable map is provided for your use at the end.



1). Identify the time zones of the following towns and cities. Pretoria has been done for you.

	City	Time zone
a).	Pretoria	+2
b).	Addis Ababa	
c).	La Paz	
d).	New York	
e).	Mexico City	
f).	Algiers	
g).	Sydney	
h).	London	

2). Find the differences in time between the cities listed below.

	City 1	City 2	Time Difference
a).	Pretoria	Abidjan	+2 – 00 (noon) = 2 hours
b).	Cape Town	Los Angeles	
c).	New York	London	
d).	Paris	Miami	
e).	Hong Kong	Dar es Salaam	
f).	Pretoria	London	
g).	Cape Town	Beijing	

Solutions

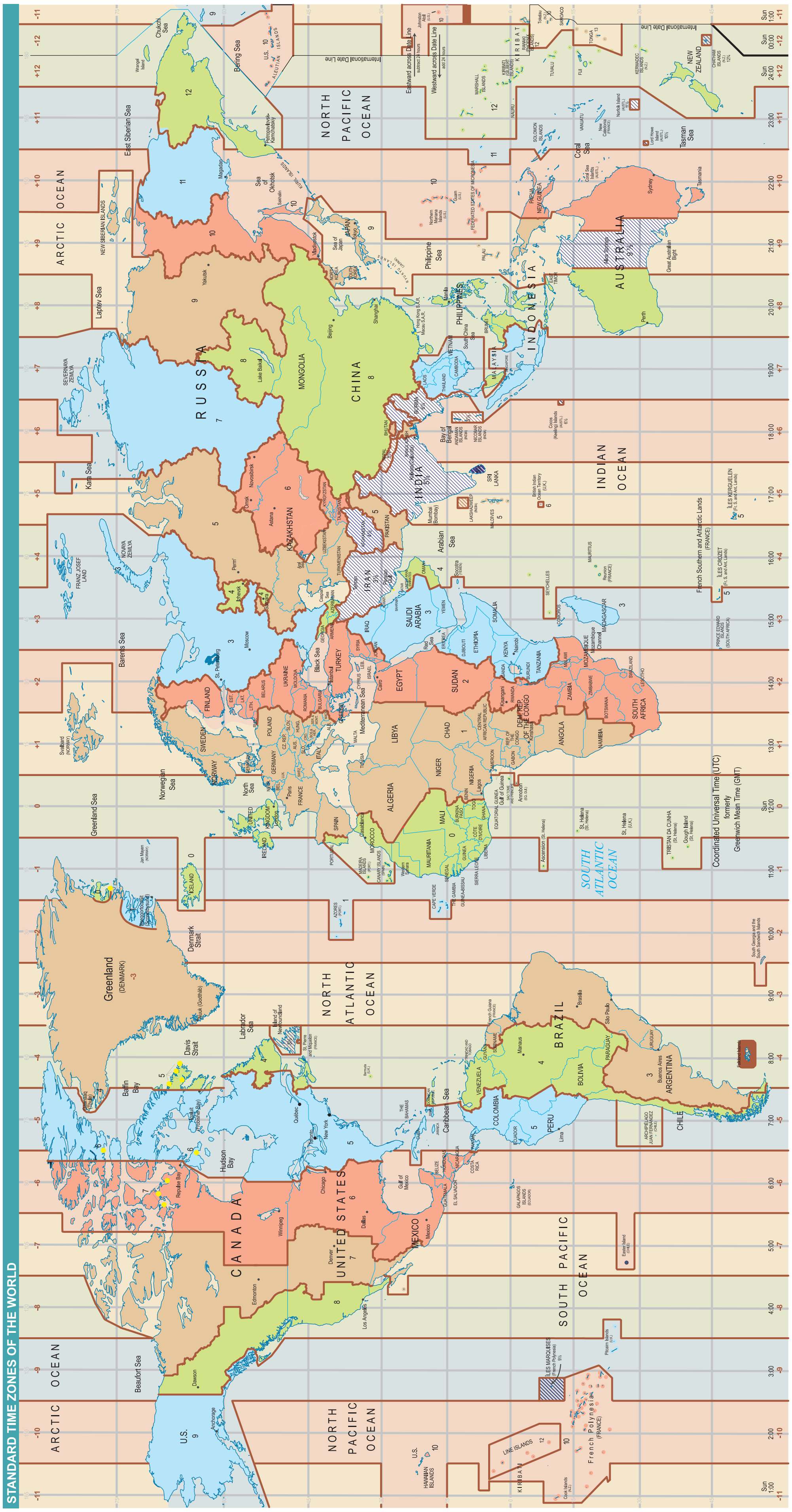
1).

	City	Time zone
a).	Pretoria	+2
b).	Addis Ababa	+2
c).	La Paz	+3
d).	New York	-4
e).	Mexico City	-5
f).	Algiers	-6
g).	Sydney	+1
h).	London	+10

2).

	City 1	City 2	Time Difference
a).	Pretoria	Abidjan	+2 – 00 (noon) = 2 hours
b).	Cape Town	Los Angeles	These 2 cities are on different sides of the zero line so we add the times. Time difference = 2 + 8 = 10 hours
c).	New York	London	London is on the zero line and New York is to the west. We just subtract the 2 values. Time difference = 5 – 0 = 5 hours
d).	Paris	Miami	1 + 5 = 6 hours
e).	Hong Kong	Dar es Salaam	8 – 3 = 5 hours
f).	Pretoria	London	2 – 0 = 2 hours
g).	Cape Town	Beijing	8 – 2 = 6 hours

Printable Time zone map



Another example of how to test time

ANA 2014 Grade 6 Mathematics Items 22.1 and 22.2

22. Study the clock faces showing the time in Rome and Tokyo.
Rome and Tokyo are in different time zones.

	
Rome Time in the morning (a.m.)	Tokyo Time in afternoon (p.m.)

22.1 Calculate the time difference between Rome and Tokyo.

____________________ [1]

22.2 If it is 17:00 in Tokyo, what time will it be in Rome? ____________________ [1]

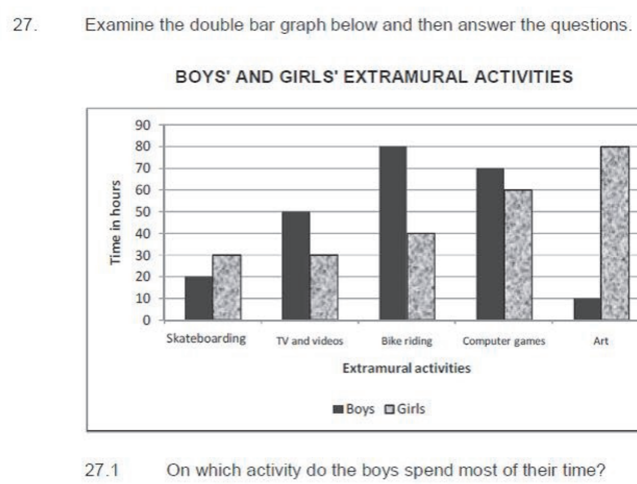
Notes:

Data handling

ANA 2013 Grade 6 Mathematics Items 1.5, 27, 28 and 30

1.5	What is the median of the data set below?						
	20	30	30	40	40	50	50
	A	20					
	B	30					
	C	40					
	D	50					[1]

27. Examine the double bar graph below and then answer the questions

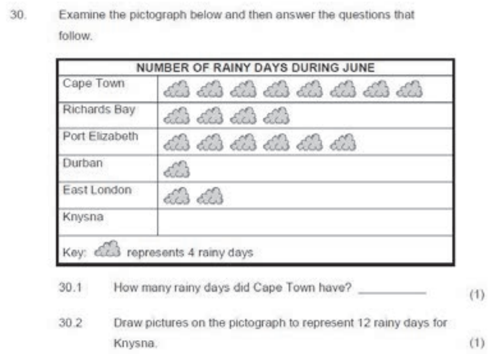


27.1 On which activity do the boys spend most of their time? [1]

27.2 How much more time do the girls spend on Art than boys? [1]

28	What is the mode of the given set of numbers?								
	21	22	22	23	23	24	25	25	25 [1]

30.	Examine the pictograph below and then answer the questions



30. Examine the pictograph below and then answer the questions that follow.

NUMBER OF RAINY DAYS DURING JUNE	
Cape Town	
Richards Bay	
Port Elizabeth	
Durban	
East London	
Knysna	

Key: represents 4 rainy days

30.1 How many rainy days did Cape Town have? ________ (1)

30.2 Draw pictures on the pictograph to represent 12 rainy days for Knysna. (1)

30.1 How many rainy days did Cape Town have? ___________ [1]

30.2 Draw pictures on the pictograph to represent 12 rainy days for Knysna. [1]

What should a learner know to answer these questions correctly?

Learners should be able to:

- Take readings from a double bar graph;
- Interpret information from a double bar graph;
- Answer questions relating to median and mode;
- Interpret the key of a pictogram.

Where is this topic located in the curriculum? Grade 6 Term 1

Content area: Data handling.

Topic: Representing, analysing and interpreting data.

Concepts and skills:

- Item 27: Critically read and interpret data represented in double bar graphs.
- Items 1.5 and 28: Analyse data by answering questions related to central tendencies (mode and median).
- Item 30: Critically read and interpret data represented in pictographs.

What would show evidence of full understanding?

Item 5.1

If the learner correctly identified the middlemost number in the data set as 40.

Item 27.1

If the learner read the bar graph correctly and deduced that boys spend the most time bike riding.

Item 27.2

- If the learner read the number of hours that the girls and boys each spend on Art correctly and subtracted the values correctly to get 70 hours as shown in the example.

28. What is the mode of the given set of numbers?

21 22 22 23 23 24 25 25 25

70 hours because 80 - 10 = 70

Item 28

- If the learner correctly chose 25 as the mode or the number that appears most frequently in the set, as shown in the following answer: in this example the learner showed the counting used to determine the number.

28. What is the mode of the given set of numbers?

21 22 22 23 23 24 25 25 25

25

Item 30.1

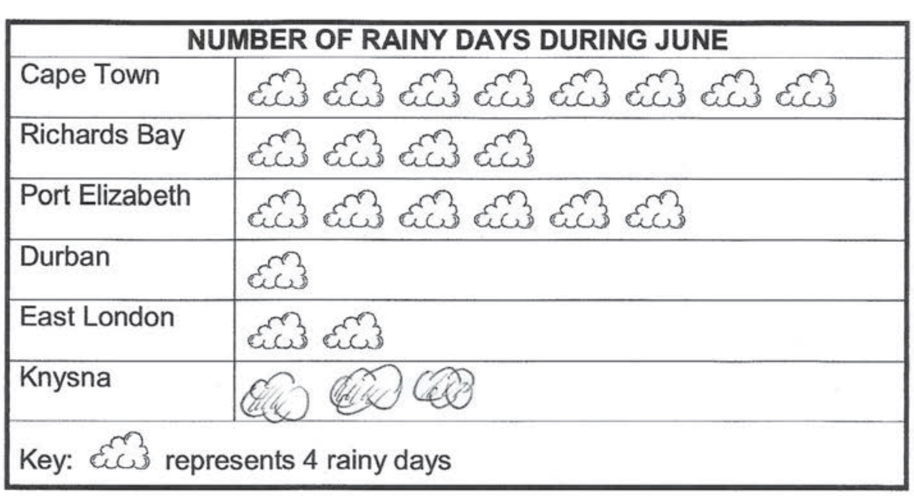
- If the learner correctly counted the number of clouds for Cape Town as 8 and then multiplied by 4 to get 32. In the example shown the learner not only provided the correct answer, but demonstrated critical reasoning by interpreting the answer, comparing it to the number of days in the month.

30.1 How many rainy days did Cape Town have? 32 more than a month (1)

Item 30.2

- If the learner drew 3 clouds for Knysna on the pictograph to indicate 12 rainy days: this indicates that the learner was able to interpret the scale and converted the "12 rainy days" to "3 clouds".

30. Examine the pictograph below and then answer the questions that follow.



What would show evidence of partial understanding?

Item 5.1

- If the learner selected B this shows partial understanding. 30 is close to 40, hence a learner may mistakenly have chosen 30, but with the intention of selecting 40.

Item 27.1

- Learners that said that Art was the activity that boys spend the most time on showed partial understanding: they did select the "highest bar", but the mistakenly selected the highest bar representing girls (instead of boys), which was not required for this answer.

27.1 On which activity do the boys spend most of their time?

Art

Item 27.2

- The language in this question seems to have been problematic for learners. The answers given indicate that learners often did not read the entire sentence, but skipped words or interpreted the question differently from what was asked.

- In the examples that follow the learners appear to have skipped some words and read the question as: "How much time do girls spend on Art?" or "How many girls spend time on Art?" They provided the following answers:

27.2 How much more time do girls spend on Art than boys?

80 girls the time in Art

27.2 How much more time do girls spend on Art than boys?

they spend more time on ~~computer on~~ (1) Art

27.2 How much more time do girls spend on Art than boys?

Art 80

- In the next example the learner subtracted 10 from 80 correctly, but gave the answer as 70 boys instead of 70 hours.

27.2 How much more time do girls spend on Art than boys?

~~80~~ 70 Boys

- The following example shows that the learner calculated correctly, but responded 70 times more, which has a different meaning to 70 hours more.

27.2 How much more time do girls spend on Art than boys?

70 times more

- These examples make it clear that learners' interpretation of the language used in this item was problematic.

Item 28

- If the learner answered 23: this shows the learner was confused by the terms mode and median. The example shown indicates clearly that the learner calculated the median instead of the mode.

28. What is the mode of the given set of numbers?

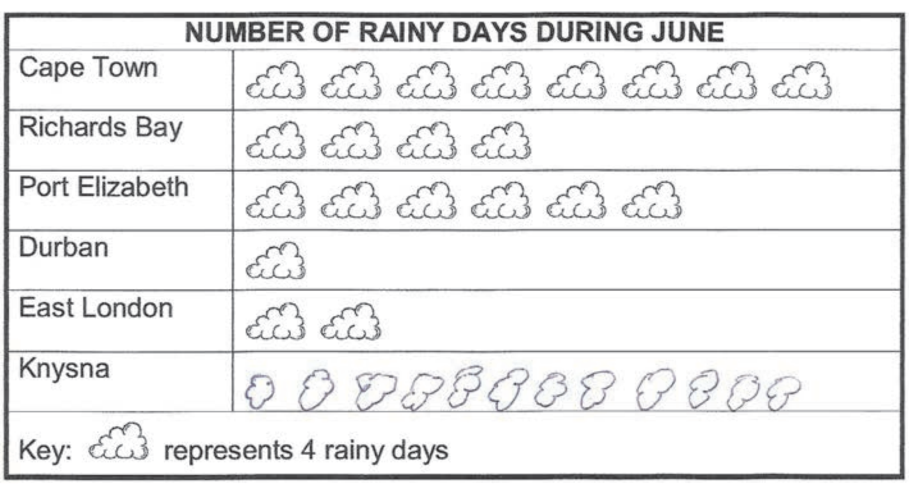
21 22 22 23 23 24 25 25 25

23

Item 30.1 and 30.2

- If a learner gave the answer 8:this indicates that the learner counted the number of clouds correctly, but ignored the scale and so did not multiply by 4. There was only one mark allocated to this question and hence this learner did not score on the test but the teacher can develop this partial understanding into full understanding by acknowledging it and taking it forward.
- If the learner drew 12 clouds for Knysna this shows the learner ignored the scale.

30. Examine the pictograph below and then answer the questions that follow.



30.1 How many rainy days did Cape Town have? 8 0

30.2 Draw pictures on the pictograph to represent 12 rainy days for Knysna. 0

What would show evidence of no understanding?

Item 5.1

- If the learner answered A or D: this indicates little or no understanding of what was asked as A is the minimum whilst D is the maximum.

Items 27.1 and 27.2

- If the learner gave an answer unrelated to the question this shows no real understanding, for example:

27.1 On which activity do the boys spend most of their time?

~~8~~ 10

27.2 How much more time do girls spend on Art than boys?

Art

Item 28

- If the learner interpreted the set of numbers as a number pattern and tried to find the next numbers in the row, this indicates that the learner had no understanding of what was being asked.

28. What is the mode of the given set of numbers?

21 22 22 23 23 24 25 25 25

26 26

28. What is the mode of the given set of numbers?

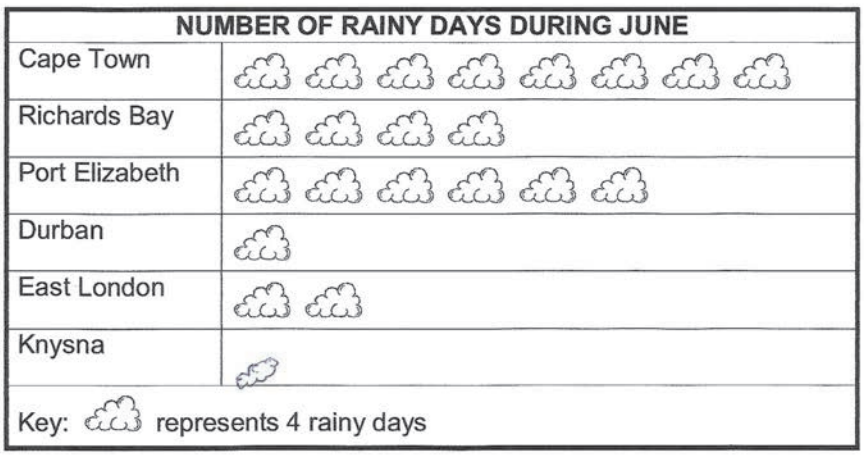
21 22 22 23 23 24 25 25 25

26

Item 30. 1 and 30.2

- If the learner did not answer the question.
- If a learner drew a number of clouds other than 4 (full understanding) or 12 (partial understanding) next to Knysna: this shows that the learner did not understand the question.

30. Examine the pictograph below and then answer the questions that follow.



What do the item statistics tell us?

Item 5

56% of learners answered the questions correctly.

Item 27.1

68% of learners answered the questions correctly.

Item 27.2

21% of learners answered the questions correctly.

Item 28

56% of learners answered the questions correctly.

Item 30.1

37% of learners answered the questions correctly.

Item 30.2

35% of learners answered the questions correctly.

Factors contributing to the difficulty of the items

- Learners may have difficulty interpreting the scale and the labelling on the y-axis (Item 27.1).
- Language barriers make it difficult for learners to interpret the questions correctly.
- Learners may not understand that they had to subtract to get the difference in time (Item 27.2).
- Learners may not remember the meaning of mode or median or may confuse the terms (Item 28).
- Learners may not understand that the key had to be considered in the pictogram (Item 30).

Teaching strategies

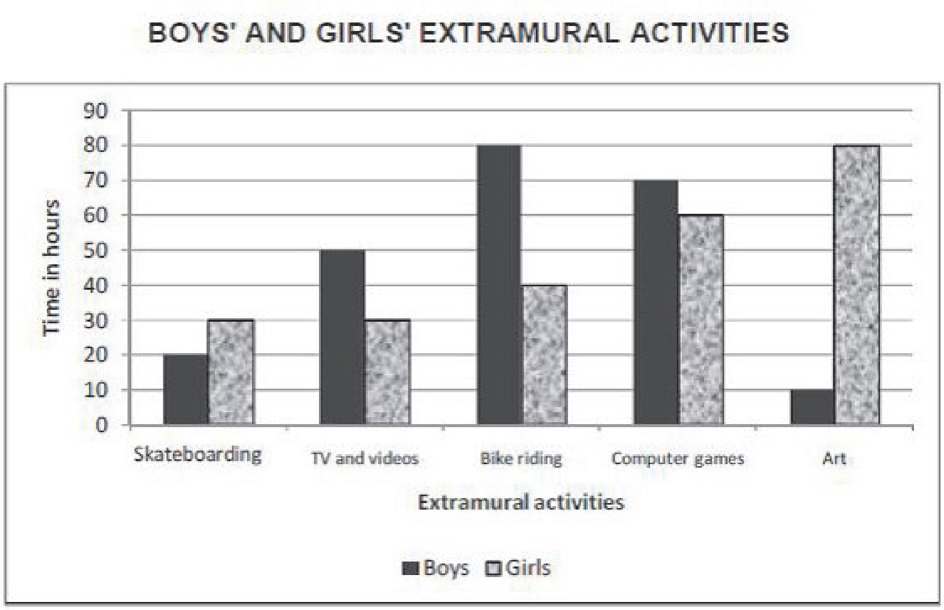
Understanding bar graphs

Learners must be able to interpret the markings on axes (or number lines) of graphs to take readings. We always label the axes so that we know what they mean.

- The horizontal axis is used to show the different items that we are measuring (counting).
- The vertical axis shows us how many of the different items have been counted.
- The "heading" or "title" of the graph tells us what the graph is about.

Example

- Show learners the following example and ask questions such as:
- What is this bar graph about?
- What is on the horizontal axis?
- What is the meaning of the markings on the vertical axis?
- What does the key below the graph tell us?



Discuss the answers:

- This bar graph is about extramural activities of boys and girls.
- The horizontal axis shows us the different kinds of extramural activities the boys and girls did.
- The vertical axis markings are for time. This axis gives us an indication of how much the boys and

girls spent on each activity.

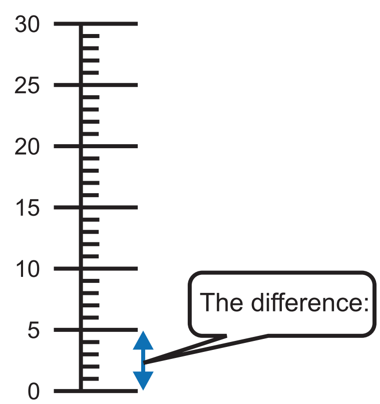
- The key tells us the colours of the bars and so it enables us to see that the darker bar is telling us about the boys and the lighter bar is telling us about the girls.

Understanding scales on the vertical axes

Learners have to be able to interpret the markings of scales. We call the markings calibrations.

- You can use the following activity to develop knowledge about scale markings on the axes.

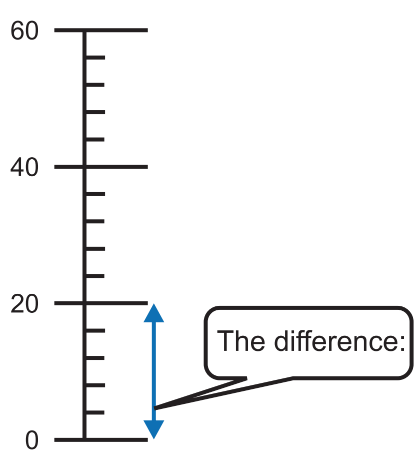
- Ask learners: Can you tell me what every small marking on this scale means?
  - Discuss the answers using the method given below.
  - The final step in the method indicates that the little markers each indicate a value of 1 unit.



Method:
① Look at any two labelled marks on the scale.
② Find the **difference** between the two markings In this example it is **5**.
③ Count the number of spaces between the little marks between the labelled markings. There are 5 spaces between 0 and 5.
④ To find out what each little mark means, calculate: $\frac{\text{the difference}}{\text{number of spaces}} = \frac{5}{5} = 1$.
⑤ Each little mark is one unit.

Example

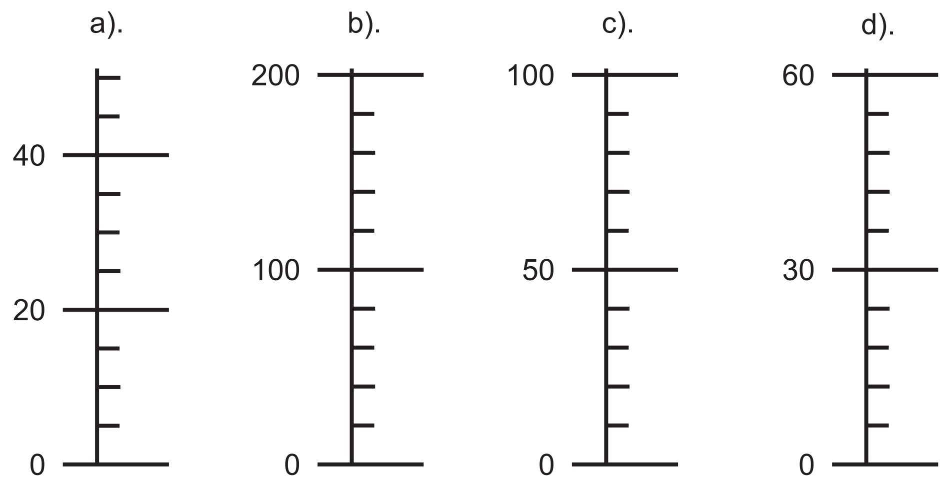
How will you read the following scale? Follow the method given.



Method:
① The **difference** between the two markings is 20.
② There are 5 spaces between 0 and 20.
③ To find out what each little mark means, calculate: $\frac{\text{the difference}}{\text{number of spaces}} = \frac{20}{5} = 4$.
④ Each little mark is 4 units.

- The final step in the method indicates that the little markers each indicate a value of 4 units.

Activity: Find the value of each little mark on the following scales



Solutions

a). $\frac{\textit{the difference}}{\textit{the number of spaces}} = \frac{20}{4} = 5$ Each little mark is 5 units

b). $\frac{\textit{the difference}}{\textit{the number of spaces}} = \frac{100}{5} = 20$ Each little mark is 20 units

c). $\frac{\textit{the difference}}{\textit{the number of spaces}} = \frac{50}{5} = 10$ Each little mark is 10 units

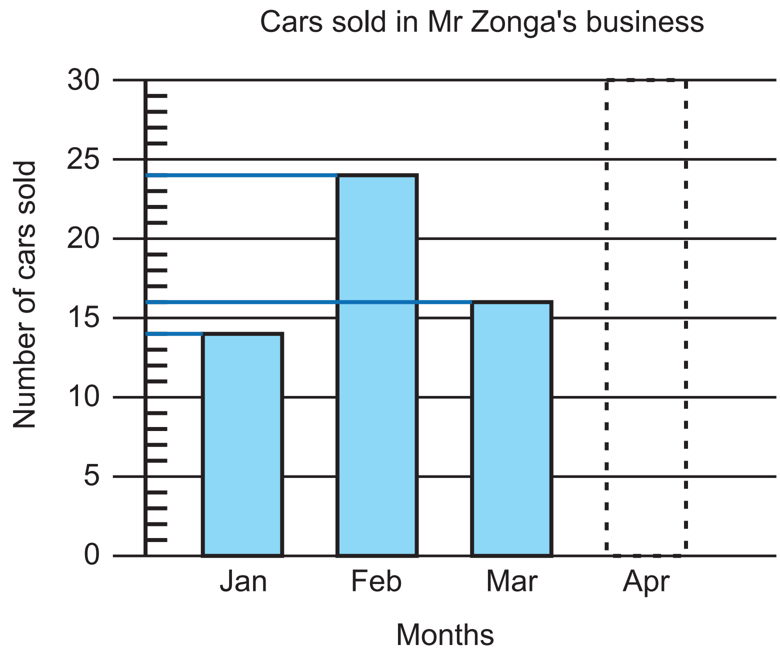
d). $\frac{\textit{the difference}}{\textit{the number of spaces}} = \frac{30}{5} = 6$ Each little mark is 6 units

Analysing and interpreting bar graphs

- The best way to develop learners' skills in analysing and interpreting graphs is for them to work through lots of examples.
- Try to find examples in the newspaper of graphs about current topics and show them to your class so that the learners can see that they are learning a skill that helps them to read information in the popular media correctly.

Example

- The bar graph below shows the number of cars that Mr Zonga sold in the first three months of 2015.



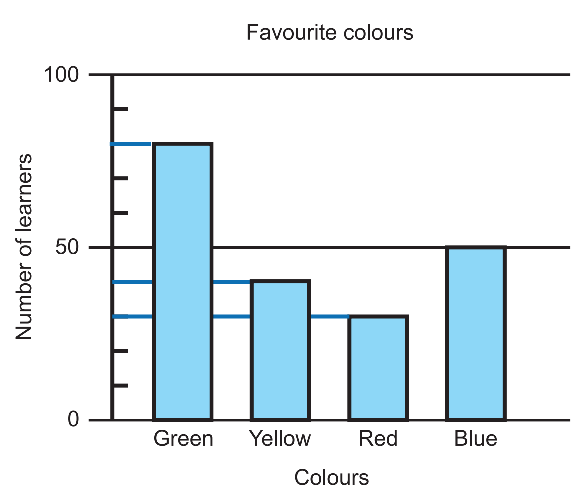
- Read the information from the graph.
- Discuss the total number of cars sold in the first three months: Month 1: 14; Month 2: 24; and Month 3: 16 cars.
- From what we can see the most cars were sold in month 2 and the least cars were sold in month 1.
- If Mr Zonga sold a total of 84 cars in four months. Draw the bar graph to show how many cars were sold in April. (The dotted bar above is the bar needed for April if Mr Zonga sold a total of 84 cars in four months.)

Activity: Graph interpretation

- A group of 200 learners were asked to name their favourite colours. The bar graph below shows their choices.

a). Read the number of learners that named each colour off the graph. Summarise this information in a table.

b). Which colour was chosen by 25% of the learners?



Solution

a). The favourite colours of the learners are summarised on the graph as follows:

Green	80
Yellow	40
Red	30
Blue	50
Total	200

b). $25\% = \frac{25}{100} = \frac{1}{4}$. 25% of $200 = \frac{1}{4}$ of $200 = \frac{50}{200}$.

There are 50 learners who said their favourite colour was blue.

The colour blue was chosen by 25% of the learners.

Interpreting keys in pictograms

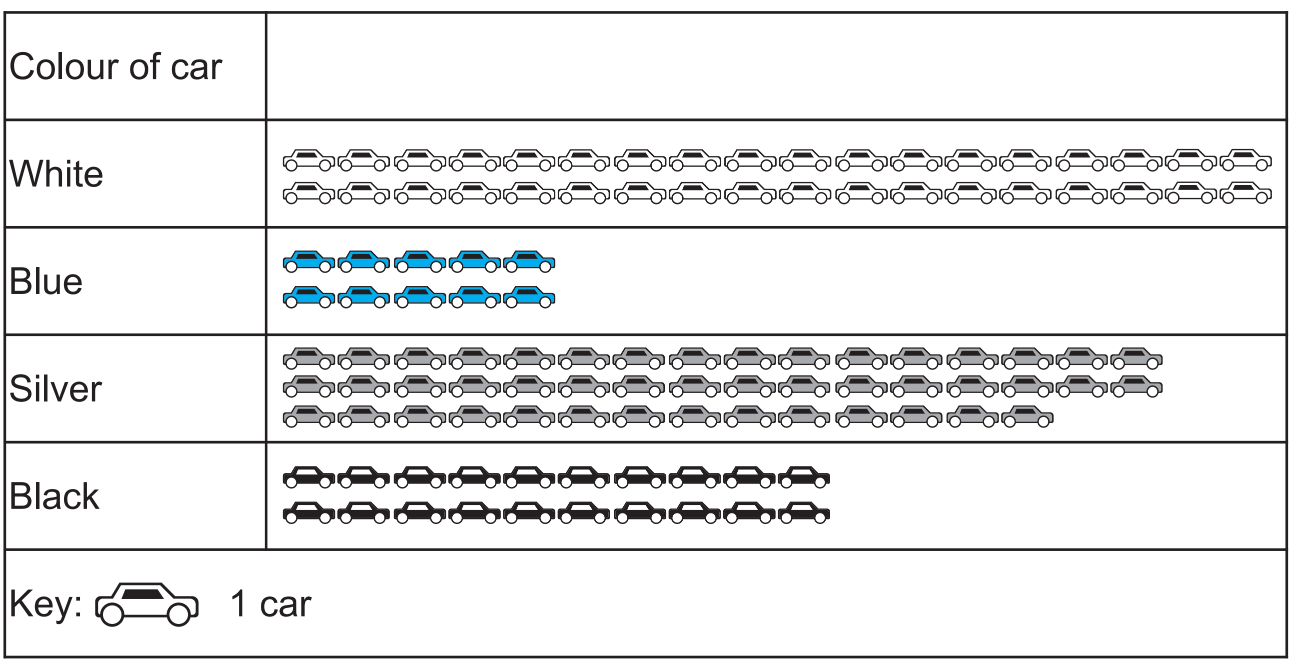
- A pictogram tells us a "story". The key that we choose must relate to the data we collected.

Example:

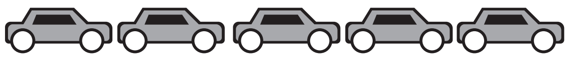
- Ask learners: What is the value of using a key in a pictograph? Show learners the following pictographs drawn by two learners in a Grade 6 class.

> • Jack and Jill counted the colours of cars in the parking garage together.
> • Each of them drew their own pictographs with the information.

- This is Jack's drawing



- This is Jill's drawing

Colour of car	
White	
Blue	
Silver	
Black	
Key:  10 cars	

- Ask the learners:
  - What is different about these two pictographs?
- Discuss:
  - Jack used one car to show one car on his pictograph and so there are lots of small cars on his graph. The cars can each be counted to find out how many cars of each colour he counted.
  - Jill used one car to show ten cars on her pictograph and so there are fewer small cars on her graph. To find out how many cars of each colour she counted we must remember to multiply by ten for each drawing of a car on her graph.

- Which one of the pictographs is easier to read? Can you explain why?
- Discuss:
  - Jack's graph is a bit cluttered with so many little cars, but we can just count each car to find out how many cars of each colour he counted. This is quite easy.
  - Jill's graph is less cluttered with little cars, but we have to remember to use her key (1 car = 10 cars) and multiply by 10 to find out how many cars of each colour Jill counted. We might forget to do this, but the graph looks clearer and is easier to read.
  - Learners might make other observations. Remember to listen to all of them and let learners add to the richness of the discussion.

Definition of terms

Mode

This is the number that appears the most often in a set of numbers.

Example

Consider the heights of 6 learners in a class:

1,54 m; 1,55 m; 1,56 m; 1,56 m; 1,62 m; 1,63 m

What is the mode of the height of the learners?

- The number 1,56 appears twice and all the other numbers just once.
- Thus the mode is 1,56

Median

This is the number that appears in the middle of a set of numbers that has been arranged from smallest to biggest (or from biggest to smallest). The median is thus a value that divides the data set into 2 equal parts.

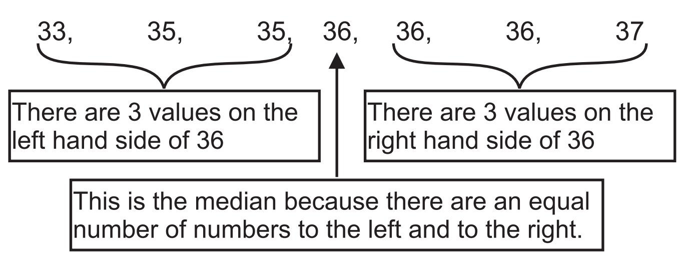
Example

Consider the following numbers of learners in 7 Grade 6 classes at a certain school:

33; 35; 35; 36; 36; 36; 37

What is the median number of learners in a class?

- If we want to find the median we must make sure that the set of data (numbers or values) we are given is arranged from smallest to biggest.
- The numbers in this example are already arranged in order from smallest to biggest (lowest to highest).
- The number that occurs at the centre of all the numbers is 36 as shown in the illustration.



Activity: Finding the mode and the median

Here are some exercises to give your learners to allow them to practice finding the mode and median.

1). Find the median of the following data set which shows the number of seeds in a peapod.

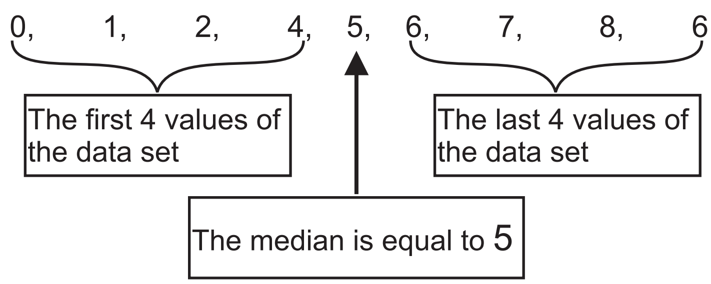
2, 6, 8, 1, 7, 8, 5, 4, 0

Solution

We can see that the data is not written from lowest value to highest value, so we need to arrange the data first, starting with the lowest value.

0, 1, 2, 4, 5, 6, 7, 8, 8

- There are 9 values in the data set.
- The median is the middlemost value.
- If we divide the number of values into 2 equal parts from either end we see that 5 is the middle most value.



2). For each of the following sets of numbers determine the median and mode.

Arrange the numbers from smallest to biggest to help you find the solutions.

a). 1; 2; 3; 4; 5
b). 21; 22; 23; 24; 25; 26; 27
c). 2; 4; 5; 7; 19; 20; 23
d). 100; 34; 267; 91; 52
e). 6; 4; 8; 7; 2; 9; 3; 7; 9; 23; 9
f). 15; 15; 16; 16; 14; 14; 14; 30; 23; 14; 33
g). 6; 0; 1; 4; 2; 5; 5; 7; 8; 9; 4; 1; 1

Solutions

a). Median: 3 Mode: there is no mode
b). Median: 24 Mode: there is no mode
c). Median: 7 Mode: there is no mode
d). Median: 91 Mode: there is no mode
e). Median: 7 Mode: 9
f). Median: 15 Mode: 14
g). Median: 4 Mode: 1

Understanding measures of central tendency, median and mode

- Learners should know what the purpose of calculating the median and mode is when we want to interpret data.
- The following examples show us data that was collected. Read the information carefully. We have to decide which of the two measurements, mode or median, will be the better to use for interpreting the data.
- Write the examples on the board and work through the solutions interactively each time, so that you are teaching about mean, median and mode at the same time and helping learners to think about which is the best one to find and use for interpreting different kinds of data.

Examples

1). Mrs Naidoo sold the following shoe sizes at school today:

4; 5; 5; 5; 5; 5; 7; 8; 8; 9; 9; 10; 12

a). What is the median of the shoe sizes?
b). What is the mode of the shoe sizes?
c). Which of these two measures will Mrs Naidoo use to buy more shoes for her clients? Explain your answer.

Solution

We can calculate the median and mode of the shoe sizes first and then think about whether the median or the mode is best to describe the data:

a). Median: 7
b). Mode: 5
c). Mrs Naidoo will look at the mode which indicates the shoes that are more in demand to order more shoes for her store. The mode tells us which shoe size she sold the most of.

2). Learners in Grade 5 were weighed by the nursing sister who visited the school.
She wrote down the learners' masses from smallest to biggest:

23; 23; 25; 27; 29; 30; 33; 34; 40; 46; 50; 72; 72; 72; 80

a). What is the median?
b). What is the mode?
c). Which of these two measures will the nurse use to describe the average mass of the learners in the class? Explain your answer.

Solution

We can calculate the median and mode of the learners' masses first and then think about whether the median or the mode is best to describe the data:

a). Median: 34

b). Mode: 72

c). There are three learners who weigh 72 kg, but the median (which is 34 kg) is a better description of the average mass of the learners. The median tells us the average.

Understanding language associated with the word "time"

The questions in item 27 dealt with the concept of time.

- This activity is included to help learners with reading questions that involve time.
- What questions can we ask learners to understand the different concepts involving time?
- How do we use language to speak about time?

Examples

"How much time?"

Ask: How much time did you spend on coming to school?

Learners should realise that the answer here has to do with time as the amount of hours, minutes or days, etc. For example, "I spent 15 minutes walking to school".

"Most of the time."

Ask: On which activity do you spend most of your time each day, walking, running, standing, sitting, or lying?

Learners should realise that they have to compare the times they spend on the different activities and answer, for example, "I spend most of my time sitting".

"How much more time?"

Ask: Jim walks 500 m to school and it takes him 20 minutes. Bonga walks 800 m to school and it takes him 40 minutes. How much more time does Bonga spend walking to school?

Learners have to understand that the question is about time. The other information about distance is not relevant to the question. Learners should answer: Bonga spends 20 minutes more than Jim does walking to school.

"What is the time?"

Ask: What is the time? At what time did you come to school?

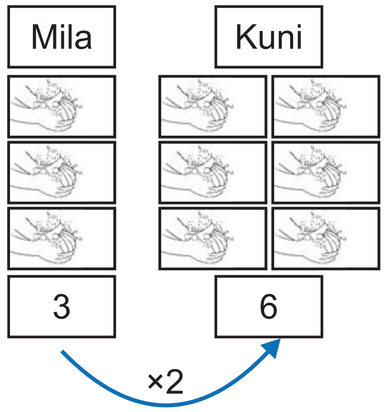
Learners should know that this question relates to the time that we read from a clock.

"How many times more?"

This statement does not relate to time as in hours or minutes.

Ask: Mila washed her hands 3 times and Kuni washed his hands 6 times. How many more times did Kuni wash his hands than Mila?

Learners have to be able to say: Kuni washed his hands twice as many times as Mila.



Other examples of how to test data handling

ANA 2014 Grade 6 Mathematics Item 1.9

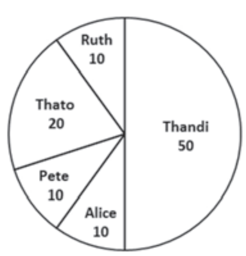
1.9 What is the median of the given set of numbers?

3 5 4 4 5 6 9 8 4

A 8
B 5
C 3
D 4 [1]

ANA 2014 Grade 6 Mathematics Items 26.1 and 26.2

26 This pie chart shows how 100 marbles were shared amongst a group of children.



26.1 Who has the same number of marbles?

____ [1]

26.2 Who received 20 marbles? ____________________ [1]

ANA 2014 Grade 6 Mathematics Item 27

27	What is the mode of the given set of test marks?									
	6	7	5	3	7	9	5	8	7	___________
										[1]

Notes:

ISBN: 978-1-4315-2128-9

Celebrating our achievements: consolidating our advances

Department of Basic Education
222 Struben Street, Pretoria, 0001
Private Bag X895, Pretoria, 0001, South Africa
Tel: (012) 357 3000 • Fax: (012) 323 0601

Private Bag X9035, Cape Town, 8000, South Africa
Tel: (021) 486 7000 • Fax: (021) 461 8110

Hotline: 0800 202 933

website
www.education.gov.za

facebook
www.facebook.com/BasicEd

twitter
www.twitter.com/dbe_sa